Towards a probabilistic approach for DAC and SAC exact reconstruction in hot emission stars

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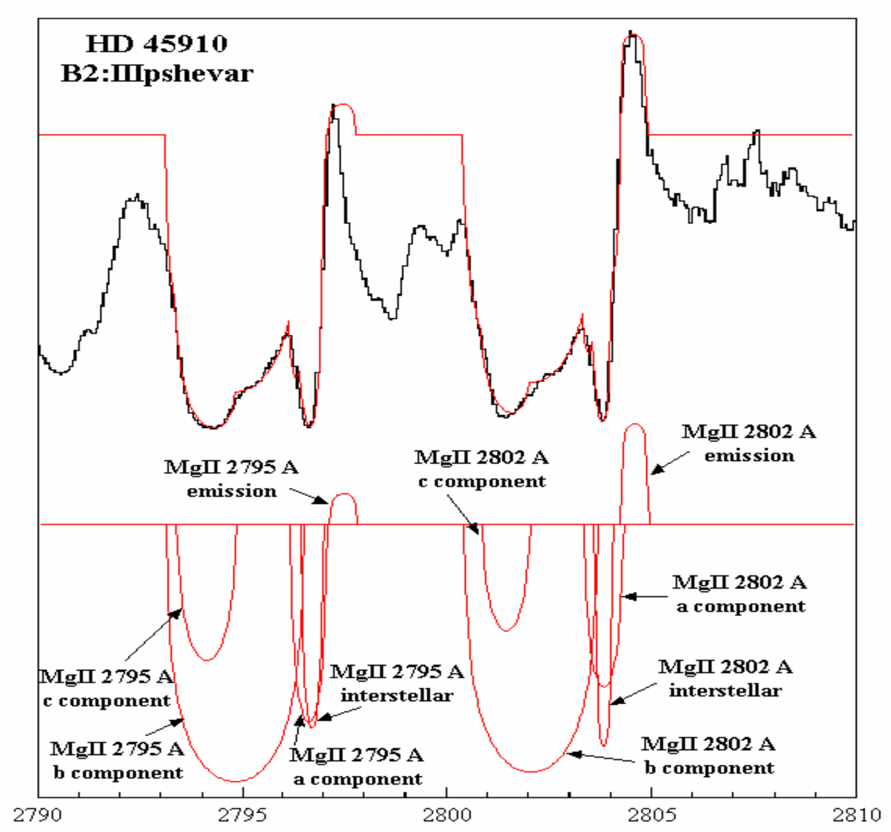
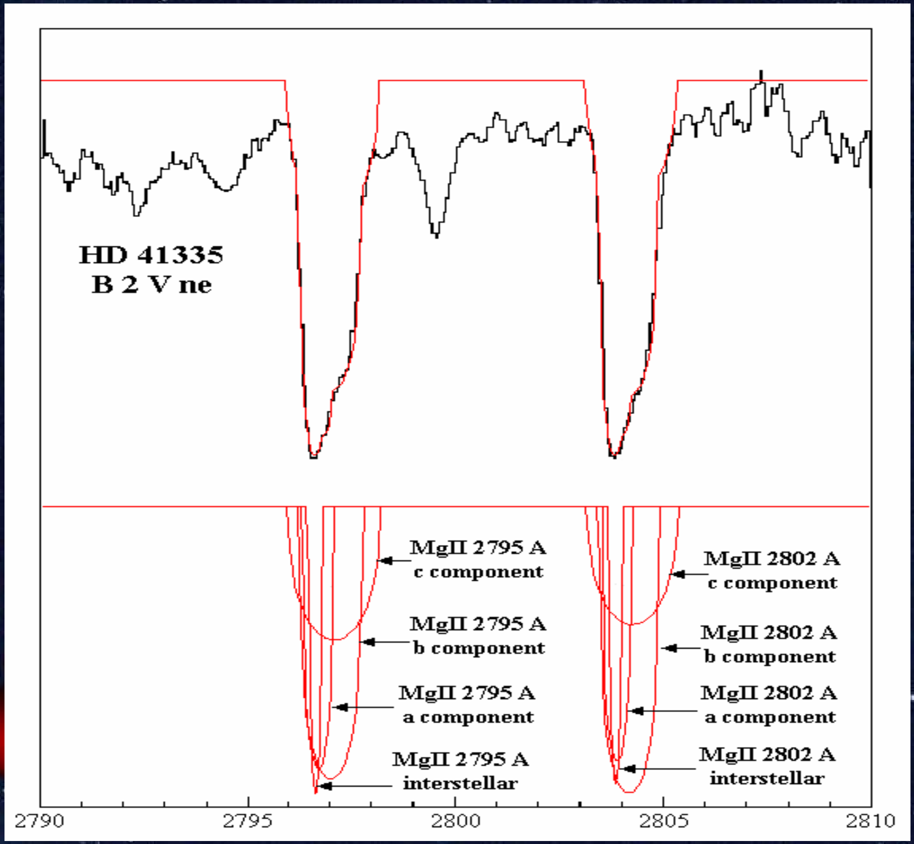
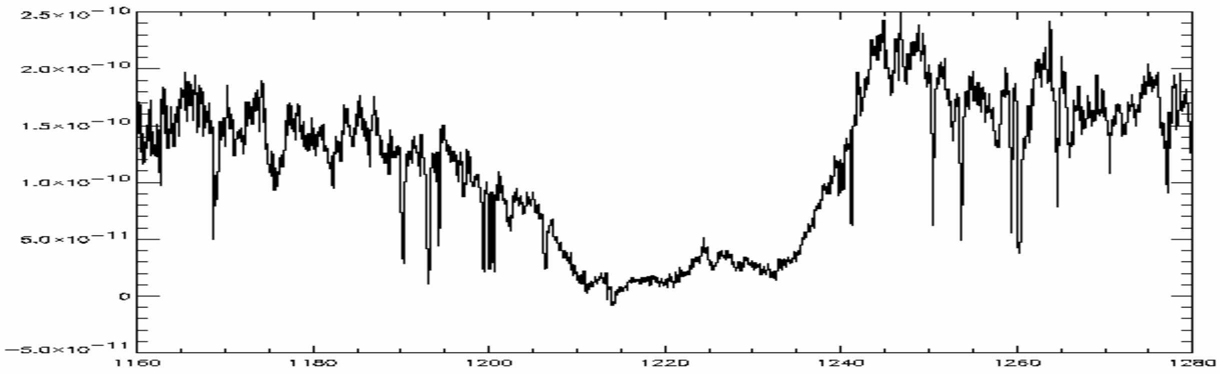
The problem of spectra reconstruction





Define the problem as a signal processing problem



$$r(x) = r_1(x) + r_2(x) + \ldots\ldots = \sum_i r_i(x)$$

$r(x)$ defined as the sum of a number of component processes $r_i(x)$

$r \ldots + r_2(x) + \ldots\ldots = \sum_i r_i(x)$

Assumptions

i. **Independent signal components** $r_i(x)$

ii. **Gaussian-shaped signal components**

$$r_i(x) = A_i e^{-B_i\left(\frac{x}{G_i}\right)^2}$$

***Note*: Generalization to non Gaussian-shaped signal is straightforward while generalization to correlated signal components requires more sophisticated methods**

Facts from probability theory - A

Defined $r(x)$ as the sum of a number of independent signal component $r_i(x)$

$$\bar{r} = \bar{r}_1 + \bar{r}_2 + \ldots.. = \sum_i \bar{r}_i$$

$$\sigma^2 = \sigma_1^2 + \sigma_2^2 + \ldots.. = \sum_i \sigma_i^2$$

Facts from probability theory - B

The variance function $\gamma(T) = s_T^2 / s^2$

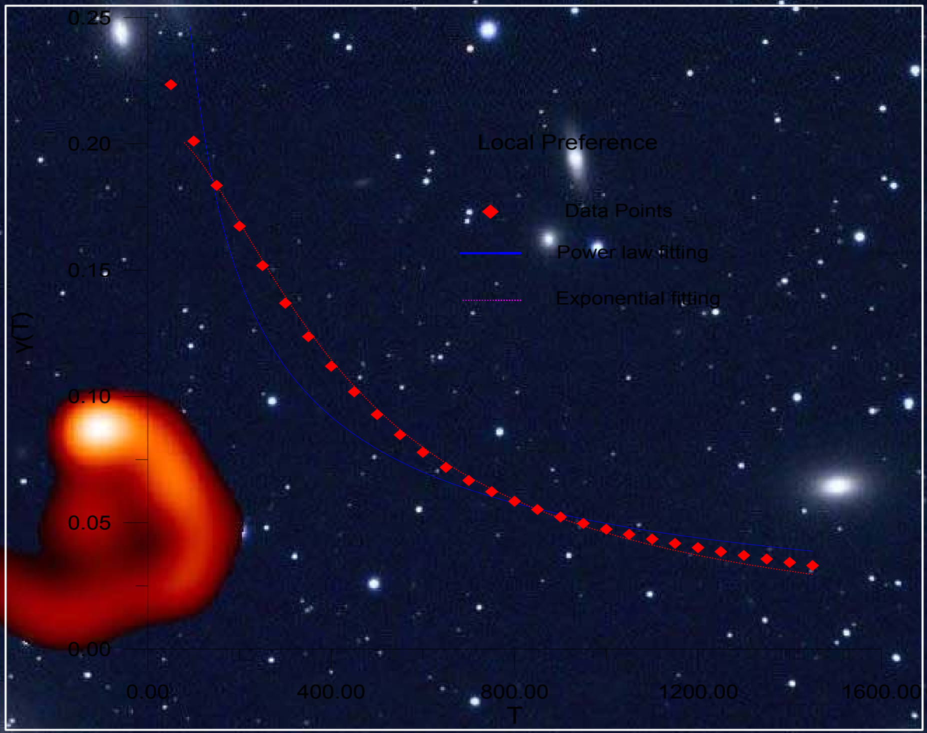
s_T^2 being the variance of the moving average process

$$r_T(x) = \frac{1}{T} \int_{x-T/2}^{x+T/2} r(x)\,dx$$

Facts from probability theory - Γ

For a general class of processes the following relation holds

$$\gamma(T) = \left[1 + \left(\frac{\lambda}{T}\right)^{m}\right]^{-1/m}$$

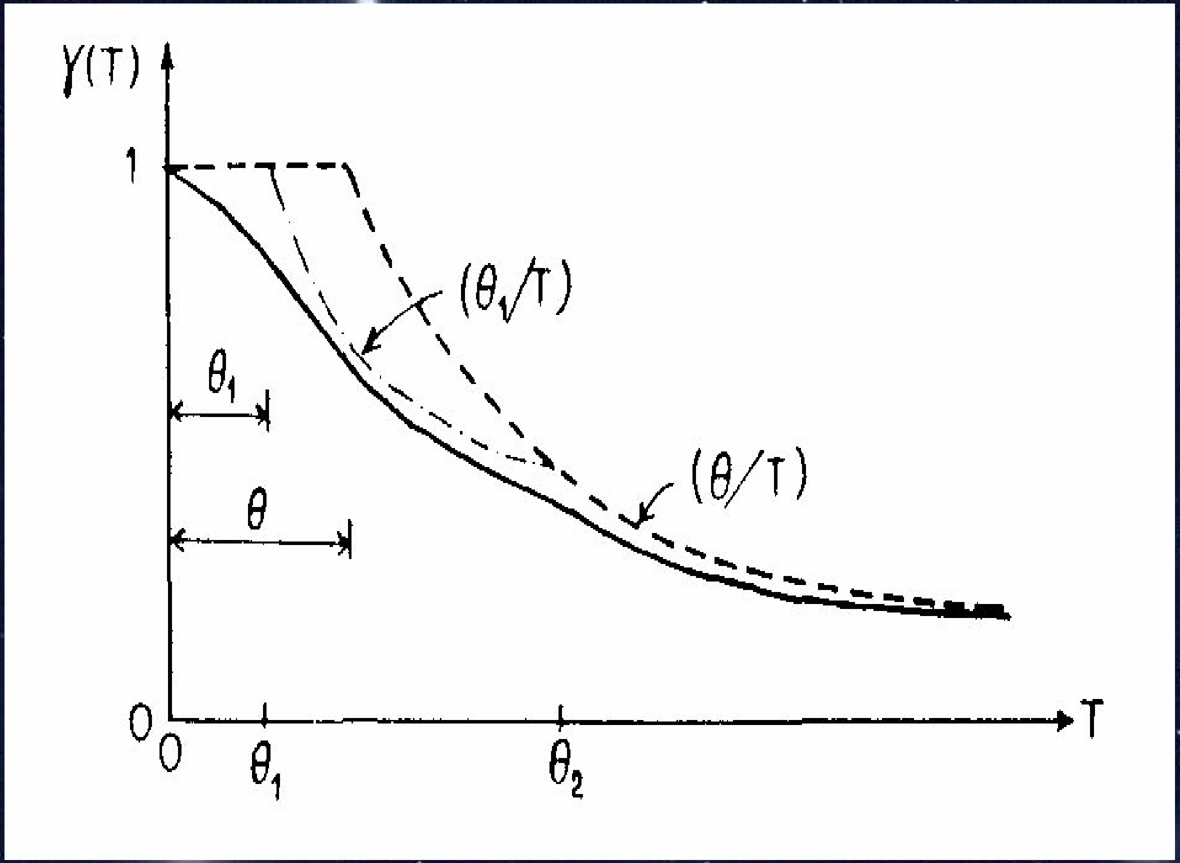


Facts from probability theory - Δ

The variance function of a two-component signal

$$\gamma(\mathrm{T}) = \sum_i q_i \gamma_i(\mathrm{T})$$

$$q_i = \frac{\sigma_i^2}{\sigma^2}$$



Proposed stochastic methodology for exact spectra reconstruction

Let us assume a three-component DAC line

$$r(x) = r_1(x) + r_2(x) + r_3(x)$$

A set of six equation is needed in order to estimate the six unknown values

$$\bar{r}_i, \sigma_i^2, \ i = 1,2,3$$

Proposed stochastic methodology for exact spectra reconstruction

The following equations are true

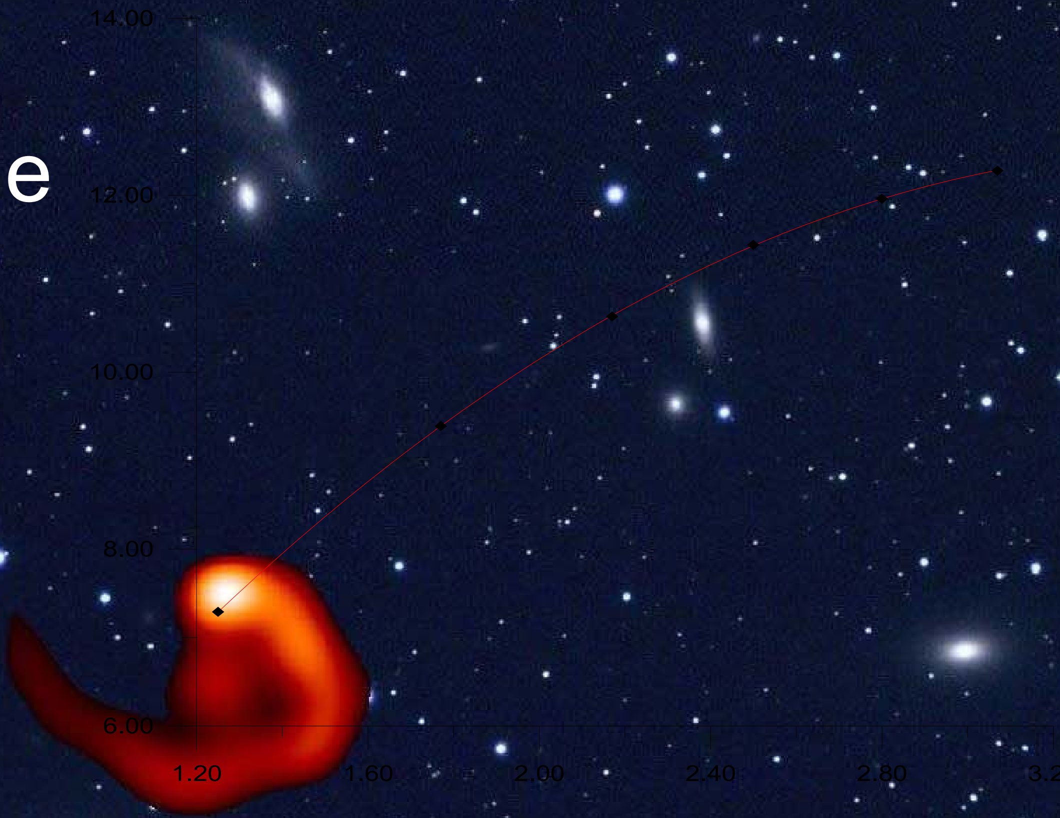
$$\bar{r} = \bar{r}_1 + \bar{r}_2 + \bar{r}_3 \qquad (1)$$

$$\sigma^2 = \sigma_1^2 + \sigma_2^2 + \sigma_3^2 \qquad (2)$$

$$\sigma_i^2 = f_i(\bar{r}_i), \quad i = 1,2,3 \qquad (3,4,5)$$

Proposed stochastic methodology for exact spectra reconstruction

Gaussian-shaped signals with the same depth and arbitrary mean values and variances



It can be seen that there is a one to one parabolic relation between the first and second moment.

Proposed stochastic methodology for exact spectra reconstruction

The sixth missing equation

$$\gamma(T)=\frac{\sigma_1^2}{\sigma^2}\gamma_1(T)+\frac{\sigma_2^2}{\sigma^2}\gamma_2(T)+\frac{\sigma_3^2}{\sigma^2}\gamma_3(T)$$