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THE ELECTRON GAS IN A MAGNETIC FIELD: THE EQUATION OF STATE

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Abstract. Continuing previous work, and using a simple change of variables proposed in existing literature, an analytical equation of state for a non-relativistic degenerate Fermi gas in a magnetic field is developed.

1. INTRODUCTION

This paper is a continuation of previous work (Celebonovic, 1998a) on the equation of state (EOS) of a degenerate non-relativistic Fermi gas. In that paper, an analytical expression for the Fermi-Dirac (FD) integrals of arbitrary order was proposed and applied in obtaining the EOS. The purpose of the present paper is to propose an analogous expression, but applicable in the case of a degenerate Fermi gas in a magnetic field. The nature of the source of the field is irrelevant.

2. CALCULATIONS

The FD integrals have the following form (for example, Cloutman, 1989)

$$F_n(\eta) = \int_0^\infty f(\varepsilon)d\varepsilon/[1+\exp[\beta(\varepsilon-\mu)]] \tag{1}$$

In this expression $f(\varepsilon)=\varepsilon^l$, l is a real number, μ is the chemical potential β denotes the inverse temperature and $\eta=\beta\mu$. The EOS of a degenerate Fermi gas is (for example, Cloutman, 1989)

$$n = (4\pi/h^3)(2mk_BT)^{3/2}F_{1/2}(\eta) \tag{2}$$

All the symbols in this equation have their standard meanings. It has recently been shown (Celebonovic, 1998a) that the FD integrals of arbitrary order can be expressed as

$$F_n(\eta) = \int_0^\mu(\varepsilon)d\varepsilon + T\sum_{n=0}^{\infty}(f^{(n)}(\mu)/n!)[1-(-1)^n]T^n[1-2^{-n}]\Gamma(n+1)\zeta(n+1) \tag{3}$$

The symbols Γ and ζ denote the gamma and zeta functions. How does this equation change when an external magnetic field is applied to the system? It has been proposed (Landau and Lifchitz, 1976) that the influence of such a field an be described by the following change of variables:

$$\mu \rightarrow \mu \mp \mu_B H = \mu(1 \mp \alpha) \tag{4}$$

where $\mu_B = e\hbar/2m_e c$ is the Bohr magneton, H is the magnetic field and $\alpha = \mu_B H/\mu$. The sign in eq. (4) depends on the orientation of the electronic magnetic moments with respect to the field. Inserting eq. (4) into eq. (3), and using the definition of $f(\epsilon)$, one gets the following form of a FD integral of arbitrary order in the presence of a magnetic field:

$$\begin{gathered} F_n(\eta) = [\mu(1 \mp \alpha)]^{l+1}/(l+1) + \\ T\sum_{n=0}^{\infty}(f^{(n)}(\mu(1 \mp \alpha))/n!)[1-(-1)^n]T^n[1-2^{-n}]\Gamma(n+1)\zeta(n+1) \end{gathered} \tag{5}$$

Using the result for the chemical potential of the electron gas (Celebonovic, 1998a), the parameter α can be expressed as follows

$$\alpha = \mu_B H/[\mu_0(1-(1/12)(\pi T/\mu_0)^2 + (1/720)(\pi T/\mu_0)^4 - (1/162)(\pi T/\mu_0)^6 + ...)] \tag{6}$$

In this equation $\mu_0 = An^{2/3}$ and $A = (3pi^2)^{2/3}\hbar^2/2m_e$. Inserting eq. (6) into eq. (5), and choosing the positive sign in eq. (4), one arrives at:

$$\begin{gathered} F(1/2)(\eta) = (2/3)[(\mu_B H) + An^{2/3} - (\pi T)^2/(12An^{2/3}) + (\pi T)^4/(720A^3n^2) - \\ (\pi T)^6/(162A^5n^{10/3}) + (\pi T)^8/(754A^7n^{14/3})]^{3/2}(1 + << 3 >>) \end{gathered} \tag{7}$$

The symbol $<< .. >>$ denotes the number of omitted terms. Expanding eq. (7) in its full form, it follows that the FD integral of the order 1/2 can be expressed as:

$$\begin{gathered} F_{1/2}(\eta) = (\mu_B H)/(54 \times 1885^{1/2})[(2442960(\mu_B H)A^7n^{14/3} + \\ << 5 >>)/A^7n^{14/3}] + << 23 >> \end{gathered} \tag{8}$$

Inserting eq. (8) in its full form into eq. (2), it follows that the number density of the electron gas can be expressed as the following function:

$$\begin{gathered} n = 23.6954(\mu_B H) \times (m_e T)^{3/2} \times [((\mu_B H)A^7n^{14/3} + A^8n^{16/3} - \\ 0.822467n^4T^2A^6 + 0.13529A^4n^{8/3}T^4 - 5.9345A^2n^{4/3}T^6 + \\ 12.5843T^8)/(A^7n^{14/3})]^{1/2}/h^3 + << 23 >> \end{gathered} \tag{9}$$

This equation, especially in its full form, is far too complex to be solvable analytically. However, its low temperature limit can be solved analytically. Developing the full form of eq. (9) in series in T up to and including terms of the order T^2, one arrives at the following implicit form of the EOS of a degenerate non-relativistic electron gas in a magnetic field

$$n = (\pi 2^{9/2}/3 \times h^3) \times (mn^{2/3}AT)^{3/2}(1 + (\mu_B H)/(An^{2/3}))^{3/2} \tag{10}$$

This can be solved analytically under the assumption that $x = \mu_B H/An^{2/3} \rightarrow 0$. Physically, this limitation means that the solution is valid when the numbre density of the electron gas is high, or when the magnetic field in which it is placed is weak. Developing now eq. (10) into series in x, up to and including terms of the order x^2, the following expression is obtained:

$$(1 - BT^{3/2})y^2 = (Fy + G)T^{3/2} \tag{11}$$

where the following symbols were used

$$\begin{aligned} &y = n^{2/3}; B = (2^{9/2}\pi/3)(m_e^{1/2}/h)^3 A^{3/2}; F = (3B/2)(\mu_B H/A) \\ &G = (3B/8)(\mu_B H/A)^2 \end{aligned} \tag{12}$$

Solving eq. (11) within S. Wolfram's MATHEMATICA 2.2 software package on a Pentium 166/MMX with a RAM of 32 MBytes, one gets that:

$$y = [-(FT)^{3/2} \pm \sqrt{F^2T^3 - 4GT^{3/2}(-1 + BT^{3/2})}]/(2(-1 + BT^{3/2})) \tag{13}$$

We have thus obtained the EOS of the non-relativistic degenerate Fermi gas in a magnetic field. This is the implicit form of the EOS. Using the changes of variables defined in eq. (12) it becomes possible to obtain the explicite form of the EOS.

What about the applications of the EOS derived in this paper? Equation (13) was derived for the low-temperature regime, assuming that $\mu_B H/An^{2/3} \rightarrow 0$. It follows from this assumption that eq. (13) is applicable to the low temperature electron gas whose number density is limited by $n \gg (\mu_B H/A)^{3/2}$. In astrophysics, situations like this can be expected to arise in accretion disks around compact objects. Having in mind laboratory work they can appear in high pressure experiments performed under external magnetic fields (like, for example, in studies of organic metals in solid state physics).

References

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