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ON THE CONNECTION BETWEEN THE MULTIFRACTALITY AND THE PREDICTABILITY FROM THE AURORAL INDEX TIME SERIES

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Abstract. We analyze the Auroral Electrojet (AE) index data from the period of Solar minimum and maximum, with respect to their predictability and intermittency. Neural networks are employed to predict the behavior of the AE- data for the different intermittent intervals, as well as recurrence plots where these intervals are visualized. We also compute the multifractal singularity spectrum as additional evidence for the existence of intermittency, and show that this spectrum is independent of the Solar activity.

1. INTRODUCTION

The Auroral Electrojet (AE) index is a measure of the aurora-related magnetic activity around the auroral oval of the Northern hemisphere. The horizontal field variations are measured at observatories along the auroral zone, and are meant to estimate the total amplitude of the ionospheric current system. This index is introduced by Sugiura and Davis, 1966, and is supposed to monitor the auroral substorm events. Since their discovery, AE index time series have been extensively studied as part of the effort to understand the dynamics of magnetosphere-ionosphere coupling. In addition to the obvious plasma physics based theories, more complex systems based approaches have attracted considerable interest. Depending on the measured AE index characteristics, one has found signatures of self-organized criticality (e.g. Kozelov et al., 2004), turbulence (Consolini et al., 1996), and low dimensional chaos (Vassiliadis, 1990, Athanasiu et al., 2001). In this brief report, we present results from an ongoing study of the AE index predictability and its connection to the intermittent intervals of the same data. We analyze the AE index minute data downloaded from the Kyoto database (http:// swdcwww. kugi. kyoto-u. ac. p/wdc/Sec3. html). We were interested in the years 1979 and 1986, since these years had the highest and lowest Solar activity, respectively. Particularly, we analyze the data from disturbed conditions, i.e., when an auroral substorm was identified. First, we use neural networks to identify the time scale of the AE index predictability. Then, we apply recurrence plot analysis which clearly shows different dynamic regimes, where the boundaries between these regimes coincide with the time scales on which the predictability from the neural network was achievable. These boundaries also match with the increase of the wavelet coefficient energy calculated for the intermittent part of the AE index time series. Finally, we show by applying multifractal singularity spectrum analysis that our time series have the signatures of the intermittent turbulence.

2. NEURAL NETWORKS

Neural networks (NNs) are often used in the analysis of the AE index (see e.g. Hernandez et al., 1993, Pallocchia et al., 2008). Generally speaking, NNs have a training and test set of data, where the first set establishes the NN, and the other is classified through the NN. NNs consist of the input, output, and layers of neurons in between. These layers learn the relationship between the input and output (training set), such that later a new input (the test set) can be added in the network and, according to the rules of the NN, can produce a new output. We use the AE index data as the input, the AE index data sampled with a time lag as the output, and one layer of 4 neurons between the input and the output. We iterate the neural network until the error $\epsilon = \frac{1}{2}\sum_{i=1}^{M}(O_i - \widetilde{O}_i)^2 \rightarrow 0$, where O_i is the desired output, $\widetilde{O}_i$ is the real output from the network and M is the number of the AE index samples. After such a network has learnt the relationship between two AE index time series, where there is a time lag between their measurements, we can predict the future of other AE index time series after the same time lag. Fig. 1(a) and Fig. 1(b) shows the original and the predicted AE index time series for the period of the Solar minimum, when the length of the series was 900 min.

For the time series lengths ≥ 900 min, different NN has to be used. In other words, this means that the data has changed its character and the rules that governed the NN up to 900 min, can not be applied anymore. Very similar results are obtained for the data in the Solar minimum as well.

3. THE LINK BETWEEN THE RECURRENCE PLOT AND THE INTERMITTENCY

Recurrence plot (RP) analysis was introduced in Eckmann et al., 1987, and has been extensively used in the last twenty years. The essential steps of the method are as follows: First the phase space is reconstructed by time-delay embedding (see Takens, 1981), where vectors $\mathbf{x}_i$ $(i = 1, \ldots, T)$ are produced. Then a $T \times T$ matrix consisting of elements 0 and 1 is constructed. The matrix element (i, j) is 1 if the distance is $\|\mathbf{x}_i - \mathbf{x}_j\| \leq R$ in the reconstructed phase space, and otherwise it is 0. The recurrence plot is simply a plot where the points (i, j) for which the corresponding matrix element is 1 are marked by a dot. The radius R is fixed and chosen such that a sufficient number of points are found to reveal the fine structure of the plot. On the RP, periodic states are visualized as diagonal, continuous lines, where the distance between the lines indicate the period. Vertically and horizontally spread black areas represent states with short laminar behavior, while abrupt changes in the dynamics as well as extreme events cause white areas in the RP. On Fig. 1(d) we show RP for the Solar minimum, obtained for the embedding dimension $D = 6$. We see that RP consists of some small black rectangular as well as white areas, which indicate changes in the dynamics (probably from laminar to intermittent states). Particularly, for $\tau \sim 10^3$ we see that mid-rectangular area finishes and exchanges with the different pattern for the larger time scales. On Fig. 1(c), we also plot

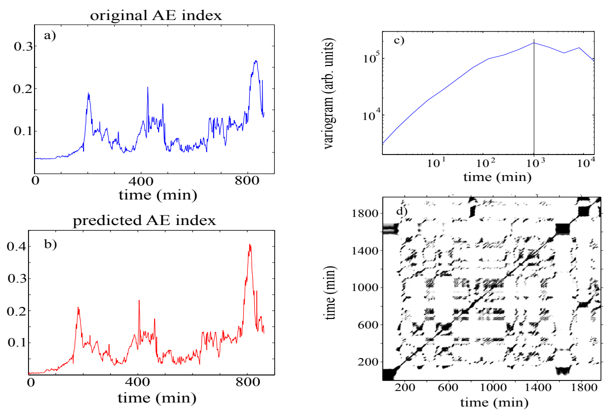

Figure 1: Data for the Solar minimum: a) original , b) predicted from NN, c) variogram, d) recurrence plot.

variogram ($V(\tau) = \frac{1}{2\ (M-\tau)} \sum_{t=0}^{M-\tau} (s(t+\tau) - s(t))^2$, where $s(t)$ is the AE index time series, and τ is a time lag), and notice that the greatest change in the variogram occurs for $\tau \sim 10^3$, which is also in agreement with the results of the neural network analysis. The differences between RP patterns are even better exposed in Fig. 2(b), where the RP for the AE index of the Solar maximum is plotted. In Fig. 2(a) on the upper part of the plot, the first 2000 points of the original AE index data for the Solar maximum are plotted. By applying mexican-hat wavelet analysis and after keeping the wavelet coefficients for the smallest scales, we are able to exclude the intermittent part of the signal, which we plot below the original signal. We also plot energies of the wavelet coefficients, which correspond to the intermittent signal. It seems like the boundaries between different patterns on the RP correspond to the peaks of the wavelet energies of the intermittent signal.

Additional evidence that AE index is intermittent is provided by direct computation of the multifractal spectrum of generalized dimensions and the singularity spectrum. First, we calculate generalized dimension spectrum $D_q = \frac{1}{q-1} \frac{\log M_q(r)}{\log r}$ when $r \to 0$. Here $M_q = \sum p_i^q$ and p_i is the probability that some point is in the box i. The generalized dimension is independent of q if the time series is monofractal. In Fig. 2 (c), we plot D_q vs. q for the AE index data at Solar maximum. Since D_q varies with q, we conclude that the time series is multifractal. Further, we define the singularity spectrum $f(\alpha)$ vs. α from the Legendre transform $(q-1)D_q = \inf\{q\alpha - f(\alpha)\}$. Here α is the Hölder exponent characterizing the local variation around a given point in the time series and $f(\alpha)$ can be thought of as the fractal dimension of the set of points which are characterized by the particular exponent α. For a monofractal, $f(\alpha)$ vanishes except for the the value of α corresponding to the (mono-) fractal dimension (see Hilborn et al., 2000). However, Fig. 2(d) shows a considerable spread in α-values for which $f(\alpha) > 0$. Very similar plots are produced for the AE index data for the Solar minimum. Since the singularity spectrum and spectrum of generalized dimensions show very little variation between the Solar maximum and Solar minimum data, it seems like solar activity does not represent a significant influence on the intermittency properties of the turbulence which give rise to the fluctuations in the AE index.

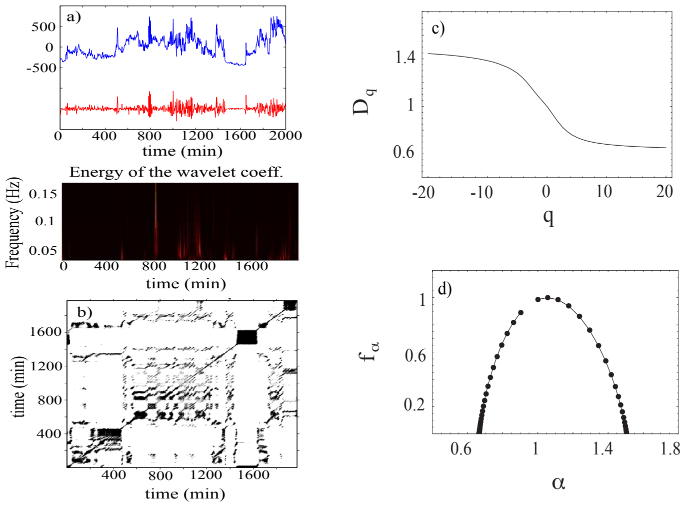

Figure 2: Data for the Solar maximum: a) original and intermittent time series with energy of the wavelet coefficients, b) Recurrence plot, c) Generalized dimension, d) Singularity spectrum.

4. CONCLUSION

We have analyzed AE index data from periods of Solar minimum as well as maximum. We investigate the predictability of the data by using neural networks, and conclude that the same neural network can not be used for all the lengths of the time series. This indicates that different dynamics govern different part of the time series, which can be clearly demonstrated via the recurrence plots. We further contemplate that these different dynamics are due to intermittent burst, which can be pictured through the wavelet energy of the small-scale wavelet coefficients. Singularity spectrum and generalized dimension also confirm that intermittent turbulence governs the dynamics of our data.

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