Letter to the Editor

Radiation-pressure instability is an artifact of constant-α closure

Implications for AGN disk tensions

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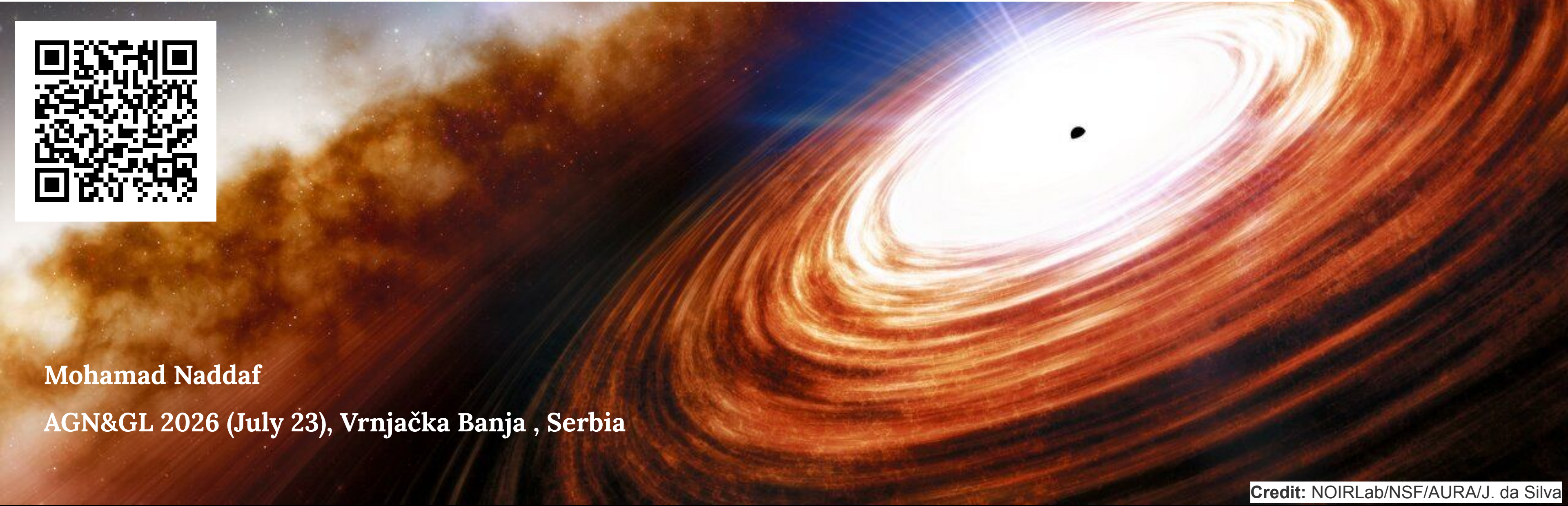

Mohamad Naddaf

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Credit: NOIRLab/NSF/AURA/J. da Silva

Standard assumption

Thin-disk equations
(conservation laws + closure)

$$\dot{M} = 3\pi\nu\Sigma$$

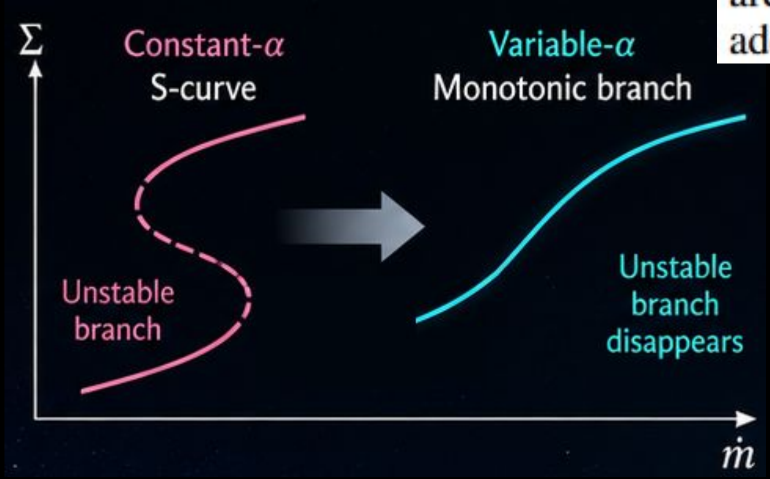
$$\nu = \alpha\frac{c_s^2}{\Omega_K}$$

$$Q^+ = \frac{9}{8}\nu\Sigma\Omega_K^2$$

$$Q^- = Q^-_{\rm rad}(\Sigma, T)$$

α is taken as constant

Proposed remedies include gas- or mixed-pressure scaling in the stress law (Sakimoto & Coroniti 1981; Stella & Rosner 1984; Merloni & Nayakshin 2006); introducing magnetically elevated or supported disks that alter the vertical structure and pressure balance (Begelman & Pringle 2007; Zheng et al. 2011; Sadowski 2016); incorporating wind-driven mass-loss that changes the local $\dot{M}$ and Σ structure (Poutanen et al. 2007; Laor & Davis 2014; Habibi & Abbassi 2019); or adopting slim-disk solutions in which advective cooling becomes important (Abramowicz et al. 1988; Chen et al. 1995). These approaches are valuable, but they generally modify the stress law, invoke additional physics, or leave the standard TD branch.



Common blame: radiation pressure

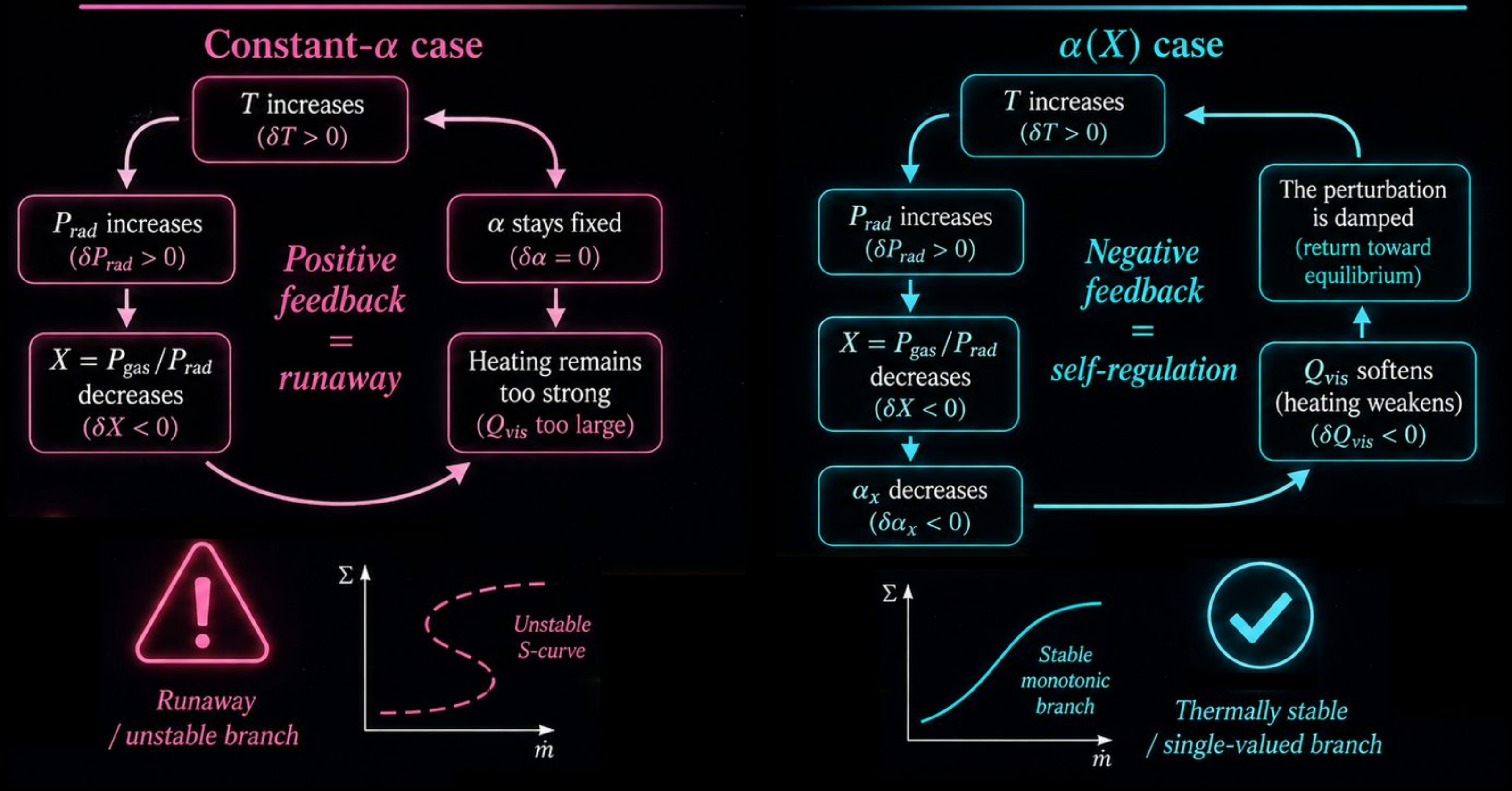
- "High P_{rad} causes the instability"
- Remove or weaken P_{rad} $\Rightarrow$ stability

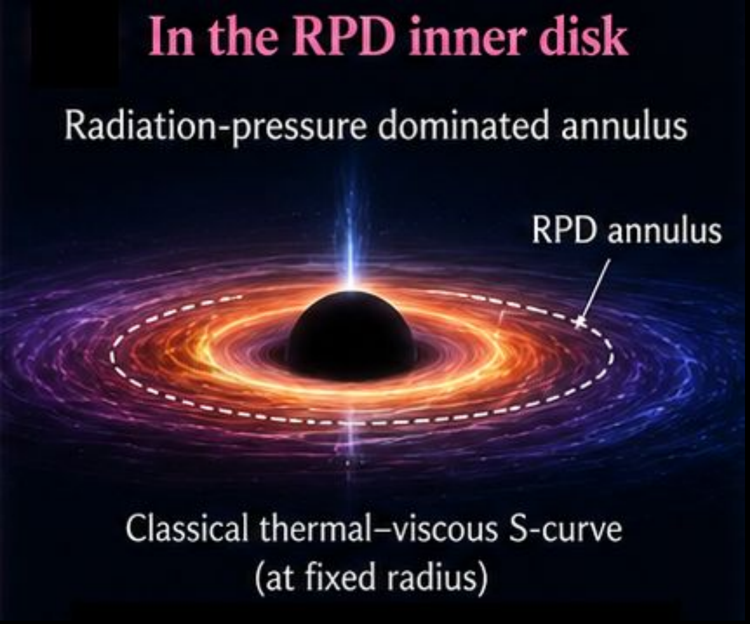
Our claim: imposing the same constant α across distinct thermodynamic regimes

- The stress response changes between gas- and radiation-pressure dominated states
- Forcing a single constant α creates an artificial S-curve

The issue is closure, not the existence of radiation pressure.

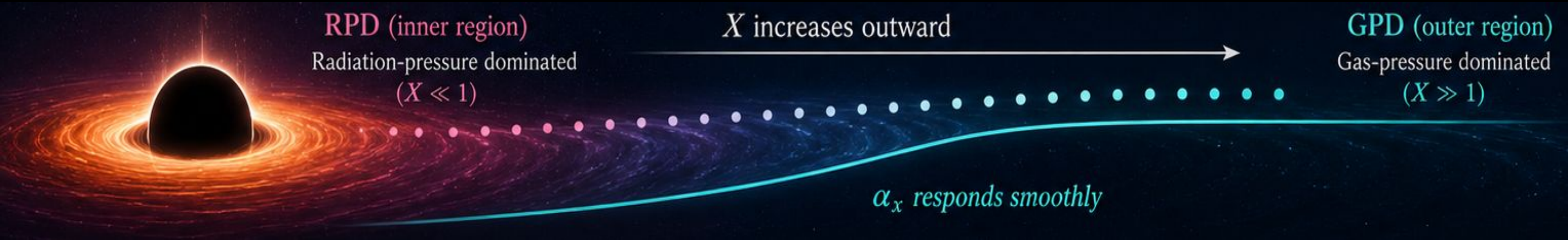
runaway or self-regulation?





- Same thin-disk framework
- Same radiative cooling
- Different stress response

$$\alpha_x \equiv \alpha(X), \quad X \equiv \frac{P_{\rm gas}}{P_{\rm rad}}$$



A.2. Thermal stability analysis with α_x

For an infinitesimal temperature perturbation while Σ is taken to be constant, thermal stability requires

$$(\partial [Q_{vis} - Q_{rad} - Q_{adv}] / \partial T)_\Sigma < 0, \tag{A.14}$$

or, since $T > 0$, simply

$$\delta Q_{vis} - \delta Q_{rad} - \delta Q_{adv} < 0. \tag{A.15}$$

Defining

$$f_a \equiv Q_{adv}/Q_{vis}, \quad f_r \equiv Q_{rad}/Q_{vis}, \quad f_r + f_a = 1, \tag{A.16}$$

Then

$$Q_{vis} [\delta \ln Q_{vis} - f_r \delta \ln Q_{rad} - f_a \delta \ln Q_{adv}] < 0 \tag{A.17}$$

Since $Q_{vis} > 0$, the sign is set only by the bracket.

The logarithmic responses of viscous heating, diffusive radiative cooling, and advective cooling are

$$\begin{aligned} \delta \ln Q_{vis} &= \delta \ln \alpha + \delta \ln P_{tot} + \delta \ln H, \\ \delta \ln Q_{rad} &= 4 \delta \ln T, \\ \delta \ln Q_{adv} &= \delta \ln \dot{M} + 2 \delta \ln H. \end{aligned} \tag{A.18}$$

In order to proceed, we define

$$\beta_r \equiv P_{rad}/P_{tot}, \quad \beta_g \equiv P_{gas}/P_{tot}, \quad \beta_r + \beta_g = 1. \tag{A.19}$$

Therefore, at a given radius for a fixed Σ,

$$\begin{aligned} \delta \ln P_{tot} &= \delta \ln H, \\ \delta \ln P_{tot} &= \beta_r \delta \ln P_{rad} + \beta_g \delta \ln P_{gas}, \\ \delta \ln P_{gas} &= \delta \ln T - \delta \ln H, \\ \delta \ln P_{rad} &= 4 \delta \ln T, \end{aligned} \tag{A.20}$$

$$\delta \ln H = A \, \delta \ln T / D, \tag{A.21}$$

where $A \equiv 4 - 3\beta_g$, and $D \equiv 1 + \beta_g$.

Using $X \equiv P_{gas}/P_{rad}$, we have

$$\delta \ln X = \delta \ln P_{gas} - \delta \ln P_{rad}, \tag{A.22}$$

which, combined with Eqs. A.2, A.20, and A.21, yields

$$\delta \ln \alpha = -7 \, \eta_x \, \delta \ln T / D \tag{A.23}$$

Using the angular-momentum equation

$$\begin{aligned} \delta \ln \dot{M} &= \delta \ln \alpha + \delta \ln P_{tot} + \delta \ln H, \\ \delta \ln \dot{M} &= (2A - 7\eta_x) \, \delta \ln T / D \end{aligned} \tag{A.24}$$

Now substituting into Eq. (A.18):

$$\begin{aligned} \delta \ln Q_{vis} &= (2A - 7\eta_x) \, \delta \ln T / D, \\ \delta \ln Q_{adv} &= (4A - 7\eta_x) \, \delta \ln T / D. \end{aligned} \tag{A.25}$$

Then

$$\left[\frac{2A - 7\eta_x}{D} - 4 f_r - f_a \frac{4A - 7\eta_x}{D} \right] \delta \ln T < 0. \tag{A.26}$$

For a positive temperature perturbation, $\delta \ln T > 0$, and as D is always positive, stability condition reads as

$$\Delta = 2A - 4D + 4 f_a (D - A) - 7(1 - f_a)\eta_x < 0, \tag{A.27}$$

Since the classical Lightman & Eardley (1974) instability is obtained in the radiatively cooled RPD branch, the critical bound is evaluated in the limit $\beta_g \to 0$ and $f_a \to 0$, thus

$$\Delta = 4 - 7\eta_x < 0, \tag{A.28}$$

$$\eta_x > 4/7 \approx 0.571. \tag{A.29}$$

A.3. Constraining α_x from the $\dot{M}$–Σ relation

We now specialize to the RPD, radiatively cooled regime,

$$P_{rad} \gg P_{gas}, \qquad Q_{adv} \ll Q_{rad}, \tag{A.30}$$

in which the classical thermal–viscous instability arises. We derive a general constraint on the functional form of the viscosity α_x from the structure of the $\dot{M}$–Σ equilibrium curve, without assuming a specific parametric form. At fixed radius, the unstable branch lies in this regime, allowing the following scalings.

From vertical hydrostatic equilibrium,

$$P_{tot} \propto \Sigma H, \tag{A.31}$$

and in the radiatively cooled RPD limit,

$$P_{tot} \approx P_{rad} \propto T^4, \tag{A.32}$$

which gives

$$H \propto T^4 \Sigma^{-1}. \tag{A.33}$$

The gas pressure scales as

$$P_{gas} \propto \rho T \propto \Sigma T H^{-1}, \tag{A.34}$$

and therefore

$$P_{gas} \propto \Sigma^2 T^{-3}. \tag{A.35}$$

Combining these expressions, we obtain

$$X \equiv P_{gas}/P_{rad} \propto \Sigma^2 T^{-7}. \tag{A.36}$$

The viscous heating rate scales as

$$Q_{vis} \propto \alpha P_{tot} H \propto \alpha T^8 \Sigma^{-1}, \tag{A.37}$$

while radiative cooling gives

$$Q_{rad} \propto T^4 \Sigma^{-1}. \tag{A.38}$$

Thermal equilibrium $Q_{vis} = Q_{rad}$ then implies αT^4 = const., hence, $T^4 \propto \alpha^{-1}$. Substituting into the expression for X yields

$$X \propto \Sigma^2 \alpha^{7/4}, \tag{A.39}$$

which determines $X(\Sigma)$ along the equilibrium branch

Taking the logarithm of the constraint equation,

$$\ln X - \frac{7}{4} \ln \alpha_x = 2 \ln \Sigma + \text{const}, \tag{A.40}$$

and differentiating with respect to $\ln \Sigma$, we obtain

$$d \ln X / d \ln \Sigma = 8/(4 - 7\eta_x). \tag{A.41}$$

From angular momentum transport,

$$\dot{M} \propto \alpha P_{tot} H \propto \alpha T^8 \Sigma^{-1}. \tag{A.42}$$

Using the thermal equilibrium condition, we obtain

$$\dot{M} \propto \alpha_x^{-1} \Sigma^{-1}. \tag{A.43}$$

Thus, the full $\dot{M}(\Sigma)$ relation is determined by the implicit dependence $X(\Sigma)$.

Differentiating $\ln \dot{M}$ gives

$$d \ln \dot{M} / d \ln \Sigma = -1 - \eta_x \, d \ln X / d \ln \Sigma. \tag{A.44}$$

Substituting, we find

$$d \ln \dot{M} / d \ln \Sigma = (\eta_x + 4)/(7\eta_x - 4). \tag{A.45}$$

This equation gives the general slope of the $\dot{M}$–Σ equilibrium curve for an arbitrary α_x. The classical instability corresponds to the denominator changing sign, producing a negative-slope segment. Requiring $d \ln \dot{M} / d \ln \Sigma > 0$, yields a necessary condition for the absence of the unstable branch. For $\eta_x > 0$, this implies

$$\eta_x > 4/7 \approx 0.571. \tag{A.46}$$

$$\eta_X \equiv \frac{d \ln \alpha_x}{d \ln X} > 4/7$$

closure-level condition for a thermally stable, single-valued steady branch

Standard assumption

Thin-disk equations
(conservation laws + closure)

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$$Q^+ = \frac{9}{8}\nu\Sigma\Omega_K^2$$

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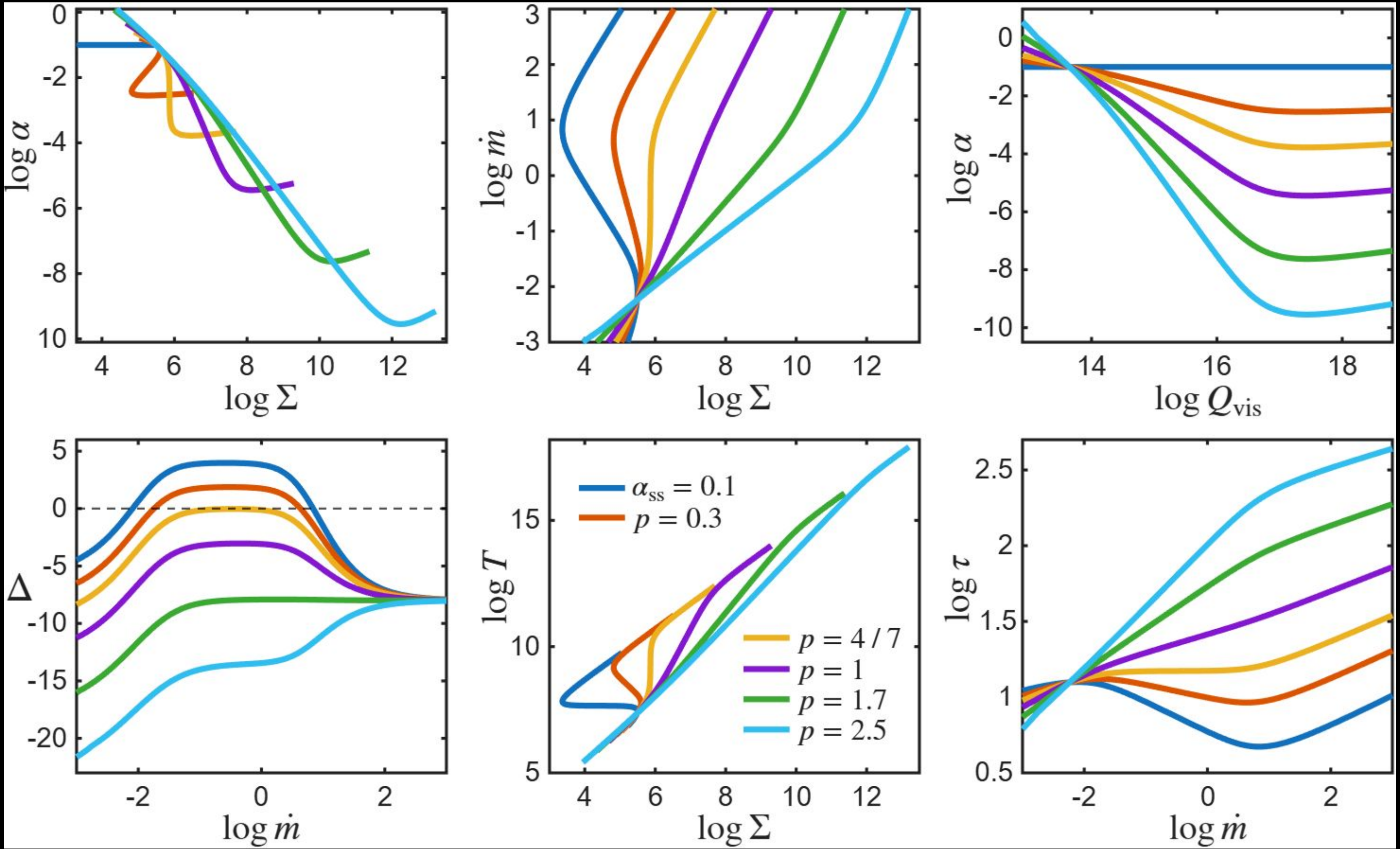
$$\eta_X \equiv \frac{d\ln\alpha_x}{d\ln X} > 4/7$$

A sufficient realization: $\alpha_x = \alpha_0 X^p$, with $p > 4/7$

! No new branch, no advection requirement, no magnetic support term.

We ask what stress response is required for the steady thin-disk branch to remain thermally stable and single-valued.



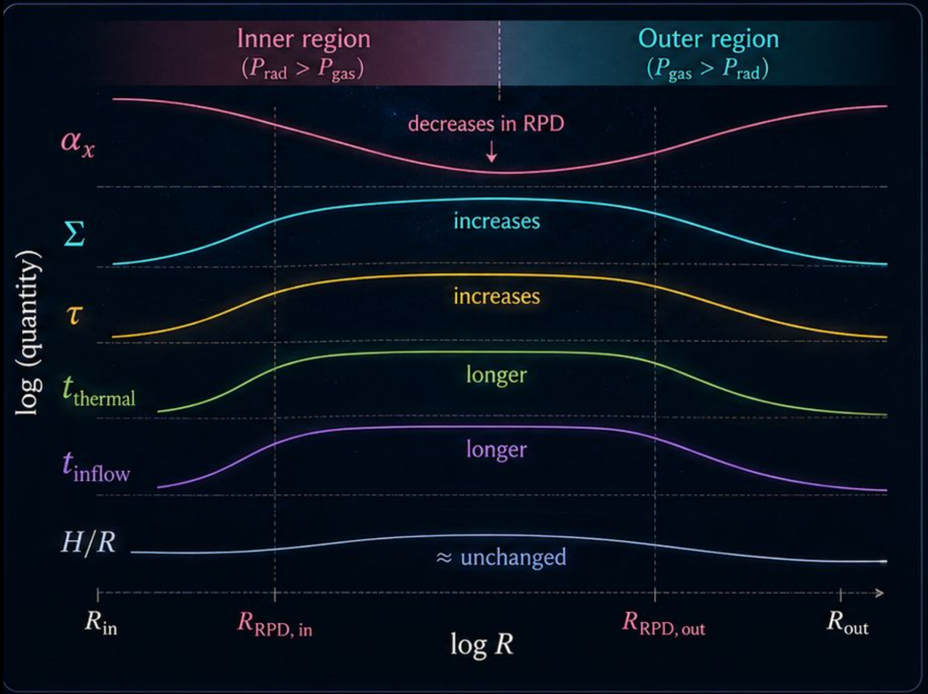
log α
log Σ
log ṁ
log Σ
log α
log Q_{vis}
Δ
log ṁ
log T
log Σ
$\alpha_{ss} = 0.1$
$p = 0.3$
$p = 4/7$
$p = 1$
$p = 1.7$
$p = 2.5$
log τ
log ṁ

$$\delta \log \Sigma = 4\,\delta \log T = -\delta \log \alpha \quad \text{(RPD limit)}$$

$$t_{\rm thermal} \sim t_{\rm inflow} \propto \alpha_x^{-1}$$

$H/R \approx$ unchanged to leading order

Radial structure across the radiation–pressure–dominaėd (RPD) region



Higher column	Larger τ	Slower inner flow	Still thin

Why this matters for AGNs

1 Variability

Longer thermal / inflow times, less need for violent radiation-pressure limit cycles.

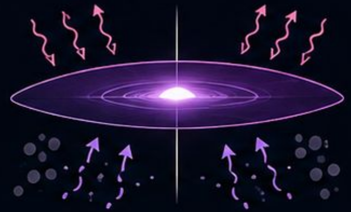
2 Disk-size tensions

Larger Σ and τ motivate revisiting microlensing sizes and continuum lags with improved radiative transfer.

3 Accretion states

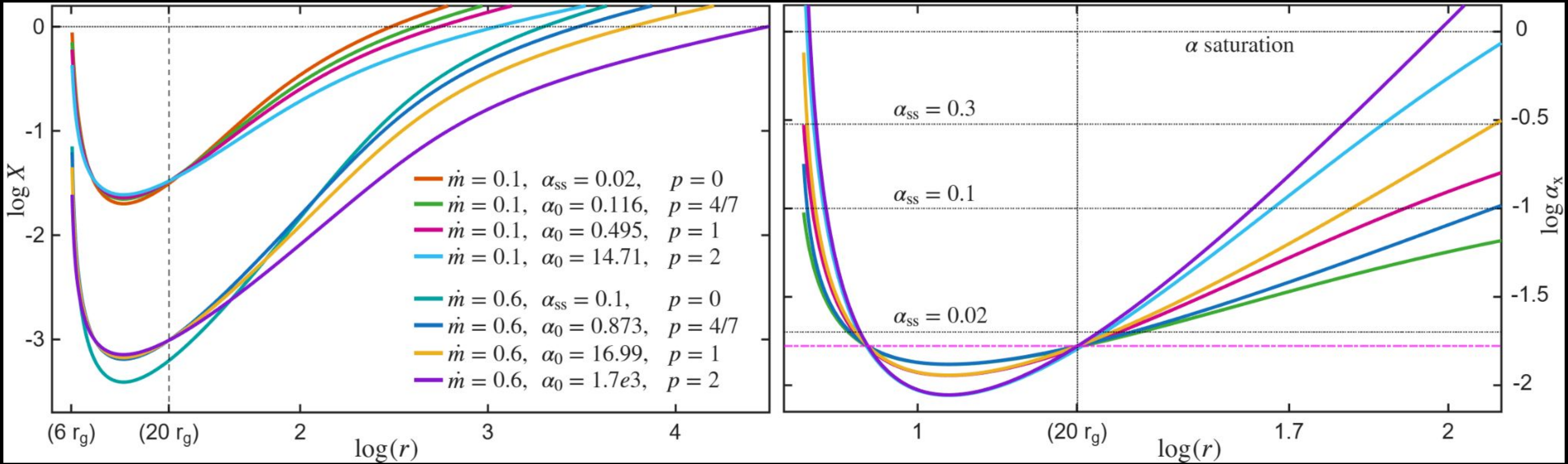
Moderate-to-high $\tilde{m}$ AGN disks may remain thin and optically thick without immediately becoming slim.

4 Geometry & BLR / winds



Smoother H/R affects irradiation, shielding, BLR illumination, and wind launching.

The closure does not remove radiation pressure; it changes the disk response to it.



log X
0
-1
-2
-3
(6 r_g)
(20 r_g)
2
3
4
log(r)
ṁ = 0.1, α_ss = 0.02, p = 0
ṁ = 0.1, α_0 = 0.116, p = 4/7
ṁ = 0.1, α_0 = 0.495, p = 1
ṁ = 0.1, α_0 = 14.71, p = 2
ṁ = 0.6, α_ss = 0.1, p = 0
ṁ = 0.6, α_0 = 0.873, p = 4/7
ṁ = 0.6, α_0 = 16.99, p = 1
ṁ = 0.6, α_0 = 1.7e3, p = 2
α saturation
α_ss = 0.3
α_ss = 0.1
α_ss = 0.02
log α_x
0
-0.5
-1
-1.5
-2
1
(20 r_g)
1.7
2
log(r)

In sufficiently ionized disks, angular momentum transport is expected to arise from MHD turbulence driven by magnetorotational instability (MRI), rather than from a universal hydrodynamic viscosity (Balbus & Hawley 1991; Hawley et al. 1995).

$$\eta_X \equiv \frac{d \ln \alpha_x}{d \ln X} > 4/7$$

closure-level condition for a thermally stable, single-valued steady branch

The MRI context sets both the motivation and the main calibration test of this approach. Since angular momentum transport in ionized disks is MRI-driven (Balbus & Hawley 1991; Hawley et al. 1995), α should be viewed as a saturated, vertically averaged effective stress, not a fundamental constant. Numerical studies often report MRI transport in terms of magnetic pressure, net magnetic flux, or plasma beta, with stress increasing as magnetization increases (e.g., Hawley et al. 1995; Salvesen et al. 2016). These scalings are not disregarded here; rather, in the present gas-plus-radiation TD branch they are absorbed into the reduced coefficient $\alpha_{\mathrm{eff}} = W_{R\phi}/(P_{\mathrm{gas}} + P_{\mathrm{rad}})$. The stability result then states that this total-pressure-normalized stress response must satisfy

$$\eta_X^{\mathrm{MRI}} \equiv \frac{d \ln \alpha_{\mathrm{eff}}}{d \ln X} > \frac{4}{7} \tag{3}$$

to remove the classical radiatively cooled RPD unstable branch. If magnetic pressure instead becomes part of the vertical support, $P_{\mathrm{mag}} \sim P_{\mathrm{gas}} + P_{\mathrm{rad}}$, the disk has moved to a magnetically supported branch, which is a complementary stabilization route (e.g., Sadowski 2016; Mishra et al. 2020; Scepi et al. 2024). Thus, the present result should be read as a closure-level stability condition for gas-plus-radiation TD disks, and as a direct calibration target for radiation-MHD simulations.

$$\eta_X^{\mathrm{MRI}}$$

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Thanks for your attention