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A POSSIBLE EXTENSION OF THE SAITOU-TAKEUTI-TANAKA ONE-ZONE STELLAR MODEL

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Abstract. The Saitou-Takeuti-Tanaka one-zone stellar model is being extended by considering that the matter in the core-surrounding shell consists of a mixture of ideal gas and radiation. The dynamical system describing the behaviour of the shell is being tackled via a bifurcation analysis of the equilibria of the corresponding linearized system.

1. BASIC EQUATIONS

Since their apparition the one-zone stellar models proved their ability to predict many features of regular and irregular pulsating star light curves (see e.g. Baker 1966, Rudd & Rosenberg 1970, Stellingwerf 1972, Auvergne *et al.* 1981, Auvergne & Baglin 1985, Auvergne 1986, Stellingwerf *et al.* 1987, Saitou *et al.* 1989).

Consider a one-zone stellar model (Saitou *et al.* 1989) of mass M, featured by a rigid core of constant radius R_c and constant luminosity L_c, and by an envelope of mass m_s. Let the stellar radius ($R \gg R_c$) be time-dependent. Furthermore, we shall consider (e.g. Baker 1966) that the matter in the shell is a mixture of ideal gas and radiation.

We shall resort to the following well-known equations of stellar structure (see Kippenhahn & Weigert 1991):

$$\partial^2 r/\partial t^2 = -4\pi r^2(\partial P/\partial m) - Gm/r^2, \tag{1}$$

$$\partial l/\partial m = -c_V(\partial T/\partial t) + (\delta/\alpha)(P/\rho^2)(\partial\rho/\partial t), \tag{2}$$

$$l = [16\pi\sigma r^2/(3\kappa\rho)](\partial T^4/\partial r), \tag{3}$$

namely motion equation, energy equation, and radiative energy transport equation in the diffusion approximation, respectively. The notations are: m = mass of the sphere of radius r, l = luminosity, P = pressure, T = temperature, ρ = density, c_V = specific heat at constant volume, κ = opacity, σ = Stefan-Boltzmann constant, G = Newtonian gravitational constant.

The goal of our paper is twofold: to establish the equations which describe the behaviour of the envelope in time, and to classify the equilibrium states with respect to the interplay among the parameters of the model.

Like Saitou *et al.* (1989) we introduce the following relations referring to the core-surrounding shell:

$$\partial P/\partial m = -P/m_s, \quad \partial l/\partial m = (L - L_c)/m_s, \quad \partial T^4/\partial r = -T^4/R, \tag{4}$$

where P, L, T stand for the pressure, radiative energy flux and temperature in the shell, respectively. Substituting (4) in (1)-(3), these ones turn to

$$\partial^2 R/\partial t^2 = -4\pi R^2 P/m_s - GM/R^2, \tag{5}$$

$$(L - L_c)/m_s = -c_V\,(\partial T/\partial t) + (\delta/\alpha)(P/\rho^2)(\partial\rho/\partial t), \tag{6}$$

$$L = 16\pi\sigma R T^4/(3\kappa\rho). \tag{7}$$

The hydrostatic equilibrium state implies $P_0/m_s = GM/(4\pi R_0^4)$, the subscript "0" corresponding to the equilibrium model with $X = X_0$, $X \in \{R, L, P, T, \rho, \kappa\}$, $L_0 = L_c$. As regards the properties of the stellar matter, we consider the following formulae for the equation of state and opacity law, respectively:

$$\rho = \rho_k P^\alpha T^{-\delta}, \tag{8}$$

$$\kappa = \kappa_k P^{\kappa_P} T^{\kappa_T}, \tag{9}$$

where ρ_k and κ_k are constants.

Let x and z be the relative variations of radius and pressure, respectively: $R = R_0(1+x)$, $P = P_0(1+z)$. We consequently derive

$$\begin{aligned}
\rho &= \rho_0(1+x)^{-3},\\
T &= T_0(1+x)^{3/\delta}(1+z)^{\alpha/\delta},\\
\kappa &= \kappa_0(1+x)^{3\kappa_T/\delta}(1+z)^{\kappa_P+\kappa_T\alpha/\delta},\\
L &= L_0(1+x)^{4(3+\delta-\kappa_T)/\delta}(1+z)^{[\alpha(4-\kappa_T)-\delta\kappa_P]/\delta},
\end{aligned}$$

with $\rho_0 = 3M/(4\pi R_0^3)$, $T_0 = \rho_k^{1/\delta}\rho_0^{-1/\delta}P_0^{\alpha/\delta}$, $\kappa_0 = \kappa_k P_0^{\kappa_P} T_0^{\kappa_T}$, and $L_0 = 16\pi\sigma R_0 T_0^4/(3\kappa_0\rho_0)$. With these relations, equations (5)-(7) lead to

$$\dot{x} = y, \tag{10}$$

$$\dot{y} = \Omega^2\left[(1+x)^2(1+z) - (1+x)^{-2}\right], \tag{11}$$

$$\begin{aligned}
\dot{z} = &-(3/\alpha)(1+x)^{2-3/\delta}(1+z)^{-\alpha/\delta}\left[(1+x)^{3(1/\delta-1)}(1+z)^{\alpha/\delta-1} + \gamma - 1\right]y - \\
&-\varepsilon(\delta/\alpha)(1+x)^{-3/\delta}(1+z)^{1-\alpha/\delta}\left[(1+x)^{4(3+\delta-\kappa_T)/\delta}(1+z)^{[\alpha(4-\kappa_T)-\delta\kappa_P]/\delta} - 1\right],
\end{aligned} \tag{12}$$

where $\Omega = GM/R_0^3$, $\gamma = c_P/c_V$, $\varepsilon = (\delta/\alpha)L_0/(m_s c_V T_0)$.

Note that in the case of a purely ideal gas shell we have $\alpha = \delta = 1$ (e.g. Kippenhahn & Weigert 1991). With these values, from equations (10)-(12) we retrieve those established by Saitou *et al.* (1989, eqs. (15a-c)).

2. EQUILIBRIA OF THE LINEARIZED SYSTEM

Here we shall limit ourselves to the classical first step in investigating the behaviour of a dynamical system: searching for the equilibria. This is a hard task in the case of equations (10)-(12), so we shall resort to the analysis of the equilibria of the linearized system. Linearizing (10)-(12), we get

$$\dot{x} = y \tag{13}$$

$$\dot{y} = 4x + z \tag{14}$$

$$\dot{z} = Ax + By + Cz \tag{15}$$

where $A = -4\varepsilon(3 + \delta - \kappa_T)/\alpha$, $B = -3\gamma/\alpha$, $C = -\varepsilon\left[\alpha(4 - \kappa_T) - \delta\kappa_P\right]/\alpha$, and we have chosen the units such that $\Omega^2 = 1$.

The corresponding characteristic equation reads

$$\lambda^3 - C\lambda^2 - (B + 4)\lambda + (4C - A) = 0. \tag{16}$$

We shall distinguish two main situations. The most general one is $A \neq 4C$. In this case, the only equilibrium is the origin $(x, y, z) = (0, 0, 0)$. It is easy enough to see that this equilibrium is hyperbolic. This is of much help for our analysis, because the local behaviour of the solutions of the linearized system in the neighbourhood of hyperbolic equilibria is the same as for the nonlinear system (Hartman-Grobman theorem). Taking into account (16), we easily see that the origin is a sink (stable equilibrium) for $C < 0$, and a source (unstable equilibrium) for $C > 0$. In case there exist at least one root (16) with negative real part and at least one root of (16) with positive real part, the equilibrium at the origin is a saddle.

A less probable case is $A = 4C$. In this situation the equilibrium at origin is no more hyperbolic, therefore we can say nothing about the behaviour of the solutions of (10)-(12) in the neighbourhood of the origin. However, the fact that $|x|$ and $|z|$ are much smaller than 1 makes us analyze this case, too.

Denoting $q = 4x + z$, we reduce (13)-(15) to the two-dimensional system

$$\dot{q} = Cq + (4 + B)y, \tag{17}$$

$$\dot{y} = q. \tag{18}$$

If $B = -4$, the equilibrium $(q, y) = (0, y_e)$, with $y_e = constant$, is stable for $C < 0$, and unstable for $C > 0$. Let us focus on the case $B \neq -4$; the only corresponding equilibrium is the origin $(q, y) = (0, 0)$, too. We differentiate several situations.

If $B < -4 - C^2/4$, the origin is a stable focus for $C < 0$, and an unstable focus for $C > 0$.

If $-4 - C^2/4 \leq B < -4$, the origin is a stable node for $C < 0$, and an unstable node for $C > 0$.

If $B > -4$, the origin is a saddle.

Finally, if $C = 0$, the equilibrium at origin is stable (a centre) for $B < -4$, and unstable for $B > -4$.

Of course, this preliminary bifurcation analysis can be made go deeper by tackling the whole possible interplay among the parameters A, B, C, especially for the most probable case of the hyperbolic equilibrium. Such a hard investigation (given the expression of A, B, C) will be done elsewhere.

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