

## A SEARCH FOR POSSIBLE UNRESOLVED COMPONENTS IN EIGHTEEN ECLIPSING BINARIES

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**Abstract.** A total of 8507 minima times (6890 visual and 1617 photographic or photoelectric ones) of 18 eclipsing binary stars (*Table 1.*) have been separated and collected from the remarkable collection of late Dieter Lichtenknecker and from the recent literature. Using the Kopal (1978) method for the analysis of the obtained O-C diagrams of these systems (belonging to different types of eclipsing variables) one can classify them into three categories :

1. 'good cases' : systems with light-time effect resulting third component with reasonable orbital and astrophysical parameters. They are AB And, TV Cas, XX Cep, AK Her. (Figs. 1.-4.)
2. 'probable cases' : good candidates of multiplicity but the observational data available up to now are insufficient for obtaining satisfactory description. Light-time analysis of these systems has resulted weaker solutions as for the previous group, but they can be held as noticeable targets for the future studies. These systems are W Del, U Peg, AT Peg, ST Per. (Figs. 5.-8.)
3. 'problematical cases' : For these systems either we do not have enough data for making unambiguous identification of the sinusoidal O-C (due to light-time effect) and thus, we could not find a corresponding good third-body orbit, or the mathematical analysis led to results which are inconsistent with other observational or astrophysical facts. They are RT And, XZ And, OO Aql, Y Cam, RS CVn, TW Cas, CQ Cep, U CrB, MR Cyg and SW Lac. (In the case of TW Cas and SW Lac we couldn't be able to find any LITE solution. ) (Figs. 9.-16.)

### 1. ANALYSIS

All data sequences were handled in the same manner. After making a first plot of the raw O-C diagram, we decided whether it is necessary to remove the effect of another type of major period variation or not. It was necessary to subtract a parabola from the O-C diagram of eight of the 18 studied systems using a least-squares method. The obtained clear, periodic pattern was analyzed by a DFT program written by one of us (T.B.) in view of the data points having equal weights. The software was checked by the MUFRA code (Kolláth, 1990). With the frequencies coming from the Fourier-analysis an optionally weighted least-squares fitting was performed which

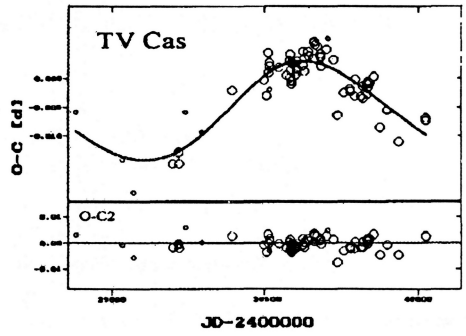
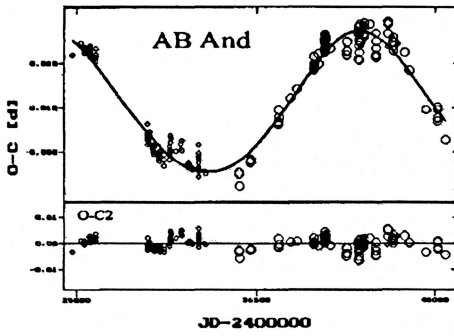


Fig. 1 & 2. Light time (LITE) solutions and residuals for Group 1. stars

The key for all figures:   
 · - visual observations   
 ○ - photographic observations   
 ◐ - "plate" minima   
 ○ - photoelectric observations

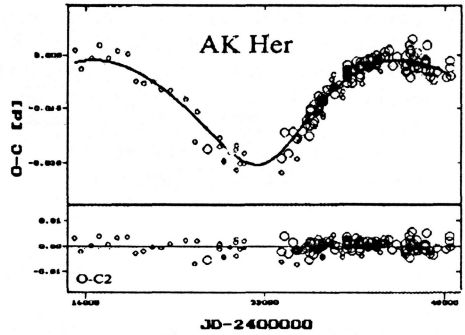
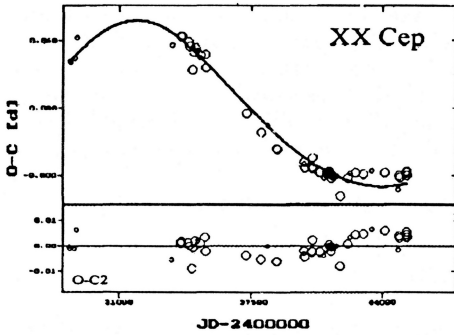


Fig. 3 & 4. Light time solutions and residuals for Group 1. stars

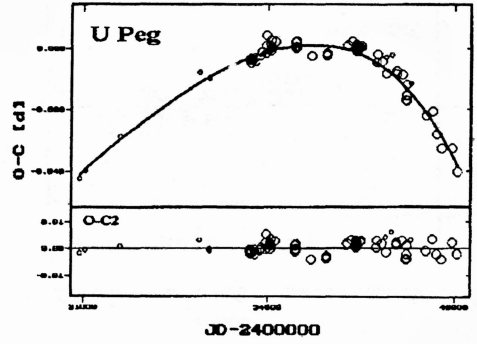
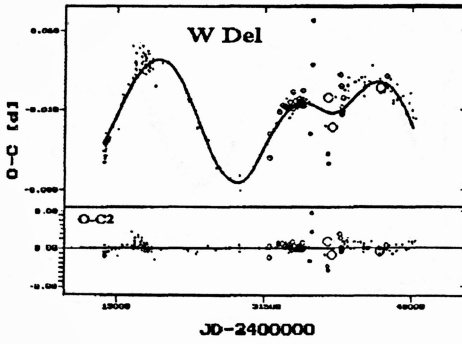


Fig. 5 & 6. Light time solutions and residuals for Group 2. stars

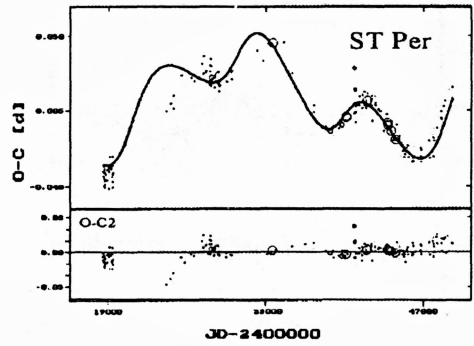
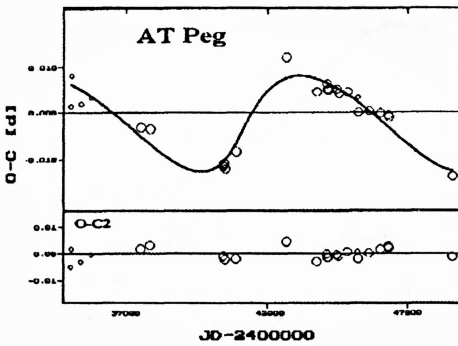


Fig. 7 & 8. Light time solutions and residuals for Group 2. stars

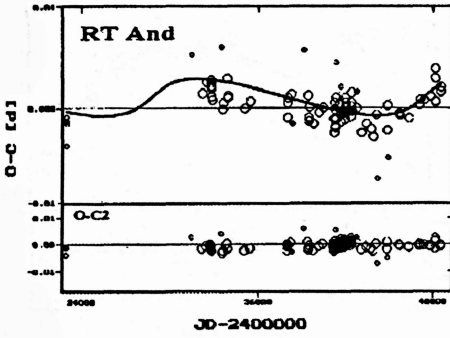


Fig. 9. PgPe points of RT And and the fourth body representation (after whitening with the third body effect)

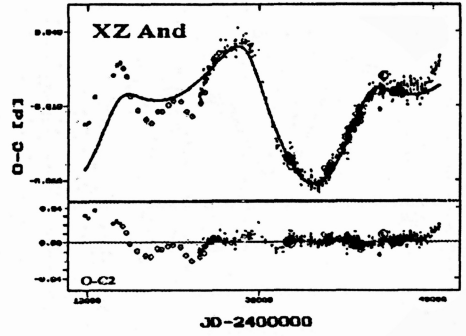


Fig. 10. TOT points of XZ And with our "best" solution

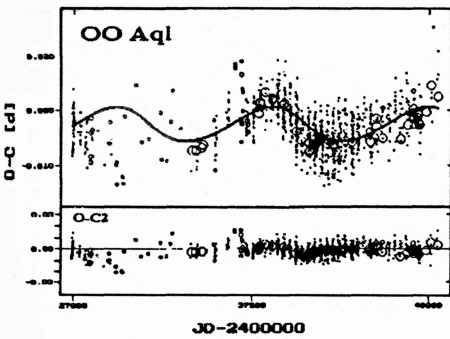


Fig. 11. TOT points of OO Aql with LITE solution made after the removal of a parabola

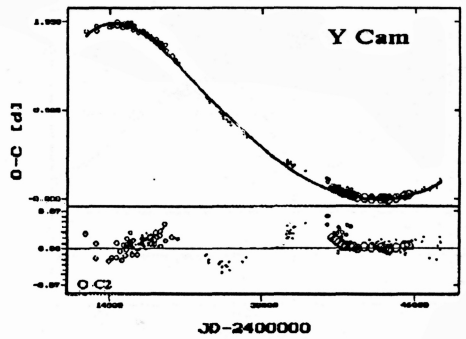


Fig. 12. TOT points of Y Cam with our formal LITE solution

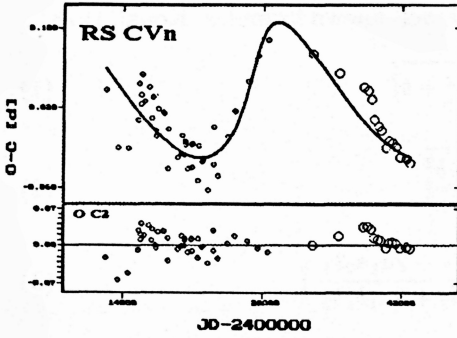


Fig. 13. PgPe points of RS CVn with LITE solution made after the removal of a parabola

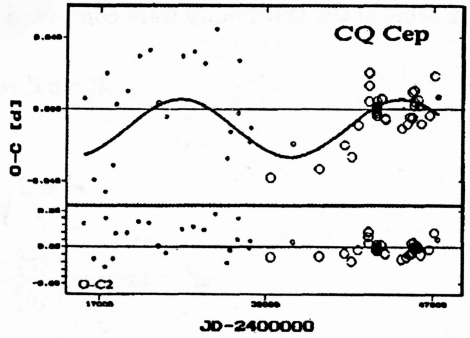


Fig. 14. TOT points of CQ Cep with LITE solution made after the removal of a parabola

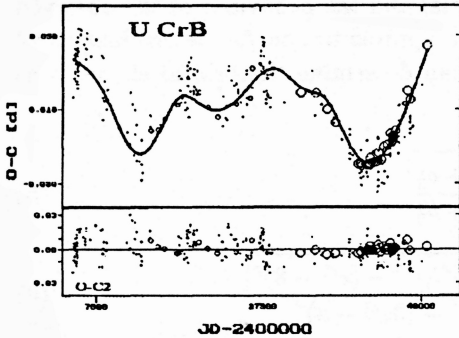


Fig. 15. A four body representation for the TOT points of U CrB

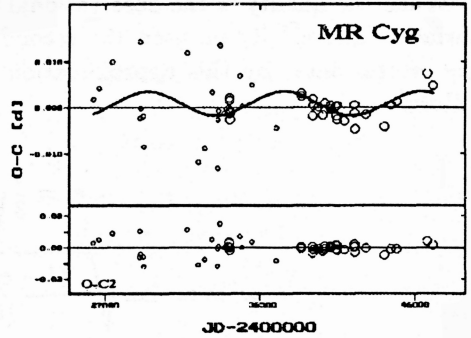


Fig. 16. PgPe points of MR Cyg with LITE solution made after the removal of a parabola

gave the necessary coefficients for the calculation of orbital elements. In general, we have repeated all procedures twice : using all available minima times (TOT), and using only the photographic (without observational results marked as 'plate minima') and photoelectric data (PGPE). There was one case (MR Cyg), when it was necessary to use only the photoelectric observations for achieving some result. The elements of the orbit of the third body were computed by the well-known formulas (Kopal, 1978) :

$$A'_1 \sin i' = c\sqrt{a_1^2 + b_1^2} \quad (1)$$

$$e' = 2\sqrt{\frac{a_2^2 + b_2^2}{a_1^2 + b_1^2}} \quad (2)$$

$$\omega' = \arctan \left( \frac{(b_1^2 - a_1^2)b_2 + 2a_1a_2b_1}{(a_1^2 - b_1^2)a_2 + 2a_1b_1b_2} \right) \quad (3)$$

$$\tau' = t_0 - \frac{P'}{2\pi} \arctan \left( \frac{a_1b_2 - b_1a_2}{a_1a_2 - b_1b_2} \right) \quad (4)$$

where  $a_{1,2}, b_{1,2}$  are the Fourier coefficients coming from the analysis as it was described above (i.e. the method needs first harmonics of the fundamental frequency),  $A'$  denotes the semi-major axis of the absolute orbit of the centre of mass of the eclipsing pair around that of the triple system,  $i', e', \omega', t'$  and  $P'$  are the usual elements of the third body orbit.  $P$  is the sidereal period of the close pair, while  $c$  is the velocity of light.

These equations can be considered to be a good approximation for the smaller eccentricities. During the test of our procedure we could obtain reliable results for  $e' < 0.6$ . Thus, for several cases we used only the first-order approximation shown above.

When the quality of the observational material and the behaviour of the observed variation allowed it, we used the second-order approximation for determination of the orbital data. By this approximation one can determine the orbital elements as follows :

$$e' = \frac{4}{3}\sqrt{\frac{a_3^2 + b_3^2}{a_2^2 + b_2^2}} \quad (5)$$

$$\omega' = \arctan \sqrt{-\frac{(1 - e'^2)[16C_1e'^2 - (e'^2 - 8)^2]}{16C_1e'^2 - (3e'^2 - 8)^2}} \quad (6)$$

where  $C_1 = \frac{a_1^2 + b_1^2}{a_2^2 + b_2^2}$  ;

$$\tau' = t_0 - \frac{P'}{2\pi} \arctan \frac{1 - \frac{a_1}{b_1} \frac{h_1}{g_1} \tan \omega'}{\frac{a_1}{b_1} + \frac{h_1}{g_1} \tan \omega'} \quad (7)$$

$$A'_1 \sin i' = c \frac{a_1^2 + b_1^2}{h_1^2 + (g_1^2 - h_1^2) \cos \omega'^2} \quad (8)$$

where

$$g_1 = \left(1 - \frac{e'^2}{8}\right) \sqrt{1 - e'^2}$$

and

$$h_1 = \frac{1 - 3e'^2}{8}$$

See e.g. at Vinkó (1989). Other designations are the same as above. Because of the features of the Fourier-analysis, sometimes it was necessary to add  $P'/2$  to the resulted value of  $t'$ . Of course, this approximation needs two harmonics of the fundamental frequency. Relevant coefficients are  $a_{1,2,3}$  and  $b_{1,2,3}$ .

From the orbital parameters determined by the above presented expressions, one can define the mass function as the following :

$$f(m_3) = \frac{m_3^3 \sin^3 i'}{(m_{12} + m_3)^2} = \frac{4\pi^2 A_1'^3 \sin^3 i'}{GP'^2} \quad (9)$$

where  $m_{12}$  is the sum of masses of the two component stars of the eclipsing binary system, while  $m_3$  is the mass of the third body.  $G$  is the gravitational constant. At this point, the right-hand side of this equation can be computed. In the sense of expression (9) we have a third order equation for the unknown  $m_3$  mass of the third body :

$$m_3 \sin^3 i' - m_2^3 f(m_3) - 2m_3 m_{12} f(m_3) - m_{12}^2 f(m_3) = 0 \quad (10)$$

Of course, the result will contain a free parameter ( $\sin i'$ ) which remains undeterminable from these kind of observations. We shall present (as it was done usually by other authors) the masses of the hypothetical satellites for a few different inclinations of its orbit.

## 2. RESULTS

*Table 2.* contains the 18 ephemeris used in our analysis, while the resultant orbital parameters can be found in *table 3.-5.* for the groups I.-III.

Table 1. Main parameters of the investigated systems

Name	T	TOT	PGPE	mag		$sp_1 + sp_2$	$M_1$	$M_2$	P	Sources
				max	min					
RT And	A	729	97	8 55	9 47V	F8V	1 52	1 00	0 63	1,2
XZ And	A	753	75	10 02	12 99p	A0+G8 · III	3 23	1 65	1 36	1,3
AB And	W	1148	187	9 50	10 32V	G5+G5V	1 06	1 71	0 33	1,4
OO Aql	W	1085	77	9.2	9 9V	G5V	sum=2 5		0 51	1,8
Y Cam	A	248	57	10 50	12 24V	A8V	2 33	0 50	3 31	1,2
RS CVn	A	93	58	7 93	9 14V	F4IV+K0IVe	1 42	1 35	4 80	1,2
TV Cas	A	657	77	7 22	8 22V	B9V+F7IV	4 04	1 62	1 81	1,5
TW Cas	A	168	72	8 32	8 98V	B9V+A0	2 90	1 18	1 43	1,2
XX Cep	A	191	59	9 13	10 28p	A8V	1 87	0 32	2 34	1,5
CQ Cep	$\beta$	58	38	8 63	9 12V	WN5 5+O7	17 5	21 1	1 64	1,6
U CrB	A	269	40	7 66	8 79V	B6V+F8III	4 7	4 4	3 45	1,11
MR Cyg	A	120	53	8 75	9 68V	B3V+B9	7 6	5 7	1 68	1,9
W Del	A	184	32	9 69	12 33V	B9 5Ve+G5	2 01	0 42	4 81	1,5
AK Her	W	330	177	8 29	8 77V	F2+F6	sum=1 5		0 42	1,7
SW Lac	A	1669	402	8 51	9 39V	G8Vp+G8Vp	0 96	1 14	0 32	1,4
U Peg	A	423	79	9 23	10 07V	F3+F3	1 29	0 86	0 37	1,2
AT Peg	A	176	26	8 97	9 75V	A7V	2 2	0 93	1 15	1,10
ST Per	A	206	11	9 52	11 40V	A3V+G-K	2 03	0 39	2 65	1,2

Remarks :

- T : type of the light changes ( A : Algol, W : W Uma,  $\beta$  :  $\beta$  Lyr.)
- TOT : total number of all accepted times of minima of the stars.
- PGPE : number of photographic and photoelectric times of minima.
- mag : brightness of the system in maximum and minimum light ( $p$  : photographic,  $v$  : V-filter photoelectric stellar magnitudes).
- $sp_1 + sp_2$  : spectral types of the components according to the GCVS.
- $M_1, M_2$  : masses of the components in Solar Masses.
- P : Approximate period of light changes (in days).

Sources

- 1 : GCVS 4th ed., Kholopov et al., 1985;
- 2 : Giannone & Gianuzzi, 1974;
- 3 : Budding, 1984;
- 4 : Rovithis-Livanoiou et al., 1990;
- 5 : Van Hamme & Wilson, 1990;
- 6 : Kartasheva & Svechnikov, 1986;
- 7 : Nagy, 1985;
- 8 : Hrivnak, 1989;
- 9 : Linnell & Kallrath, 1987;
- 10 : Hill & Barnes, 1972;
- 11 : Heintze, 1990.



**Table 2. Ephemerides used for O-C diagrams**

AB And	Min I pgpe	2436109 58041	+0 33188985 E	+5 335 10 <sup>-11</sup> E <sup>2</sup>	recent paper
TV Cas	Min I pgpe	2444662 27198	+1 81260 E	-8 697 10 <sup>-10</sup> E <sup>2</sup>	recent paper
XX Cep	Min I pgpe	2444839 8022	+2 33732665 E		GCVS 1985
AK Her	Min I pgpe	2442186 460	+0 42152227 E		Barker&Herczeg (1979)
U CrB	Min I tot .	2437844 37911	+3 45220552 E		Mayer et al (1991)
W Del	Min I tot	2443328 52755	+4 80610015 E	+7 253 10 <sup>-9</sup> E <sup>2</sup>	recent paper
U Peg	Min I pgpe	2436511 66821	+0 374781439E		GCVS 1985
AT Peg	Min I pgpe	2445219 85614	+1 1460796E	-1 05 10 <sup>-9</sup> E <sup>2</sup>	recent paper
ST Per	Min I tot .	2442436 577919	+2 6483418 E		recent paper
RT And	Min I pgpe :	2441141 88901	+0 628929513 E		GCVS 1985
XZ And	Min I tot	2423977.1915	+1 357278 E		GCVS 1985
OO Aql	Min I tot :	2438613.21434	+0 5067914 E	-1 618 10 <sup>-10</sup> E <sup>2</sup>	recent paper
Y Cam	Min I tot	2442961 9276	+3 3056244 E		GCVS 1985
RS CVn	Min I pgpe	2422811 69133	+4 7978765 E	-6 593 10 <sup>-9</sup> E <sup>2</sup>	recent paper
TW Cas	Min I tot	2442008 3873	+1 4283240 E		GCVS 1985
CQ Cep	Min I tot .	2432456 706	+1 641247 E	-1 05 10 <sup>-9</sup> E <sup>2</sup>	recent paper
MR Cyg	Min I pgpe	2433396 4096	+1 67703362 E		GCVS 1985

**Table 3. Solutions for group I. systems**

	AB And	TV Cas	XX Cep	AK Her
Remarks	1,3,5	1,3,5	3,5	3,5
$P'_{orb}$	19764.71	21412 46	21888 38	27243 24
$e'$	0 14	0 16	0 26	0 33
$\omega'$	1 30	0 53	1 32	5 10
$\tau'$	22570 12	37774 42	31054 65	32442 26
$a' \sin i'$	413 19	219 95	968 44	381 88
$f(m_3)$	0 0072	0 0009	0 0751	0 0030
$m_3$ 90°	0 42	0 32	0 89	0 21
60°	0 49	0 37	1 07	0 24
30°	0 92	0 61	2 30	0 45
dev Fr	0 0022	0 0023	0 0026	0 0021
dev O-C	0 0022	0 0023	0 0029	0 0021

**Table 4. Solutions for group II. systems**

Remarks	W Del		U Peg	AT Peg	ST Per	
	3 *	4 *	4	1,4,5	3 *	4 *
$P'_{orb}$	21444 26	13432 78	49721 00	9168 64	27000 00	9411 77
$e'$	0 36	0 18	0 43	0 39	0 19	0 24
$\omega'$	4 44	4 30	3 55	6 05	5 80	6 36
$\tau'$	29105 99	38835 92	50631 38	41011 26	48543 06	39904 00
$a' \sin i'$	1094 55	567 70	1188 50	290 70	678 64	426 87
$f(m_3)$	0 1130	0 0402	0 0269	0 0116	0 0170	0 0348
$m_3$ 90°	1 12	0 74	0 59	0 54	0 53	0 70
60°	1 36	0 88	0 69	0 63	0 62	0 80
30°	2 98	1 79	1 39	1 20	1 22	1 67
dev Fr	0 0114		0 0021	0 0013	0 0062	
dev O-C	0.0125		0 0022	0 0021	0 0062	

Table 5. Solutions for group III. systems

Remarks	RT And		XZ And		OO Aql	Y Cam	RS CVn	CQ Cep	U CrB		MR Cyg
	3		2		1,2	2	1,3	1,2	2		1,4
	3 *	4 *	3 *	4 *							
$P'_{\text{orb}}$	40373 22	18680 15	25500 00	13000 00	9214 39	50407 61	21808 53	19974 44	31954 55	18680 15	8343 24
$e'$	0 56	0 51	0 15	0 33	0 22	0 35	0 54	0	0 24	0 30	0
$\omega'$	4 38	0 03	3.77	1 47	2 57	1 04	0 42	0	2 69	1 67	0
$\tau'$	20201 70	28717 62	7347 38	15204 81	39897 27	62637 82	5519 42	19358 91	30958 56	35900 12	27544 13
$a' \sin i'$	768 70	60 33	874.28	496 24	125 97	13750 00	2036 02	422 17	1008 97	787 82	71.24
$f(m_3)$	0 0110	0 00003	0 0391	0 0266	0 0009	40 0128	0 7034	0 0076	0 0845	0 0567	0 0002
$m_3$ 90°	0 46	0 05	1 09	0 94	0 19	45 18	2 79	2 34	1 73	1 48	0 34
60°	0 54	0 06	1 29	1 11	0 22	66 92	3 49	2 71	2 05	1 75	0 39
30°	1 04	0 11	2.53	2 15	0 40	325 69	9 42	4 86	4 14	3 46	0 69
dev Fr					0 0045		0 0157		0 0070		
dev O-C	0 0019		0 0063		0 0048	0 0147	0 0201	0 0123	0 0072		0 052

Remarks :

- 1 : results were obtained by subtracting quadratic ephemeris
- 2 : TOT type results (they were obtained using all kind of minima times)
- 3 : PGPE type results (they were obtained using only photoelectric and photographic minima)
- 4 : PE type result (they were obtained using only photoelectric minima)
- 5 : final results contain the exclusion of the data out of the  $3\sigma$ 
  - $P'$  (in days),  $e'$ ,  $\omega'$  (in rad),  $\tau'$  (in JD-2400000),  $a' \sin i'$  are the orbital elements of the orbit of the eclipsing binary in the triple (multiple) system.
  - $f(m_3)$  : mass function of the third (fourth) body
  - $m_3$  : mass of the third (fourth) body at different inclinations (in Solar Mass)
  - dev Fr are the standard deviation of the data from the Fourier-fitting
  - dev O-C are the standard deviation of the data from the LITE orbit of above elements

3. CONCLUSION

We studied a group of stars for achieving modified or new interpretation of their O-C diagram. We could verify and/or improve the orbital elements of third or further satellites of the eclipsing systems studied by earlier authors. For several cases our results are in contradiction with those of the previous investigators.

LITE is a very attractive explanation for rather curious O-C behaviour. However, one must be very careful at the decision whether the results can be acceptable (physically reasonable) or not. We can say for every star that the crucial point will be the new observational data in the future. We shall continue this investigation, by taking into account further eclipsing binaries, and also by monitoring the systems studied here for checking whether the newer times of minima will follow our theoretical approximation or not.

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