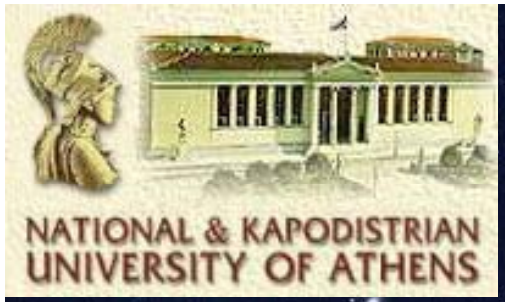

A new modelling approach for DACs and SACs regions in the atmospheres of hot emission stars

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The GR model

One of the main hypotheses when we constructed an old version of our model (rotation model), was that the line's width is only a rotational effect and we considered spherical symmetry for the independent density regions.

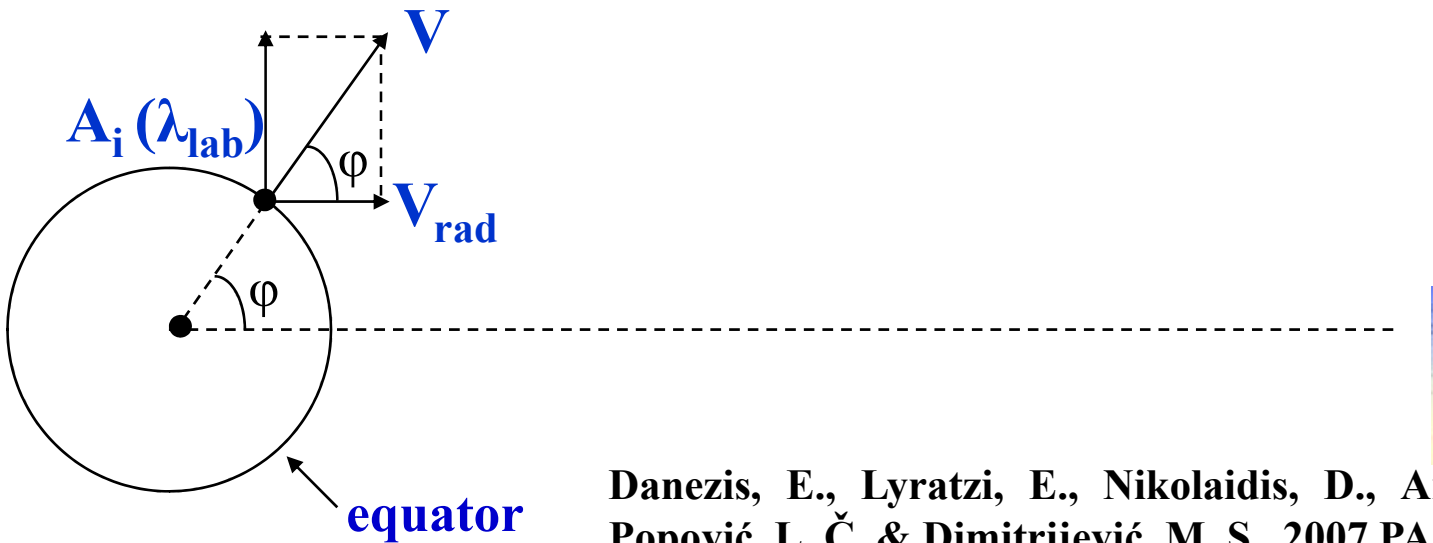
In a new approach of the problem we also consider the random velocities in the calculation of the distribution function L that we can detect in the line function.

$$I_\lambda = \left[I_{\lambda 0} \prod_i \exp\{-L_i \xi_i\} + \sum_j S_{\lambda ej} \left(1 - \exp\{-L_{ej} \xi_{ej}\}\right) \right] \prod_g \exp\{-L_g \xi_g\}$$

This new L is a synthesis of the rotational distribution Lr that we had presented in the old rotational model and a Gaussian that well defines the random velocities. This means that the new L has two limits, the first one gives us a Gaussian and the other the old rotation distribution Lr.

The new calculation of the distribution functions L

Let us consider a spherical shell and a point A_i in its equator. If the laboratory wavelength of a spectral line that arises from A_i is λ_{lab}, the observed wavelength will be $\lambda_0=\lambda_{lab}+\Delta\lambda_{rad}$



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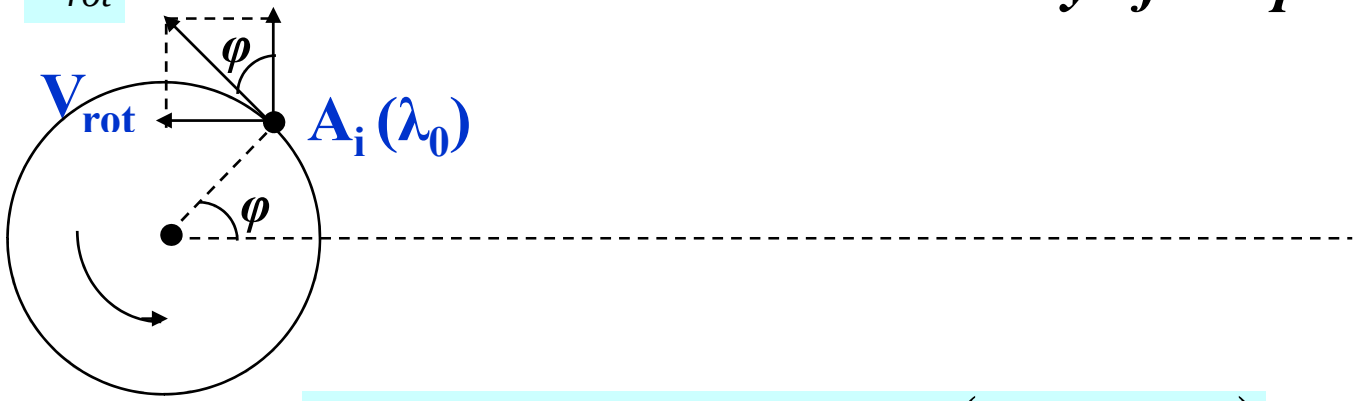
The calculation of the distribution functions L

If the spherical density region rotates, we will observe a displacement $\Delta\lambda_{rot}$ and the new wavelength of the center of the line λ_i

is: $\lambda_i = \lambda_0 \pm \Delta\lambda_{rot}$ ***where*** $\Delta\lambda_{rot} = \lambda_0 z \sin\varphi$

$$z = \frac{V_{rot}}{c} = \frac{\Delta\lambda_{rot}}{\lambda_0 \sin\varphi}$$

where V_{rot} ***is the observed rotational velocity of the point*** A_i,



This means that $\lambda_i = \lambda_0 \pm \lambda_0 z \sin\varphi = \lambda_0 (1 \pm z \sin\varphi)$

and if $-\frac{\pi}{2} < \varphi < \frac{\pi}{2}$ ***then*** $\lambda_i = \lambda_0 (1 - z \sin\varphi)$

If we consider that the spectral line profile is a Gaussian, then we have:

$$P(\lambda) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\left[\frac{\lambda-\kappa}{\sigma\sqrt{2}}\right]^2}$$

where κ is the mean value of the distribution and in the case of the line profile it indicates the center of the spectral line that arises from A_i.

***This means that*:**

$$P(\lambda) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\left[\frac{\lambda-\lambda_0(1-z\sin\varphi)}{\sigma\sqrt{2}}\right]^2} = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{[\lambda-\lambda_0(1-z\sin\varphi)]^2}{2\sigma^2}}$$

For all the semi-equator we have

$$L(\lambda)=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{[\lambda-\lambda_0(1-z\sin\varphi)]^2}{2\sigma^2}}\cos\varphi d\varphi.....(4)$$

If we make the transformation $\sin\varphi = x$

and $u=\dfrac{\lambda-\lambda_0(1-zx)}{\sigma\sqrt{2}}$

the above function (4) will be transformed and finally we have the function (5) :

$$L(\lambda) = \frac{1}{\lambda_0 z\sqrt{\pi}} \int_{\frac{\lambda-\lambda_0(1+z)}{\sigma\sqrt{2}}}^{\frac{\lambda-\lambda_0(1-z)}{\sigma\sqrt{2}}} e^{-u^2}\, du$$

$$L(\lambda) = \frac{1}{\lambda_0 z\sqrt{\pi}} \left[\int_{0}^{\frac{\lambda-\lambda_0(1-z)}{\sigma\sqrt{2}}} e^{-u^2}\, du - \int_{0}^{\frac{\lambda-\lambda_0(1+z)}{\sigma\sqrt{2}}} e^{-u^2}\, du \right] \ldots(5)$$

The above integrals have the form of a known integral erf(x)

$$erf(x)=\frac{2}{\pi}\int_0^x e^{-u^2}\,du$$

that has the following properties:

1. $erf(-x)=-erf(x)$
2. $erf(0)=0$
3. $erf(+\infty)=1$ **as** $\lim_{x\to+\infty} erf(+\infty)=1$
4. $erf(-1)=-1$
5. $$erf(x)=\frac{2}{\sqrt{\pi}}\left(x-\frac{x^3}{3\cdot 1!}+\frac{x^5}{5\cdot 2!}-\frac{x^7}{7\cdot 3!}+\ldots\right)$$
6. $$erf(x)=1-\frac{e^{-x^2}}{\sqrt{\pi}x}\left(1-\frac{1}{2x^2}+\frac{1\cdot 3}{(2x^2)^2}-\frac{1\cdot 3\cdot 5}{(2x^2)^3}+\ldots\right)$$

$$L(\lambda)=\frac{\sqrt{\pi}}{2\lambda_0 z}\left[erf\left(\frac{\lambda-\lambda_0(1-z)}{\sqrt{2}\sigma}\right)-erf\left(\frac{\lambda-\lambda_0(1+z)}{\sqrt{2}\sigma}\right)\right] \quad (6)$$

The distribution function from the semi-spherical region is:

$$L_{final}(\lambda)=\frac{\sqrt{\pi}}{2\lambda_0 z}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\left[erf\left(\frac{\lambda-\lambda_0}{\sqrt{2}\sigma}+\frac{\lambda_0 z}{\sqrt{2}\sigma}\cos\theta\right)-erf\left(\frac{\lambda-\lambda_0}{\sqrt{2}\sigma}-\frac{\lambda_0 z}{\sqrt{2}\sigma}\cos\theta\right)\right]\cos\theta d\theta \quad (7)$$

(Method Simpson)

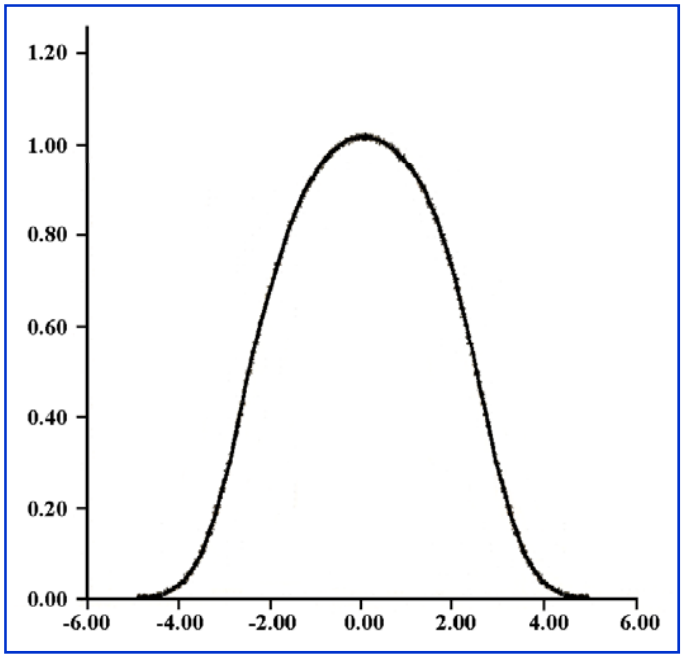
This $L_{final}(\lambda)$ is the distribution that replaces the old rotational distribution L that our group proposed some years ago (Danezis et al 2001).

Discussion

In the proposed distribution an important factor

is $m = \dfrac{\lambda_0 z}{\sqrt{2}\sigma}$

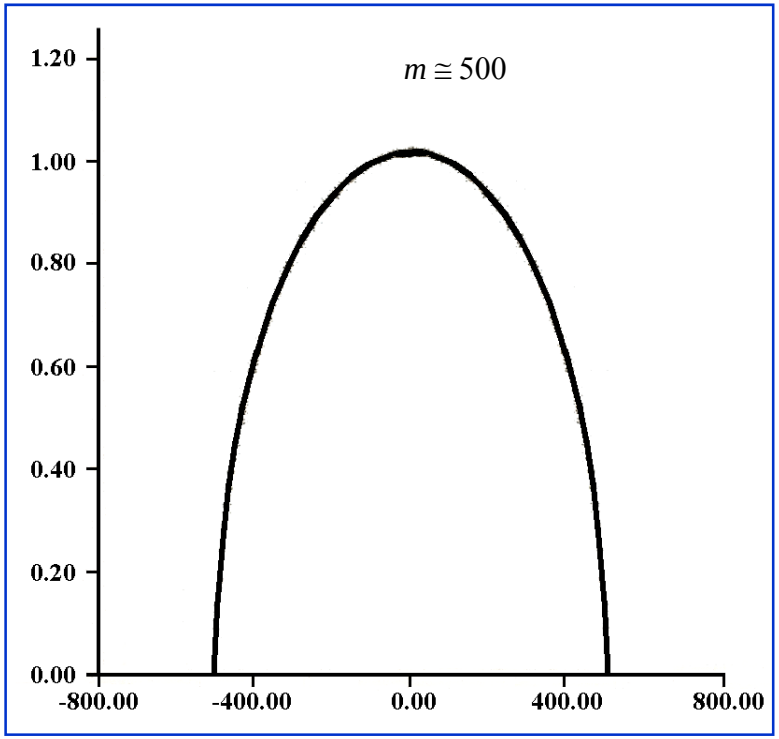
This factor indicates the kind of the distribution that fits the line profile.



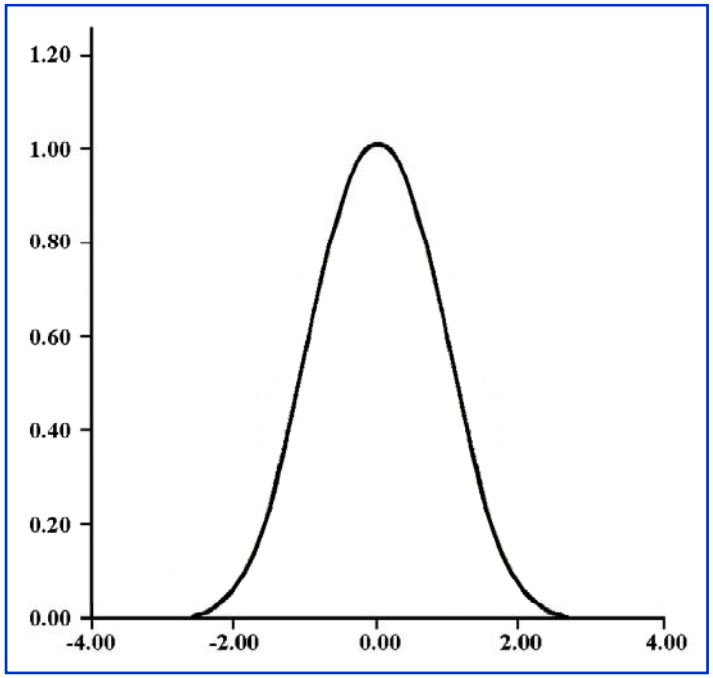
1. ***If*** $m \cong 3$ ***we have a mixed distribution. The line broadening is an effect of two equal reasons:***

a. The rotational velocity of the spherical region and
b. The random velocities of the ions.

2. If m ≈ 500 *the line broadening is only an effect of the rotational velocity* ***and the random velocities are very low.*** ***In this case*** *the profile of the line is the same with the profile that we can produce using the old rotation model* ***(Danezis 2001, 2003).***



3. Finally, if m<1 *the line broadening is only an effect of random velocities*
and the line distribution *is a Gaussian.*



The column density

An important point of our study is the calculation of the ***column density from our model.***

Lets start from the definition of the optical depth:

$$\tau = \int_0^s k\rho ds$$

where τ *is the optical depth (no units),*

k *is the absorption coefficient (* $\frac{cm^2}{gr}$ *),*

ρ *is the density of the absorbing region (* $\frac{gr}{cm^3}$ *),*

s *is the geometrical depth (cm)*

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In the model we set $k = L\Omega$ *, so* $\tau = \int_0^s L\Omega\rho ds$

where **L** *is the distribution function of the absorption coefficient* **k** *and has no units,*
Ω *equals 1 and has the units of* **k** *(* $\Omega = 1\frac{cm^2}{gr}$ *)*

We consider that for the moment of the observation and for a significant ion, **k** *is constant, so* **k** *(and thus* **L** *and* **Ω***) may come out of the integral. So:* $\tau = L\int_0^s \Omega\rho ds$

We set $\xi = \int_0^s \Omega\rho ds$

and **τ** *becomes*

$$\tau = L\xi$$

Absorption lines

For every one of ξ along the spectral line (henceforth called ξ_i)

we have that: $$\xi_i = \int_0^s \Omega \rho ds \Rightarrow \xi_i = \Omega \int_0^s \rho ds \Rightarrow \frac{\xi_i}{\Omega} = \int_0^s \rho ds$$

We set $$\sigma_i = \frac{\xi_i}{\Omega} = \int_0^s \rho ds$$

As $\Omega.(1\frac{cm^2}{gr})$ ***contributes only to the units, σ_i takes the value of ξ_i .***

For each of λ_i along the spectral line, we extract a σ_i from each ξ_i.
The program we use calculates the ξ_i for the centre of the line.
This means that from this ξ_i we can measure the respective σ_i .

If we add the values of all σ_i along the spectral line then we have

$$\sigma = \sum_i \sigma_i \qquad (\text{ in } \frac{gr}{cm^2}),$$

which is the surface density of the absorbing matter, which creates the spectral line.

If we divide σ with the atomic weight of the ion which creates the spectral line, we extract the number density of the absorbers, meaning the number of the absorbers per square centimetre

$$(\quad n = \frac{\sigma}{AW} \qquad (\text{in } \quad cm^{-2} \quad)).$$

This number density corresponds to the energy density which is absorbed by the whole matter which creates the observed spectral line ($\frac{E}{AW}$ *(* *in* $\frac{erg}{cm^2}$ *)) and which is calculated by the model.*

It is well known, that each absorber absorbs the specific amount of the energy needed for the transition which creates the specific line.
This means that if we divide the calculated energy density ($\frac{E}{AW}$ *) with the energy needed for the transition, we obtain the column density (in* cm^{-2} *).*

Emission lines

*In the case of the emission lines we have to take into account not only ξ_e, but also the source function **S**, as both of these parameters contribute to the height of the emission lines. So in this case we have:*

$$S\xi_e = \frac{j}{k}\int_0^s \Omega\rho_e ds$$

*where: **j** is the emission coefficient (* $\frac{erg}{gr \cdot s \cdot rad \cdot A}$ *),*

***k** is the absorption coefficient (* $\frac{cm^2}{gr}$ *)*

ρ_e *is the density of the emitting region (* $\frac{gr}{cm^3}$ *)*

***s** is the geometrical depth (cm)*

We set $k = L\Omega$

***where L** is the distribution function of the absorption coefficient **k** and has no units,*
***Ω** equals **1** and has the units of **k** (* $\Omega = 1\frac{cm^2}{gr}$ *)*

And $j = L_e\Omega_e$

***where L_e** is the distribution function of the emission coefficient **j** and has no units,*

Ω_e** equals **1** and has the units of **j $(\Omega_e = 1\frac{erg}{gr \cdot s \cdot rad \cdot A})$

As we did before, in the case of the absorption lines, we may consider that Ω may come out of the integral.

So: $$S\xi_e = \frac{j}{k}\int_0^s \Omega\rho_e ds = \frac{L_e\Omega_e}{L\Omega}\int_0^s \Omega\rho_e ds = \frac{L_e\Omega_e}{L\Omega}\Omega\int_0^s \rho_e ds = \frac{L_e\Omega_e}{L}\int_0^s \rho_e ds$$

As in the model we use the same distribution for the absorption and for the emission, $L_e = L$ **.**

So: $$S\xi_e = \Omega_e\int_0^s \rho_e ds \Rightarrow \frac{S\xi_e}{\Omega_e} = \int_0^s \rho_e ds$$

We set $$\sigma_e = \frac{S\xi_e}{\Omega_e} = \int_0^s \rho_e ds$$

As $$\Omega_e\left(=1\frac{erg}{gr\cdot s\cdot rad\cdot A}\right)$$ ***contributes only to the units,***

σ_e ***takes the value of*** $S\xi e$.

For each λ_i along the spectral line, we extract a σ_i from each $S\xi_e$.
The program we use calculates the ξ_e for the center of the line and the S. This means that from this ξ_e and S we can measure the respective σ_i.
If we add the values of all σ_i along the spectral line then we have

$$\sigma = \sum_i \sigma_i \quad \textit{(in} \quad \frac{gr}{cm^2} \quad \textit{),}$$

which is the surface density of the emitting matter, which creates the spectral line. If we divide σ with the atomic weight of the ion which creates the spectral line, we extract the number density of the emitters, meaning the number of the emitters per square centimetre

$$n = \frac{\sigma}{AW} \quad \textit{(in} \quad cm^{-2} \textit{).}$$

This number density corresponds to the energy density which is emitted by the whole matter which creates the observed spectral line

($\frac{E}{AW}$ *(in* $\frac{erg}{cm^2}$ *)) and which is calculated by the model.*

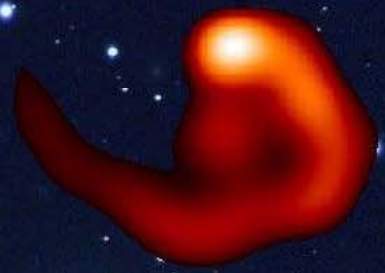
It is well known, that each emitter emits the specific amount of the energy needed for the transition which creates the specific line.

This means that if we divide the calculated energy density($\frac{E}{AW}$ *)*

with the energy needed for the transition, we obtain the column

density (in cm^{-2}*).*

The next presentation is about some important remarks and applications of GR model



Thank you very much for your attention