

**Natural broadening of atomic spectral lines and
generalized operator of electromagnetic interaction**

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It is well known that in quantum electrodynamics (QED) the ultraviolet divergences can be removed from S-matrix and the Green's functions but cannot be removed from quantities characterising the temporal evolution of the processes since regularisation of the scattering matrix leads to a situation in which divergent terms automatically appear in the Schrödinger equation and the Tomonaga-Schwinger equation. For this reason these equations are of only formal importance to quantum field theory.

Thus the description of temporal evolution of QED systems, which determines the energy and other characteristics of bound states, encounters unregularizable ultraviolet (UV) divergences. This leads, in particular, to difficulties in finding a consistent QED description of natural broadening of spectral line profiles in atomic systems. This problem, however, has only minor importance for most applications. Recent progress in heavy-ion-beam techniques has made it possible to begin experimental studies of multiply-charged heavy ions whose interaction with their own radiation field can no longer be considered as a small perturbation and, accordingly, the radiative broadening (RB) of spectral lines may be much greater than other types of broadening. The difficulties that a researcher encounters in describing the RB of spectral lines and that are associated with surface divergences were discussed in [1,2]. This problem seems can be resolved by means of T-matrix equation that is the basic dynamical one in the framework of the method of relativistic T-matrix [3]. It has been shown that the boundary condition for this equation can be chosen in such a way that no difficulties involving surface divergences arise in the description of the temporal evolution of QED systems [2].

In this paper it has been offered a generalised operator of electromagnetic interaction (EMI). This means that EMI is regarded as a non-local one. It determines that boundary condition. In this modified case the theory maintains its gauge invariance. The generalised interaction operator involves a form-factor. In such a way this idea opens new possibilities in a temporal description of QED systems. It is discussed the problem of phenomenological investigation of the form-factor by means of analysis of the natural broadening (NB) of

atomic spectral lines. It has been shown that the study of the NB in heavy highly ionised atoms touches the more delicate aspects of regarded QED problems. Experimental alternatives to study such accurate effects are discussed. The ion-laser-beam technique allows to be promising method in the question.

On the basis of the method of relativistic T-matrix the formula of the NB of spectral line profiles has been derived [2]. The later is determined by the probability of a photon being emitted with energy ω as the atomic system goes from the i -th excited state to the ground one:

$$\frac{dW_{ii}}{d\omega} = A\omega \sum_{\lambda} \int d\Omega_k \left| \frac{\langle 1, k, \varepsilon_{\lambda} | M(z) | i \rangle}{z - E_i - C_i(z)} \right|^2, \quad (1)$$

where A is a normalisation form-factor, k and ε_{λ} are, respectively, the photon's momentum and polarisation. The ground-state energy E_i already incorporates the correction due to the interaction of the atom in the ground state with the vacuum. Equation (1) holds when the contribution of interference terms can be ignored. If within the limits of the spectral line profile we can ignore the z -dependence of $C_i(z)$ we can put:

$$C_i(z) = C(E_i) = \Delta E_i + i \frac{\Gamma_i}{2}, \quad (2)$$

where ΔE_i and Γ_i can be interpreted, accordingly, as a shift in energy caused by the interaction of the atom with the vacuum and the width of the energy level. But if we don't ignore the z -dependence we obtain dramatically changed picture [4]. Not the actual surface divergences appear. To overcome this difficulty we have chosed a boundary condition determined by the minimal scale of certain, in particular QED, interaction. The T matrix is defined as follows [2]:

$$\begin{aligned} \langle n_2 | T(z) | n_1 \rangle &= i \int_0^{\infty} d\tau \exp(iz\tau) \langle n_2 | \tilde{T}(\tau) | n_1 \rangle, \\ \langle n_2 | T(t_2 - t_1) | n_1 \rangle &= \exp(iE_{n_2} t_2) \langle n_2 | \tilde{S}(t_2, t_1) | n_1 \rangle \exp(-iE_{n_1} t_1), \end{aligned} \quad (3)$$

where $\langle n_2 | \tilde{S}(t_2, t_1) | n_1 \rangle$ is the probability amplitude that if at $t \rightarrow -\infty$ the state of the system was $|in; n_1\rangle$ then the interaction in the system begins at time t_1 , finishes at t_2 and at $t \rightarrow +\infty$ the system is found in the state $|out; n_2\rangle$. By assuming the boundary condition of the $\langle n_2 | \tilde{S}(t_2, t_1) | n_1 \rangle$ exist it have been obtained the difference equation for the T matrix. If the boundary condition has the form:

$$\langle n_2 | \tilde{S}(t_2, t_1) | n_1 \rangle \rightarrow -2\pi i \delta(t_2 - t_1) H_i(t_1), \quad (4)$$

the dynamics determined by that equation proves to be equivalent to the Hamiltonian one. But in this case the UV divergences are actual. In more general case one can define the boundary condition (BC) as following:

$$\langle n_2 | \tilde{S}(t_2, t_1) | n_1 \rangle \rightarrow H_{in}(t_2, t_1),$$

$$H(t_2, t_1) = \int_{t_1}^{t_2} dt \int_{R_1 \cup R_2} d^3\vec{r} d^3\vec{r}' \mathfrak{R}(\vec{r}_2, t; r_1, t_1) F(t_2 - t, t - t_1) \quad (5)$$

where $\mathfrak{R}(x, x) = ie\bar{\psi}(x) \gamma^\mu \psi(x_1) A_\mu((x_2 + x_1)/2) \exp(ie \int_{x_1}^{x_2} d^\mu y A_\mu(y))$ [5]. We call the $H(t_2, t_1)$ as a generalized interaction operator. An advantage of the operator consists of three aspects. First, this allows to remove all difficulties associated with ultraviolet divergences. Second, such operator's form maintains the gauge invariance of the theory, and, third, when this generalized operator is applied to describe such quantities as S-matrix and Green's functions it leads to the usual results being in local QED. In such a way the operator $H(t_2, t_1)$ allows to describe consistently the temporal evolution of atomic QED systems. Therefore, this operator have to provide with fundamental information of electromagnetic interaction. This operator depends on arbitrary form-factor $F(t_2 - t, t - t_1)$. To determine this quantity, we allow, that the more effective way is comparison of theoretical results with experimental data of heavy multicharged ions. The more pure object of study is the natural broadening of atomic spectral lines in such atomic systems since QED effects and relativistic corrections have scale about Z^4 . Note, the high energy experiments are senserable to the Born's diagrams only. But accurate and precision experiments allow to catch QED of 4-th term of perturbation theory and relativistic corrections of the nucleus reflection arising in covariant describing of bound state of atomic systems.

Measurements of hydrogen-, helium-, lithium- and natrium-like ion spectra lead to the unique tests of the multielectron QED effects and relativistic corrections like the anomal self-magnetic momentum and other relativistic. These circumstances stimulis and rise priority of precision experiments which exclusively possible in spectroscopy only. Since the shape of SL is Lorentzian it is equivalently to regard both width life-time of the level. The most precise mean lifetime measurements have been carried out on fast atoms by observing the optical decay in flight following excitation by using laser beam which crossed the ion beam. This technique avoided the measurement complications associated with optical cascades from higher excited levels, since the excitation using the laser was selective. Another advantage of this method arises from the use of the Doppler effect to tune the ions into resonance with the

laser, thus permitting the optical decays to be resolved from the exciting laser wavelength, reducing background effect. Jin and Church has modernised this method: They have applied collinear ion- and laser-beams to measure the mean-lifetime of the $4p^2F^0_{7/2}$ level of ArII. We suppose that this idea can be applied in experiments of heavy multicharged ions mentioned above. QED is senscible for electrons closer to the nucleus. Therefor the most efficient tests of the theory come from the measurement of transitions ending on the (1s) ground state i.e., from the study of the Lyman lines. There are some similar ways to produce excited states of ions. The most useful technique of production is the so-called beam-foil technique: The ions, after being accelerated at large velocities (velocities comparable to those of the K electron of the projectile to optimise ionisation), are sent into light targets where they capture an electron into an excited state. But as far as H-like or bare uranium are concerned their production is extremely difficult, requiring many ionising collisions with small cross sections. More over x-ray measurement has had to deal with substantial Doppler shift corrections. Recently high velocity U^{91+} and U^{92+} ions have been stored in a ring, and deceleration to lower velocities has been proposed. More improved method uses electron-beam ion trap (EBIT). Obtaining x-ray spectrum one can estimate a value of minimal scale of QED [4]. In the case of few-electron ions U^{90+} , U^{89+} , U^{88+} one can use the EBIT technique as well. Excited levels can be "scanned" by means of laser-ion-beam method. Similar experiments were carried out by Cowan et al., but they have measured the $3s_{1/2} - 3p_{1/2}$ transition energy in Na-like $79Pt^{67+}$ only.

In such a way a majority of the experiments is devoted to the determination and closer definition of energy levels. Our work has, in fact, two advantage aspects. First, we closer define the natural broadening, second, the experiment investigations stimulated in this direction will allow to determine a frame of consistent QED. More over such experimental information can be useful in QCD as well and in understanding the basic laws governing the nature.

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