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A REVIEW ON THE EFFECT OF FINITE TARGET THICKNESS IN ABLATIVE RAYLEIGH TAYLOR-INSTABILITY

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Abstract. We consider the ablative Rayleigh-Taylor instability at a steady ablation front of a finite thickness target. In the present work a more accurate prediction is obtained based on the sharp boundary model (SBM). One the advantages of the simple model here presented is that offers the possibility of analyzing the effect of finite target thickness, which is stabilizing for long-wavelength modes. On the other hand, it shows that the cut-off wave number k_c is independent of target thickness.

1. INTRODUCTION

In Inertial Confinement Fusion (ICF), a cold target material is accelerated by a hot, low density plasma (Lindl 1995). The surface (front ablation) between the heavy and light materials is Rayleigh-Taylor (RT) unstable similar to the interface between two fluids of constant densities ρ_2 and ρ_1 subject to a gravitational field $\vec{g}$ pointing toward the lighter fluid ρ_1 (Rayleigh 1883, Taylor 1950). The Rayleigh–Taylor instability (RTI) in ICF is one of the principal physical processes inhibiting the achievement of inertial fusion. In this work, the linear stability analysis of accelerated ICF targets is carried out including the finite thickness effect.

2. LINEAR STABILITY ANALYSIS

In This section, we study the RTI in a finite thickness target. We consider the following planar configuration: the heavier target material of constant density ρ_2 (heavy-fluid region, $-d < y < 0$) is accelerated against the lighter blow-off plasma of constant density ρ_1 (blow-off region, $y > 0$). The acceleration $\vec{g}$ is pointing in the y direction $\vec{g} = g\ \vec{u}_y$. In order to determine the growth rate of unstable perturbations, one needs to solve the hydrodynamic equations describing mass, momentum and energy conservation. We use one-fluid equations in the frame moving with the unperturbed ablation front (Piriz et al. 1997):

$$
\begin{aligned}
&\frac{\partial\rho}{\partial t} + \vec{\nabla}\cdot(\rho\vec{v}) = 0 \\
&\rho\frac{\partial\vec{v}}{\partial t} + \rho\left(\vec{v}\cdot\vec{\nabla}\right)\ \vec{v} = -\vec{\nabla}p + \rho\vec{g} \\
&\frac{\partial}{\partial t}\left[\rho\left(\frac{v^2}{2}+\varepsilon\right) + \vec{\nabla}\cdot\left[\rho\ \vec{v}\left(\frac{v^2}{2}+\frac{5}{3}\varepsilon\right)+\vec{Q}\right]\right] = \rho\vec{v}\cdot\vec{g} \\
&\vec{Q} = -\kappa_D\vec{\nabla}\varepsilon
\end{aligned}
$$

Where we have assumed that the fluid is an ideal gas and ρ, $\vec{v}$, and ε are, respectively, the density, the velocity, and the specific internal energy of the fluid. Moreover, $p = (2/3)\,\rho\varepsilon$ is the fluid pressure, Q is the energy flux corresponding to a diffusive process of transport, and $\kappa_D = \chi\varepsilon^n$ is the thermal conductivity, where for electronic heat conduction in laser fusion n=5/2. The stability of such an equilibrium is investigated by imposing a small perturbation of the interface between the two fluids in the following form: $\psi = \psi_o + \delta\psi$, where ψ_o is the unperturbed value and $\delta\psi << \psi_o$ is a small perturbation $\delta\psi \prec \exp(\gamma t + qy + ikx)$); q represents the longitudinal wave number and γ is the instability growth rate.

2. 1. RESULTS AND DISCUSSION

The growth rate of the ablative RT can be calculated using a sharp boundary model (SBM). We perform a first-order perturbation expansion on the above equations. After a tedious but simple algebra (see e.g. Piriz et al. 1997), we obtain the following dispersion relation that includes the effect of finite shell in terms of the density jump r_D

$$
\gamma^2 + \frac{kv_2\left[3+\coth(kd)\right]}{r_D+\coth(kd)}\gamma + kg\left(\frac{k\ {v_2}^2}{g\ r_D} - A_T\right)\left[\frac{1+r_D}{r_D+\coth(kd)}\right] = 0
$$

Where v_2 is the fluid speed ahead the front (ablation speed) and A_T is the Atwood number: $A_T = \frac{1-r_D}{1+r_D}$

The former equation can be easily solved for obtaining the instability growth rate:

$$
\begin{aligned}
\gamma \quad &= \quad \sqrt{\left\{\frac{kv_2}{2}\left[\frac{3+\coth(kd)}{r_D+\coth(kd)}\right]\right\}^2 - kg\left(\frac{k\ {v_2}^2}{g\ r_D} - A_T\right)\left[\frac{1+r_D}{r_D+\coth(kd)}\right]} \\
&- \quad \frac{kv_2}{2}\left[\frac{3+\coth(kd)}{r_D+\coth(kd)}\right]
\end{aligned}
$$

We can also find the following analytical expression for the cut-off wave number k_c:

$$
\frac{k_c{v_2}^2}{g} = \frac{(1-r_D)\,r_D}{1+r_D}
$$

This one is exactly the result obtained for the semi-infinite target model: the cut-off wave number is invariant to target thickness.

It may be interesting, from a qualitative point of view, taking the limit $kd >> 1$, where, $\coth(kd) \rightarrow 1$. Our model reduces to semi-infinite target one, and the relation dispersion from Piriz is recovered:

$$\gamma = \sqrt{\left(\frac{2kv_2}{r_D+1}\right)^2 - kg\left(\frac{k\ v_2{}^2}{g\ r_D} - A_T\right)} - \frac{2kv_2}{r_D+1}$$

By introducing a simple corona model based on diffusive process of energy transport, the parameter r_D as a function of the Froude number results (Piriz et al. 1997, Betti et al. 1996):

$r_D = \left(\frac{4\alpha}{5F_r}\right)^{\frac{2}{5}}$, where α is the dimensionless parameter $\alpha = \frac{k\ v_2{}^2}{g}$. We can plot the dispersion relations which are shown in figures 1-2, for Fr=5. The solid lines plotted are the results that would be obtained with our model (finite thickness target); dashed lines are the results from Piriz model (semi-infinite target). The dimensionless parameter kd is varied as indicated in the figures for kd=1 and kd=2. The solid line (figure 1) show a considerable reduction of the growth rate due to finite thickness for long-wavelength perturbations. Also notice that the cut-off wave number is the same in both models. Therefore, the finite thickness does not affect the cut-off wave number.

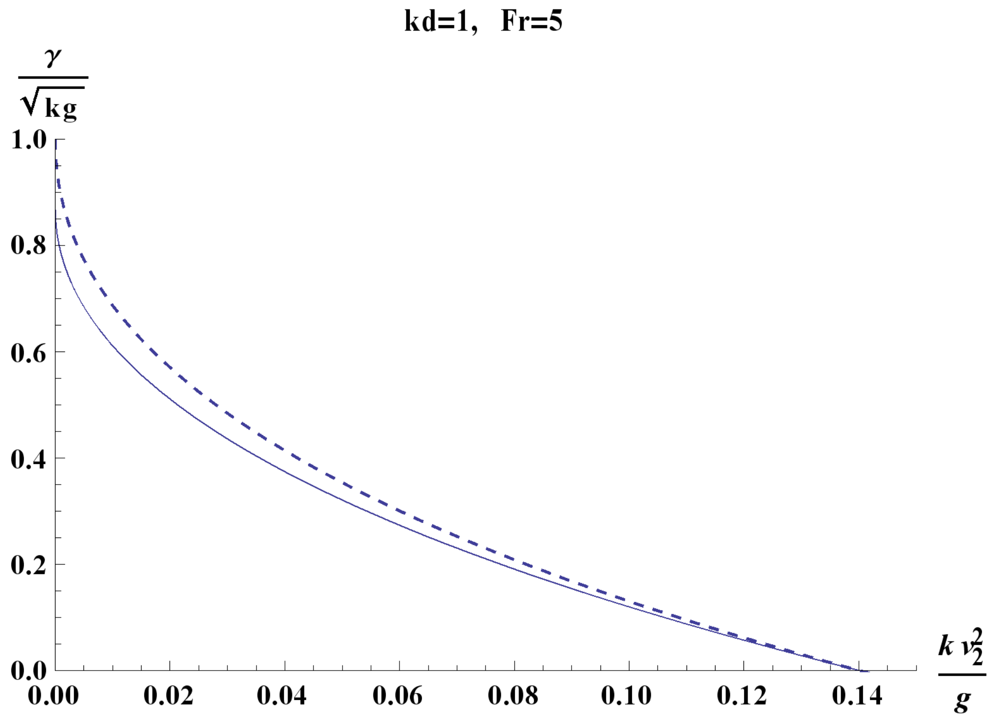

Figure 1: Dimensionless normalized growth rate versus dimensionless normalized wave number. Dashed line corresponds to the Piriz's results; solid line corresponds to our model. Finite-thickness corrections are most important for long-wavelength perturbations.

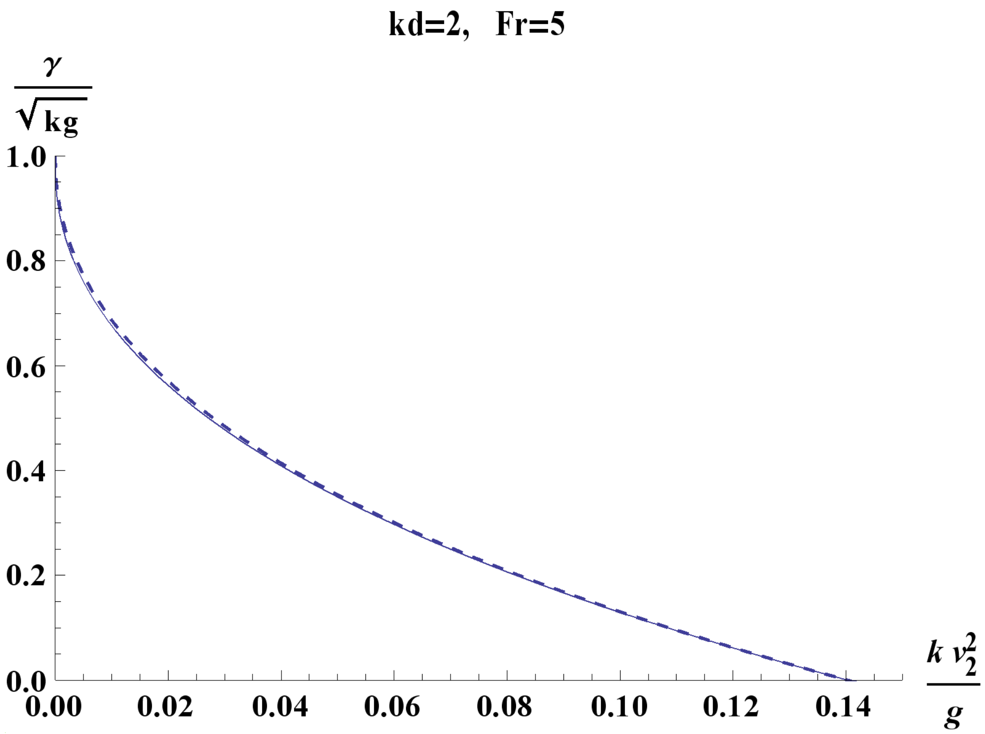

Figure 2: Dimensionless normalized growth rate versus dimensionless normalized wave number. Dashed line corresponds to the Piriz's results; solid line corresponds to our model. For kd$\geq$ 2 and Froude number Fr=5 there is an excellent agreement with semi-infinite target model.

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