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COUPLED GAS ACOUSTIC AND ION ACOUSTIC WAVES IN WEAKLY IONIZED PLASMA

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Abstract. Gas acoustic and ion acoustic modes are investigated in a collisional, weakly ionized plasma in the presence of un-magnetized ions and magnetized electrons. In such a plasma, an ion acoustic mode, driven by an electron flow along the magnetic field lines, can propagate almost at any angle with respect to the ambient field lines as long as the electrons are capable of participating in the perturbations by moving only along the field lines. The electron-ion collisions are shown to modify the previously obtained angle dependent instability threshold for the driving electron flow. The inclusion of the neutral dynamics implies an additional neutral sound mode which couples to the current driven ion acoustic mode, and these two modes can interchange their identities in certain parameter regimes.

1. INTRODUCTION

Weakly ionized plasmas with three or more plasma/gas species include a plethora of effects that are normally absent in the simpler one-fluid or two-fluid models. If such a plasma-gas mixture is placed in an external magnetic field, it may result in the un-magnetized ions while the electrons may remain magnetized. An example of such plasma-gas mixture is the solar atmosphere where, for protons, the ratio Ω_i/ν_i changes between $\sim 10^{-4}$ and 0.2 for the altitudes $h = 0$ km and $h = 1000$ km, respectively. In the same time, for electrons the value of the ratio Ω_e/ν_e varies between $\sim 10^{-2}$ and 16 (see e.g. Vranjes et al. 2007a, 2008).

In our recent works Vranjes and Poedts (2006a,b), Vranjes et al. (2006), the excitation of the dust-acoustic (DA) and ion-acoustic (IA) modes by a parallel or perpendicular (with respect to the ambient magnetic field) flow of the light plasma species (electrons in the case of IA mode and ions in the DA case) has been studied. The driving current instability is associated with the collisions and represents a purely fluid effect. In the IA case, the electron collisions are shown to be necessary for the instability to develop, while the ion collisions introduce an angle dependent dip in the profile for the threshold value of the flow. In the present study we investigate the behavior of an ion acoustic (IA) mode driven by an equilibrium electron flow along the magnetic field lines, in the presence of the ion-neutral, electron- neutral, and electron-ion collisions. Neutrals can also be perturbed due to various external reasons (a common situation in the lower solar atmosphere, in the terrestrial atmosphere, etc.), or due to interaction with the perturbed plasma species. This introduces an additional (neutral) gas acoustic (GA) mode.

2. MODEL AND RESULTS

Acoustic perturbations of the neutral gas component include the friction force due to collisions with the ions of the form $-m_n n_{n0}\nu_{ni}(\vec{v}_{n1}-\vec{v}_{i1})$, with the momentum conservation due to friction $m_\alpha n_{\alpha 0}\nu_{\alpha\beta}=m_\beta n_{\beta 0}\nu_{\beta\alpha}$. Using the neutral momentum and continuity equations, one finds that the perturbed velocity of neutrals is coupled to the perturbed ion velocity as $v_{n1}=[i\omega\nu_{ni}/(\omega\omega_n-k^2v_{Tn}^2)]v_{i1}$. This describes an acoustic wave in the neutral gas which is coupled to the ions due to collisions. We have assumed small longitudinal perturbations of the form $\sim\exp(-i\omega t+i\vec{k}\cdot\vec{r})$, propagating in an arbitrary direction $\vec{r}$ which makes an angle ψ with the magnetic field vector $\vec{B}_0=B_0\vec{e}_z$. Here, $\omega_n\equiv\omega+i\nu_{ni}$, $v_{Tn}^2=\kappa T_n/m_n$, and v_{i1} is the perturbed ion velocity in the same $\vec{r}$ direction.

The ion momentum equation reads

$$m_i n_{i0}\frac{\partial v_{i1}}{\partial t}=-en_{i0}\frac{\partial\phi_1}{\partial r}-\kappa T_i\frac{\partial n_{i1}}{\partial r}-m_i n_{i0}\nu_{in}(v_{i1}-v_{n1})+\mu_L\frac{\partial^2 v_{i1}}{\partial r^2}. \qquad (1)$$

The ions are not magnetized, $\nu_{in}\gg\Omega_i$, and hence there is no Lorentz force term ($\sim\vec{v}_i\times\vec{B}$) in this equation. Here, $\mu_L\nabla^2\vec{v}_i$ is an effective viscosity term that accounts for the Landau damping (D'Angelo et al. 1979). The parameter μ_L is chosen to quantitatively describe the well-known properties of the Landau effect. Taking note of the fact that the ratio between the attenuation length δ and the wavelength λ is independent both of the wavelength and the plasma density n, and dependent on the electron/ion temperature ratio $\tau=T_e/T_i$ in a prescribed way, one writes $\mu_L=m_i n_{i0}v_s\lambda/(2\pi^2\delta/\lambda)$. Here, $v_s=(c_s^2+v_{Ti}^2)^{1/2}$ is the ion sound speed, $c_s^2=\kappa T_e/m_i$, while the dependence of the ratio δ/λ on τ is such that the attenuation is strong at $\tau\approx 1$ and weak for higher values of τ. The 'fluid' attenuation length can be expressed by the following approximate fitting formula to give the same damping as the corresponding kinetic expression: $d\equiv\delta/\lambda\approx 0.2751+0.0421\,\tau+0.089\,\tau^2-0.011785\,\tau^3+0.0012186\,\tau^4$. Using the ion continuity we obtain:

$$\frac{n_{i1}}{n_{i0}}=\frac{ek^2}{m_i(\omega\omega_2-k^2v_{Ti}^2)}\,\phi_1. \qquad (2)$$

Here $\omega_2=\omega+i(\nu_{in}+\mu_0k^2)+\nu_{in}\nu_{ni}\omega/(\omega\omega_n-k^2v_{Tn}^2)$, $\mu_0=\mu_L/(m_i n_{i0})$.

Electron collisions with both neutrals and ions should be included in the momentum equation which is of the form

$$m_e n_e\left[\frac{\partial\vec{v}_e}{\partial t}+(\vec{v}_e\cdot\nabla)\vec{v}_e\right]=en_e\nabla\phi-en_e\vec{v}_e\times\vec{B}-\kappa T_e\nabla n_e$$
$$-m_e n_e\nu_{en}(\vec{v}_e-\vec{v}_n)-m_e n_e\nu_{ei}(\vec{v}_e-\vec{v}_i). \qquad (3)$$

The electrons are assumed to be magnetized, i.e., $\Omega_e>\nu_{ei}+\nu_{en}$. In this case, their perpendicular dynamics is negligible and the electron continuity yields

$$\frac{n_{e1}}{n_{e0}}=\frac{\nu_e\omega_0}{\nu_e\omega_0+ik_z^2v_{Te}^2}\left(\frac{iek_z^2}{m_e\nu_e\omega_0}\,\phi_1+\frac{\nu_{en}}{\nu_e}\frac{k_z^2}{k\omega_0}\,v_{n1}+\frac{\nu_{ei}}{\nu_e}\frac{k_z^2}{k\omega_0}\,v_{i1}\right). \qquad (4)$$

Here, $\omega_0 = \omega - k_z v_0$, $\nu_e = \nu_{ei} + \nu_{en}$, and the electron inertia terms are omitted implying a Doppler shifted wave frequency below the electron collision frequency, and a Doppler shifted wave phase velocity below the electron thermal speed. We have assumed a constant equilibrium electron flow $\vec{v}_0 = v_0 \vec{e}_z$. Using the quasi-neutrality, we finally obtain the following dispersion equation:

$$\left[\omega^2 - k^2 c_s^2 \left(1 + \frac{1}{\tau}\right)\right] (\omega\omega_n - k^2 v_{Tn}^2) = -\nu_{ni}\omega^2 \left(\nu_{in} + \nu_{en}\frac{m_e}{m_i}\right)$$
$$-i(\omega\omega_n - k^2 v_{Tn}^2) \left\{\omega(\nu_{in} + \mu_0 k^2) + \frac{m_e}{m_i}\left[\omega\left(\nu_{ei}\left(\frac{k^2}{k_z^2} - 1\right) + \nu_{en}\frac{k^2}{k_z^2}\right)\right.\right.$$
$$\left.\left. -k_z v_0(\nu_{ei} + \nu_{en})\frac{k^2}{k_z^2}\right]\right\}. \tag{5}$$

We discus first the case of static neutrals. From Eq. (5) we find a modified ion sound mode that is unstable provided that

$$V > \frac{\kappa}{1+\nu}\left(1 + \frac{1}{\tau}\right)^{1/2} \left[\nu\left(\frac{1}{\kappa^2} - 1\right) + \frac{1}{\kappa^2} + \frac{\mu}{\widehat{\nu}_{en}}\left(\frac{\widehat{\nu}_{en} b}{(\mu\tau)^{1/2}} + \frac{(1 + 1/\tau)^{1/2}}{\pi d}\right)\right]. \tag{6}$$

Here, $V = v_0/c_s$, $\widehat{\nu}_{en}$ is normalized to $\omega_r \simeq kc_s$ and should be chosen in accordance with the model. From $\nu_{en} = \sigma_{en} n_{n0} v_{Te}$ and $\nu_{in} = \sigma_{in} n_{n0} v_{Ti}$, we obtain $\widehat{\nu}_{in} = \widehat{\nu}_{en} b/(\mu\tau)^{1/2}$, where $\mu = m_i/m_e$, $b = \sigma_{in}/\sigma_{en}$, $\kappa = k_z/k$, $\nu = \widehat{\nu}_{ei}/\widehat{\nu}_{en}$, $\widehat{\nu}_{ei}$ is also given in units of kc_s. For larger values of ν, the second and third terms in Eq. (6) are reduced, the latter implying that the minimum in the threshold velocity profile reduces. This behavior is presented in Fig. 1 (left) for a hydrogen plasma in a neutral hydrogen gas (Vranjes et al. 2007b). Here, $\tau = 1$, and it follows that $\sigma_{en} = 2.5 \cdot 10^{-19}$ m^2, $\sigma_{in} = 9.24 \cdot 10^{-19}$ m^2 at the temperature of 1 eV, and we have chosen $\widehat{\nu}_{en} = 30$. For these parameters $d \simeq 0.4$, and $\widehat{\nu}_{in} = 2.6$. The electron-ion collisions drastically reduce the velocity threshold at small angle of propagation (i.e., for k_z/k close to 1). The limit $\nu \sim 1$ still implies a much larger neutral number density, and the neutral hydrogen gas can be assumed as a static non-moving background. This is due to the much larger collision cross section for Coulomb collisions.

When the neutral gas is perturbed or when the perturbations in the ionized component induce (due to the friction) perturbations of the neutral background, Eq. (5) in dimensionless form becomes

$$\widehat{\omega}^2 = 1 + \frac{1}{\tau} - \frac{\widehat{\omega}^2 \widehat{\nu}_{ni}\widehat{\nu}_{en}\left[b(\mu\tau)^{-1/2} + \mu^{-1}\right]}{\widehat{\omega}(\widehat{\omega} + i\widehat{\nu}_{ni}) - \frac{\mu_n}{\tau_n}}$$
$$-i\left\{\widehat{\omega}\left[\widehat{\nu}_{in} + \frac{(1+\tau^{-1})^{1/2}}{\pi d}\right] + \frac{1}{\mu}\left[\widehat{\omega}\widehat{\nu}_{en}\left(\nu\left(\frac{1}{\kappa^2} - 1\right) + \frac{1}{\kappa^2}\right) - \frac{V\widehat{\nu}_{en}(1+\nu)}{\kappa}\right]\right\}. \tag{7}$$

All frequencies are normalized to kc_s, and we have introduced new parameters $\tau_n = T_e/T_n$, $\mu_n = m_i/m_n$. The number of parameters can be reduced by using as before $\widehat{\nu}_{in} = \widehat{\nu}_{en} b/(\mu\tau)^{1/2}$, and from the momentum conservation in the friction force terms here we have $\widehat{\nu}_{ni} = \mu_n X\, b\widehat{\nu}_{en}/(\mu\tau)^{1/2}$, $X = n_{i0}/n_{n0}$. As a demonstration Eq. (7) is solved for $\tau = 4$, $\tau_n = 4$, $\mu = 1838$, $\mu_n = 1$, $\widehat{\nu}_{en} = 30$, and $V = 30$, $n_{e0} = n_{i0} =$

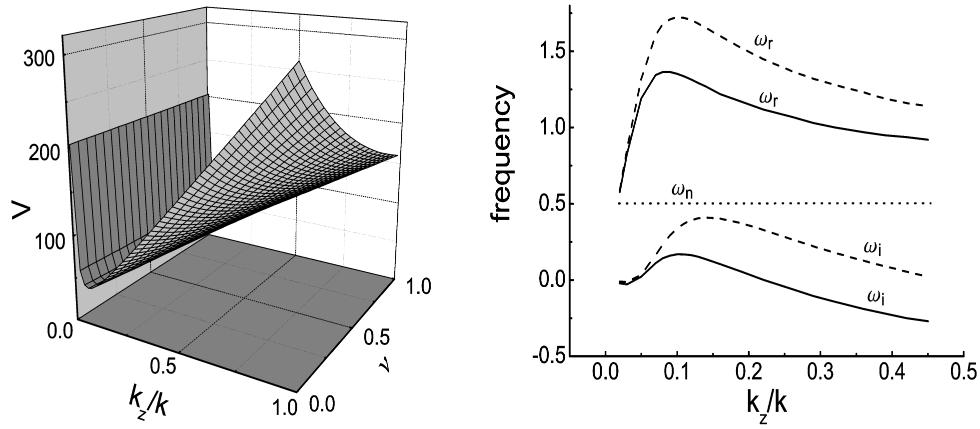

Figure 1: *Left:* The normalized threshold velocity $V \equiv v_0/c_s$ for the instability in terms of k_z/k and $\nu \equiv \nu_{ei}/\nu_{en}$. The unstable values are located above the surface. *Right:* Normalized real ω_r and imaginary ω_i parts of the angle dependent ion acoustic frequency for $\nu = 0$ (full lines) and $\nu = 0.916$ (dashed lines), in terms of k_z/k. The dotted line ω_n describes the neutral acoustic mode.

$6 \cdot 10^{16}$ m^{-3} and $n_{n0} = 10^{19}$ m^{-3}, which yields $X = 0.006$ and $\nu = 0.916$. The results are presented in Fig. 1 (right), with the remarkable angle dependent behavior of the IA mode. The neutral acoustic mode has nearly a constant frequency $\omega_n \simeq 0.5$ and a very small decrement $\simeq -0.005$. The real and imaginary parts of the ion acoustic mode frequency change in the presence of electron-ion collisions ν although the ionization is relatively small. Note that the assumed value of $\widehat{\nu}_{en} = 30$ in principle fixes the wavelength of fluctuations. For example, assuming $T = 5000$ K, one has $c_s = 6.4$ km/s and $\nu_{en}/(kc_s) = 30$ implies a wavelength of 0.7 m.

Some additional features of the mode behavior is given in Vranjes et al. (2007b). These include the interchange of their identities in certain parameter regimes.

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