Publ. Astron. Obs. Belgrade No. 61 (1998), 163 – 166 *Poster paper*

POWERFUL SHOCK WAVES RADIATION IN AIR I

K.L. STEPANOV, L.K. STANCHITS, Y.A. STANKEVICH

Heat & Mass Transfer Institute, ul. Brovky, 15, 220072, Minsk, Belarus
E-mail kls@hmti.ac.by

Abstract. Problems related to the structure and radiation of a strong shock waves propagating in the Earth's atmosphere with velocities D≤50 km/s at heights H≤50 km are considered in this paper on the basis of real thermodynamic and optical properties of air. Data on shock adiabatic curves of air obtained both in the supposition of complete thermodynamic equilibrium of air behind the shock wave front, and with account of possible transparency of gas for self radiation are presented. The hierarchy of scales of various physical processes in a shock wave permitting to simplify solution of the problem concerning the front structure is considered. The modes of SW propagation, profiles of parameters, spatial scales, spectral and integrated radiation fluxes have been determined for various heights and velocities.

1. Hugoniot adiabatic curves. The condition of gas behind the shock wave front (SWF) is determined by thermodynamic parameters prior to break and the magnitude characterizing the wave's amplitude (for example its velocity D). If the equation of state is represented as that for ideal gas $\varepsilon = (\gamma - 1)^{-1} p/\rho$, the jump of pressure, density and temperature at the shock wave is determined by equations:

$$\frac{p_1}{p_0} = \frac{\gamma_0 M^2 + 1}{\gamma_1 + 1} + \frac{1}{\gamma_1 + 1}\sqrt{\gamma_0^2 M^4 - 2\gamma_0\left(\frac{\gamma_1^2 - 1}{\gamma_0 - 1} - 1\right) + \gamma_1^2}\,, \qquad (1)$$

$$\frac{\rho_1}{\rho_0} = \frac{(\gamma_1 + 1)p_1 + (\gamma_1 - 1)p_0}{(\gamma_0 + 1)p_0 + (\gamma_0 - 1)p_1}\frac{\gamma_0 - 1}{\gamma_1 - 1}, \qquad \frac{T_1}{T_0} = \frac{\mu_1}{\mu_0}\frac{p_1}{p_0}\frac{\rho_0}{\rho_1}, \qquad M = \frac{D}{\sqrt{\gamma_0 p_0/\rho_0}}$$

Here M is the Mach number, index 0 corresponds to the state in front of the shock wave, 1 – to the state behind its front. The equation of the shock adiabatic curve represented as $\varepsilon_1 - \varepsilon_0 = (p_0 + p_1)(\rho_0^{-1} - \rho_1^{-1})/2$, can be numerically calculated. The parameters of undisturbed air were taken from the CIRA atmosphere model. Composition and thermodynamic functions of air were determined according to equations of ionized and chemical equilibrium in Debye approximation of the large canonical ensemble (Romanov 1995). The results are presented in figs.1,which illustrates the dependence of temperature T_1 and compression $\delta = \rho_1/\rho_0$ on shock wave velocity at various altitudes.

At sufficient transparency of plasma for its own radiation when absorption and emission processes are not compensated, spontaneous decay of excited levels, the processes of a photo- and dielectron recombination result in deviation from boltzmann population and shift ionization equilibrium in plasma to smaller ionization degree (Romanov 1988, Panasenko 1995). The shock adiabatic curves calculated as per the radiative-collision model in the supposition of a full transparency of plasma are shown in fig.2. Their comparison with equilibrium data is presented here in as well. As seen from the curves, noticeable differences in temperature and compression in SWF are observed at altitudes over 40-50 km. If partial reabsorbtion of radiation, which reduces the contribution of radiative transitions in level population, is taken into account, the distinctions of shock adiabatic curves are maintained at altitudes H =60 km.

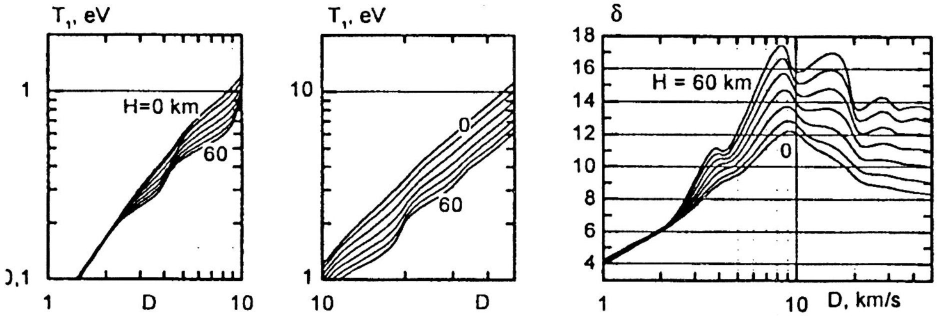

Fig. 1. Dependence of T_1 and compression δ from SW velocity D at various altitudes.

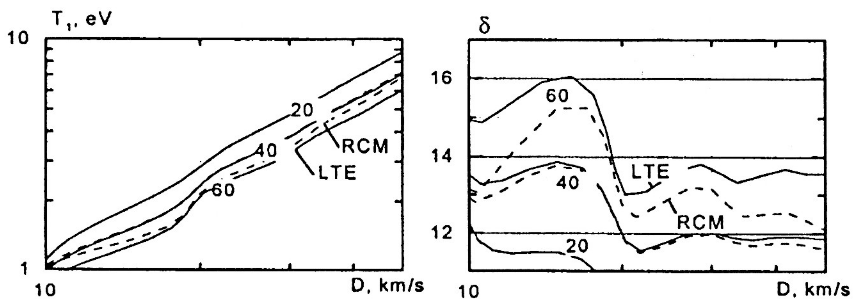

Fig. 2. Shock adiabatic curves of air at H=20, 40 and 60 km, obtained in LTE approximations and as per the RCM- model for transparent plasma.

2. The structure of a powerful SW front. Analysis of hierarchy of scales describing various physical processes indicates (Stepanov et al. 1998) that the largest scale of length is associated with plasma heat radiation free lengths. The radiation free length exceeds the scale of lengths of dissociation and ionization, the area of relaxation of electrons and heavy particles temperatures to uniform plasma temperature as well as the length of the zone, where electronic thermal conduction is considerable. Further it is supposed, that the dissociation and ionization processes «monitor» the medium's temperature and density, that the temperatures T_e=T_i, electronic thermal conduction is small as compared to radiation transfer. Therefor the problem concerning the structure can be formulated as a problem concerning radiation relaxation in the front of a powerful SW (Zeldovich 1966). Further we shall assume that the thickness of shock compressed gas behind the front is adequately for it to be optically dense. The structure of a SW having velocity D, is described by a set of gas dynamics and radiation transfer equations

$$\rho u = \rho_0 D\,, \quad p + \rho u^2 = p_0 + \rho_0 D^2\,, \quad \varepsilon + \frac{p}{\rho} + \frac{u^2}{2} + \frac{S}{\rho_0 D} = \varepsilon_0 + \frac{p_0}{\rho_0} + \frac{D^2}{2} + \frac{S_0}{\rho_0 D} \qquad (2)$$

$$\mu \frac{dI_\nu}{dx} = \kappa_\nu (I_{\nu p} - I_\nu)\,, \qquad S_\nu = \int_{(4\pi)} I_\nu \Omega \, d\Omega\,, \qquad S = \int_0^\infty S_\nu d\nu \qquad (3)$$

As the laws of conservation of mass and impulse determine linear dependence of pressure on the magnitude of inverse compression $p = p_0 + \rho_0 D^2 (1-\eta)$, $\eta = \rho_0 / \rho$, it is possible to construct the tabulated equations $\varepsilon(T)$, $p(T)$, $\rho(T)$, dependent from one variable and describing the modification of gas characteristics in a wave. Then we find the relationship between T and S

$$S = S_0 - \rho_0 D F(T), \quad F(T) = \varepsilon(\eta) - \varepsilon_0 - \frac{p_0}{\rho_0}(1-\eta) - \frac{D^2}{2}(1-\eta)^2, \quad \eta = \eta(T) \qquad (4)$$

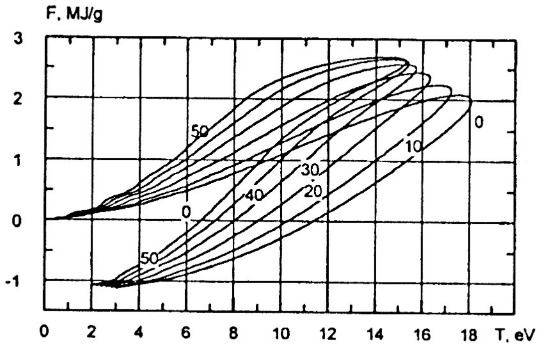

Fig. 3. F(T) for SW with D=50 at various H.

Function F(T) for a SW with a given velocity for various H is shown in fig.3. Flux $S = S_0$, i.e. $F(T_0) = 0$ corresponds to the initial state; in equilibrium state behind the SW front $S_1 = 0$ and $F(T_1) = S_0/(\rho_0 D)$. As $S_0 < 0$, the final temperature behind the wave front will be less than if there was no outgoing radiation. If the energy emitted is neglected in comparison with the hydrodynamic energy flux ($S_0 << \rho_0 D^3/2$) T_1 is determined by condition $F(T_1) = 0$.

3. Diffusion "gray" approximation. In order to find a solution of the equations of radiation transfer and hydrodynamics it is necessary to set "good" initial profiles of parameters, to solve equations of spectral transfer, to determine radiation flux S and to construct a converging iterative process. Such a distribution of parameters can be found by considering the transfer in a gray diffusion approximation. The appropriate equations

$$\frac{dS}{d\tau} = c(U_p - U), \qquad S = -\frac{c}{3}\frac{dU}{d\tau}, \qquad U_p = \frac{4\sigma T^4}{c}, \qquad (5)$$

where τ is the "average" optical variable ($d\tau = \kappa dx$), with account for of energy conservation law (5) can be reduced to a phase plane equation

$$\frac{dU}{dT} = -3\left(\frac{\rho_0 D}{c}\right)^2 \frac{F'(T)\left(F(T) - S_0/(\rho_0 D)\right)}{U_p(T) - U} \qquad (6)$$

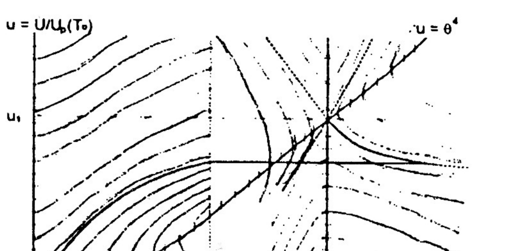

Fig. 4. The phase plane of (6) at $S_0 = 0$.

The phase plane of (6) is given in fig.4. Points of initial (0) and final (1) states are singular for (6) should satisfy conditions $T = T_i$: $U = U_p(T_i)$ $F(T_i) = 0$ $i = 0,1$. After the problem on the phase plane is solved, it is possible to find the distribution of T, ρ, S along τ counted from the SWF. The profiles of T and S along τ are shown in figs. 5 at H=50 km for SWs having velocities D=50, 30 and 20 km/s. Note a considerable difference in optical thickness of the warming up layer and relaxation area behind SWF.

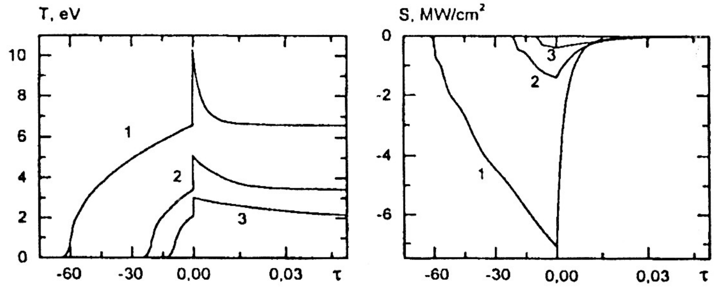

Fig.5. Distributions of T and S along τ . H= 50 km. 1-D=50 km/s, 2-D=30 km/s, 3-D=20 km/s.

4. Radiation transfer in real spectrum. The recalculation of gas-dynamic profiles from the optical variable τ to a spatial coordinate x was performed with account of the character of radiation transfer in various layers. In super critical amplitude waves temperature $T(\tau)$ varies weakly on scale $\Delta\tau = 1$ in the warming up zone. Radiation heat transfer occurs here in the mode of radiation thermal conductivity. Therefore it is natural to use the Rosseland free length. The distances from the viscous jump upwards along the flow were found by ratio $dx = L_R d\tau$. The Planck coefficient was used as the recalculation scale in the vicinity of the SW front and in the front part of the warming up zone ($dx = K_P^{-1} d\tau$). In the iterative process the most complicated problem is to obtain a self-consistent solution in the neighborhood of the front and periphery of the warming up zone. Here it is necessary to determine the average absorption coefficient $\overline{K}$, $(\Delta x = \Delta\tau / \overline{K})$ by averaging κ_ν on the field of self radiation.

Integral representation of the flux was used to describe radiation transfer

$$S^+_\nu(\tau_\nu) = 2\int_{\tau_0}^{\tau_\nu} S_{\nu p}(\tau')\left[\exp(-(\tau_\nu - \tau')) - (\tau_\nu - \tau')E_1(\tau_\nu - \tau')\right]d\tau' + S^+_{\nu 0}(\tau_\nu) \quad (7)$$

$$S^-_\nu(\tau_\nu) = 2\int_{\tau_\nu}^{\tau_1} S_{\nu p}(\tau')\left[\exp(-(\tau' - \tau_\nu)) - (\tau' - \tau_\nu)E_1(\tau' - \tau_\nu)\right]d\tau' + S^-_{\nu 0}(\tau_\nu) \quad (8)$$

The integral radiation was determined by direct integrating on spectrum. It overlapped the range of quantum energies, in which plasma radiation is considerable. In the range of quantum energies $E < 17.35$ eV, 560 spectral groups were taken, the averaging interval was 250 cm^{-1} (0.031 eV). Starting at 17.35 eV scale was used, on which the averaging spacing was set by a ratio $\Delta E_i = E_0 \cdot 10^{(i-560)/500}$, where $E_0 = 8.\cdot 10^{-2}$ eV. Thus 1300 spectral groups in the range of radiation transfer energy were used (fig.6).

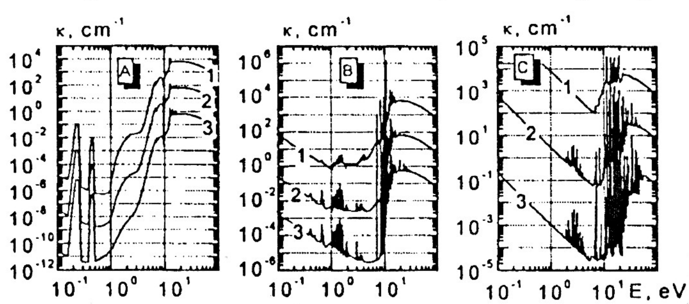

Fig. 6. Absorption coefficients of air. T=$3\cdot 10^3$ K (A), 10^4 B), 3.16 eV (C). 1 - $\rho_1 = 1.23\cdot 10^{-3}$ g/cm^3, 2 - $\rho_2 = 10^{-2}\rho_1$,3 - $\rho_3 = 10^{-4}\rho_1$.