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THE USE OF ITERATION FACTORS IN THE LINE FORMATION PROBLEM WITH SPATIAL VARIATIONS IN PROFILE FUNCTION

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Abstract – In a previously published paper a simple and fast–convergent method using iteration factors is developed to solve two–level line transfer problem in a constant property medium. In this paper a spatial variation of the profile function is taken into account and a new iteration factors family is considered.

1. Introduction

In a previously published paper (Simonneau and Atanacković-Vukmanović, 1991), henceforth reffered to as Paper I, a simple and fast–convergent iterative method is developed to solve the two–level line transfer problem. The method represents an efficient way to accelerate Λ iteration scheme by the use of the quasi-invariant depth -dependent functions, so- called iteration factors.

In the two–level atomic case where the explicit form of the line source function enables a straightforward derivation of the angular and frequency integrated moments of the radiative transfer (RT) equation, the factors are defined as the ratios of the relevant radiation field intensity moments. Computed at the beginning of each iteration step from the formal solution of the RT equation with the given (old) source function, the factors are then used to close RT moment equations and, hence, to give new radiation field, i.e. new source function.

In order to check the convergence properties of the method in Paper I we considered the case of the constant property medium whose exact solutions are well known. The assumption made throughout the paper about depth independence of the profile function φ_x enabled frequency integration of the differential RT equation over profile function and definition of the corresponding closure relations.

Here we consider the case when φ_x is some specified function of depth. The iterative procedure described in Paper I can be directly applied. The only difference is that in getting RT equation moments the operator $\int[..]dx$ and not $\int[..]\varphi_x dx$ must be used.

2. The line formation with spatial variations in Doppler width

For the sake of simplicity in presentation we shall consider the time independent RT equation for a one- dimensional, planar and static medium with no background opacity. Using the standard notation, RT equation has the form:

$$\mu\frac{d}{d\tau}I(\mu,x,\tau) = \varphi(x,\tau)[I(\mu,x,\tau) - S(\tau)] \quad , \tag{1}$$

where the line source function $S(\tau)$ under the assumption of complete redistribution is given by:

$$S(\tau) = \varepsilon B(\tau) + (1-\varepsilon)\int_{-\infty}^{\infty}\varphi(x,\tau)J(x,\tau)dx \quad . \tag{2}$$

For pure Doppler broadening, $\varphi(x,\tau)$ is given by the Gauss normalized profile function:

$$\varphi(x,\tau) = \frac{1}{\sqrt{\pi}\delta(\tau)} e^{-x^2/\delta(\tau)^2} \quad , \tag{3}$$

where $x = (\nu - \nu_0)/\Delta\nu_D^*$ is frequency displacement from line center in some standard frequency interval $\Delta\nu_D^*$ (Doppler width at some reference depth point), and the parameter $\delta(\tau)$ is given by:

$$\delta(\tau) = \frac{\Delta\nu_D(\tau)}{\Delta\nu_D^*} \quad . \tag{4}$$

In order to solve eqs. (1) and (2) with $\varphi(x,\tau)$ given by (3) we proceed like we did in Paper I. After getting the angular moments of eq. (1) by applying the operators $\int ...d\mu$ and $\int ...\mu d\mu$, we perform their integration over line frequencies $\int_{-\infty}^{\infty} dx$ to get:

$$\frac{dH}{d\tau} = J_\varphi - S = \varepsilon(J_\varphi - B) \tag{5a}$$

$$\frac{dK}{d\tau} = H_\varphi \quad . \tag{5b}$$

In the above expressions we used eq.(2) and the following notation for the frequency moments:

$$H = \int H_x dx \ , \quad K = \int K_x dx \ , \quad J_\varphi = \int J_x \varphi_x dx \ , \quad H_\varphi = \int H_x \varphi_x dx \quad .$$

The system of two differential equations (5) with four unknown intensity moments needs two additional relationships to be solved.

Here we consider the most straightforward way to close the above system, i.e. the following iteration factors family:

$$F = \frac{K}{J_\varphi} \quad , \quad f_H = \frac{H}{H_\varphi} \quad . \tag{6}$$

The two factors take into account the anisotropy of the radiation field as well as the repartition of the energy over frequencies within the line profile. The factors are to be computed according to their definitions using the formal solution of eq. (1). Given the factors, the system (5) that can be rewritten using (6) as:

$$\frac{dH}{d\tau} = \frac{\varepsilon}{F} K - \varepsilon B \tag{7a}$$

$$\frac{dK}{d\tau} = \frac{1}{f_H} H \quad , \tag{7b}$$

is easily solved for the unknown moments H and K. New source function

$$S(\tau) = \varepsilon B + (1 - \varepsilon)\frac{K(\tau)}{F(\tau)}$$

is then used to start the next iteration step.

3. Results and discussion

For the case of pure Doppler broadening, depth dependence of the profile function $\varphi(x,\tau)$ means a depth variation in Doppler width $\Delta\nu_D(\tau)$. We shall make some tests of the above described procedure and the proposed family of the iteration factors specifying the spatial variations in Doppler width $\Delta\nu_D(\tau)$ in the form similar to the one given in Rybicki and Hummer (1967) and in Athay (1972). Namely, we consider two cases: (a) "cool" and (b) "hot" surface layer defined, respectively, by:

$$\Delta\nu_D(\tau) = 2 - e^{-A\tau} \tag{8a}$$

$$\Delta\nu_D(\tau) = 1 + e^{-A\tau} \tag{8b}$$

For $\Delta\nu_D^*$ we take Doppler width at great optical depths. The behaviour of parameter $\delta(\tau)$ for both cases and for three different values of $A = 10^{-1}, 10^{-2}$ and 10^{-3} is shown in Fig.1.

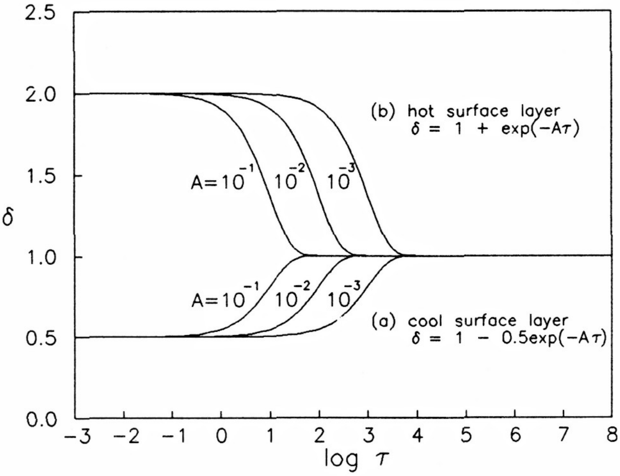

Fig.1. Parameter $\delta(\tau)$ characterizing spatial variation of the profile function

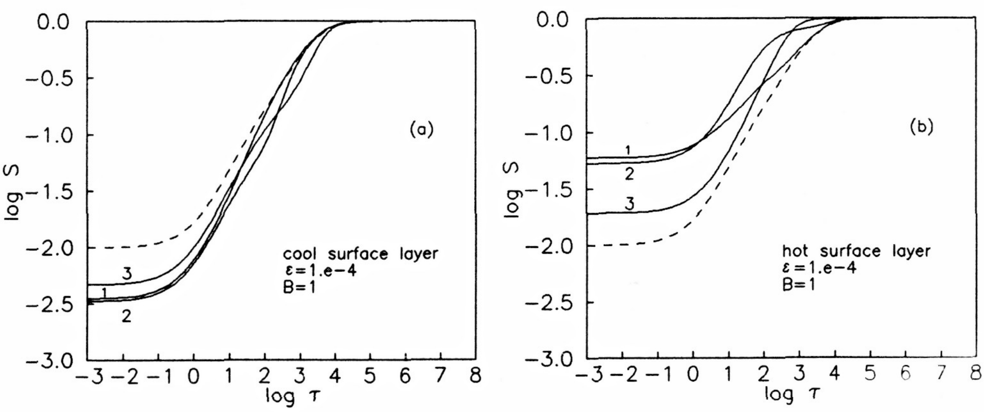

Fig.2. Source function vs. $log\tau$ in variable property media

The source functions obtained for the two cases (a) and (b) (Fig.1.) and for the semi-infinite medium with $\varepsilon = 10^{-4}$ and $B = 1$ are shown in Fig.2. The case of depth independent profile function ($\delta = 1$) is presented by the dashed curve. Curves labeled by $n = 1, 2, 3$ correspond to three different values of coefficient $A = 10^{-n}$ in depth varying Doppler width (eq.(8)).

The effects of depth–variations in profile upon the line source function are widely studied by Rybicki and Hummer (1967), Athay (1972). The behaviour of $S(\tau)$ obtained by our method and shown in Fig.2. is in a good agreement with the solutions considered therein. For a "cool" surface layer in all three cases $S(\tau)$ lies below the value $S(\delta = 1)$ due to a greater escape probability in the line wings (the profile φ_x is much narrower than in the $\delta = 1$ case). Besides, the thermalization length increases when $\Delta\nu_D$ grows deeper in the medium. The rate of convergence grows also with the thermalization length (see Table 1). With relation to 39 iterations needed for the convergence in $\delta = 1$ case, cases in which the increase of $\Delta\nu_D$ happens deeper require more iterations. In a "hot" surface layer, values of $S(\tau)$ are much larger than in the case $\delta = 1$ due to wider wings in the absorption profile that intercept the emergent photons. The so–called reflector efect on the radiation flowing up implies a decrease in thermalization length. The corresponding rate of convergence is very high (only 8 iterations are needed for the case $A = 10^{-3}$).

Table 1. Number of iterations necessary to achieve the convergence ($\varepsilon = 10^{-4}, B = 1$, (a) "cool" surface layer, (b) "hot" surface layer)

A	(a)	(b)
10^{-1}	42	28
10^{-2}	59	13
10^{-3}	73	8

The above results are obtained by the use of the most straightforward family of iteration factors. According to the results given in papers by Atanacković- Vukmanović and Simonneau (1991, 1993), we expect that the significant improvement in the rate of convergence would be achieved by an explicit treatment of the non-local (active) part of the radiation field.

References

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