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THE BROKEN TITIUS-BODE LOW FOR THE SOLAR PLANETS AND FOR THE SATELLITE SYSTEMS OF THE JOVIAN PLANETS

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Abstract. In this paper we build and compare TBL models for the planets and dwarf planets in the Solar System, as well as for regular and dwarf satellites of the Jovian planets. The TBL models of the planetary system and satellite systems of the Jovian planets seem to be broken or dualistic. While the solar planets and the regular satellites of the Jovian planets obey TB relations within relative standard error about 10 %, the small inner satellites of the Jovian planets obey their own TB relations with low TBL gradient and standard errors about 5 %. In the cases of Jupiter, Saturn and Neptune the bounds of the planetary rings, marked along the ordinate axes of log *P* (orbital period) envelop the rotational periods of the planet under TBL No. 0. (at Saturn – together with the missing satellite No.1). The rings of Jupiter, Saturn and Neptune correspond to the missing small satellites under No.0 (at Saturn – together with the missing satellite under No.1). However, the rings of Uranus is placed above the known inner satellites. The rings of Uranus may be associated with a missing satellite under No.6

1. INTRODUCTION

The Titius-Bode low (TBL) is a generalization of the Titius-Bode rule found in the last third of 18th century (Goldreich 1965, Dermott 1968, Nietto 1972). The TBL contains 4 basic topics:

First. In every system of orbiting bodies (solar planets, planet satellites, exoplanet systems) the orbital size (or period) grows up with the distance from the gravitational center near-commensurability, following a power function on the number of the orbital size/period. The conventional model of the TBL for the periods *P* today is

(1) $P_n = P_0.P_c^n$ or $\log P_n = \log P_0 + \log P_c.n$,.

Here $n = 1, 2, \ldots, N$ is the period number, P_n is the n^{th} period, P_0 and P_c are constants. P_0 is the scale factor (amplitude coefficient) with meaning of period under number 0 and P_c is the power factor of the regularity (near-commensurability) of the orbital periods. The TBL may be written by exponential function and natural logarithms too (Poveda & Lara 2008, Panov 2009). Other models of the distributions of the planetary distances exists too (Carry 1988, Kotliarov 2008, Ignatovich 2014, Ignatovich 2018).

Second. Every orbiting system has its own constants P_0 and P_c, which might be derived empirically. The best way is the deriving of the regression line of the logarithmic form of the TBL.

Third. In principle the TBL concerns the "regular" objects in the system – relatively massive bodies with almost circular and almost complanar orbits.

The TBL model (Eq.1) has been applied firstly occur unique and conclusive. So,this problem has been settled for decades past. Recently on the solar planets and on the regular satellites of Jupiter (4 satellites), Saturn (7) and Uranus (5) by Dermott (1968). The TBL models TBL models for the regular satellites of Neptune (3) and Pluto (3) were derived on contemporary data by Georgiev (2016).

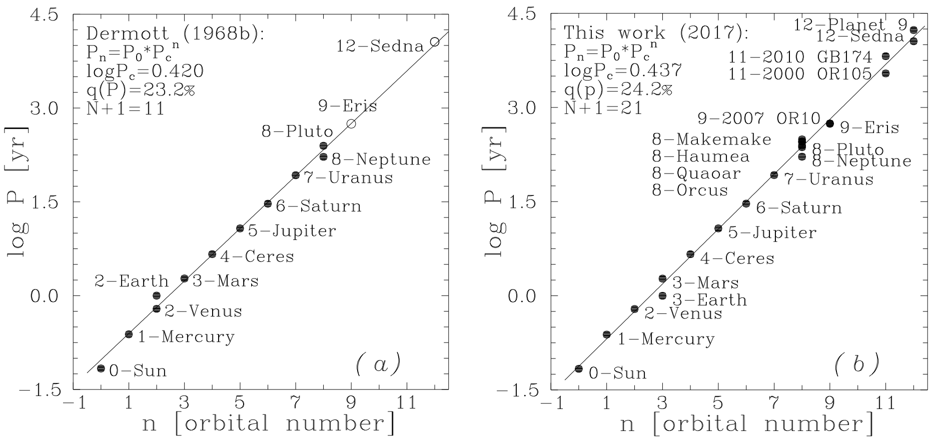


Figure 1: TBL regression models including the rotational period of the Sun under No. 0. (*a*) Classic TB relation (Dermott, 1968). (*b*) Full TB relation, based on contemporary data (Georgiev 2017). The standard errors are large, about 23 % and 24 %, respectively. Note that the Earth seems to be misplaced in both cases.

Regular bodies among the solar planets are 4 Jovian planets and (not obligatory) 4 Terrestrial planets, but Dermott (1968) was used 10 bodies – 8 planets plus Ceres and Pluto (Fig.1,*a*). The upper mentioned regular planet satellites were used in the papers of Georgiev (2016, 1917). The full contemporary model includes 8 planets and 12 dwarf planets with computer defined optimal numbering of the orbits (Fig.1,*b*).In the presented other TB relation of the solar planets are shown in Fig.2.

In the Solar System the TBL reveals one important particularity. The rotational period of the central body, under TBL No. 0, supports approximately the TBL model and this fact is used in the papers of Dermott (1968) и Georgiev (2016, 2017). However, such support is not observing even once among all 8 exoplanet systems with known rotational period of the star (Georgiev 2018). The reason for this particularityis not clear. Though, in the presented paper the rotational periods of the central bodies are not used in the TB regressions (Fig.1, left panel of Fig.2, Fig. 3, Fig.4).

The TBL is not explained conventionally yet (Hayes & Tremaine 1998, Lynch 2003), but it occur fulfilled for all, more than 200 exoplanet systems with known at least 3 exoplanets. About 100 holes in the orbital sequences of these systems, corresponding by presumption to stable orbits, are known (Bovaird & Lineweaver 2013, Bovaird et al. 2015). Georgiev (2018) regarded 30 orbital systems (17 exoplanet systems, 4 versions of the Solar systems, 5 systems of regular planetary satellites and 4 systems of small inner satellites).

Today any example of not-performance of the TBL would be regarded as unexpected and important news. For this reason the interest in the TBL is increasing. So, the TBL is considered as a fundamental natural law, which realizations, especially in the Solar System, need new additional attention.

One basic question is: Whether the constants of the TBL correlate with other, more fundamental parameters of the system, such as the mass of the central body and the total mass of the orbiting bodies? Georgiev (2016, 2017) regards 6 realizations of the TBL in the Solar System and 17 such among the exoplanet systems. The answer is “Yes”. The correlations might be useful for the conventional explaining of the TBL.

Another basic question is: Whether the TBL model, derived through the regular bodies, is valid for small distant bodies? In the Solar system numerous (but not all) dwarf planets with elliptic and arbitrary oriented orbits, even with retrograde orbits, follow well the TBL, derived by the regular bodies (Dermott 1968, Georgiev 2016). This interesting circumstance hampers additionally the understanding of the TBL.

Third question, which is regarding in the presented paper is: Whether the TBL model derived through the regular satellites, is valid for the small inner satellites of the Jovian planets? The results in the presented paper claim “No”. The inner small satellites obey their own TBL model. Thus the details about the TBL become more complicated, as follows.

2. THE BROJEN TITIUS-BODE RELATIONS

The left panel of Fig.2 shows separate TBL models (solid lines) for 2 group planets (dots) with implicit numbering, derived without participation of the solar rotational period. The TBL model for all 8 planets is shown too (dashed line). The positions of the rotational period of the Sun and orbital periods of 3 dwarf planets are shown by circles. The relative standard deviations of the models are 12 %, 15

% and 21 %, respectively. Thus, the TBL model for the regular solar planets seems to be broken in the region of the main asteroid belt.

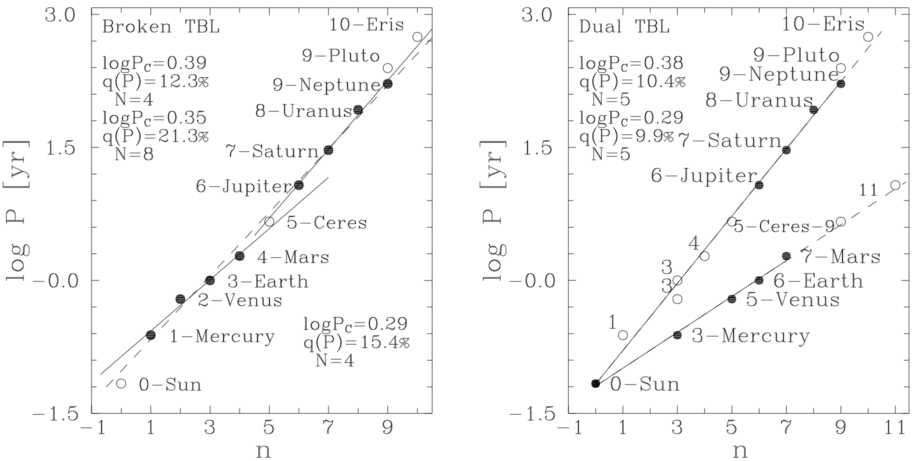


Figure 2: TBL regression models for the regular solar planets (dots) and the positions of other objects (circles). Abscissa axes – numbers of the period (invariant in respect to additive number); Ordinate axes – logarithms of the periods in Earth years. Here log P_c is the TBL gradient, $q(P)$ is the relative standard deviation of the model, N is the number of used periods. (See the text.)

The right panel of Fig.2 shows separate TBL models (solid lines) for the Jovian and Terrestrial planets (dots), derived with participation of the solar rotational period under No.0 (dot). Such kind of diagram is known so far (Rubčić & Rubčić 1995). Here the optimal numbers of the planets are derived by suitable computer method (Georgiev, 2018). The positions of the orbital periods of other objects are shown by circles. The relative standard deviations of both models are about 10 %. Thus, the TBL model for the regular solar planets seems to be dual.

The gradient of the TBL model in the right panel, derived by the Jovian planets (which are just the regular objects in the Solar System) is close to the "standard" one, with log $P_c \approx 0.4$. According to this model and other similar models (Dermott 1968, Georgiev 2016) Venus and the Earth receive one number, here No.3. However, here Venus seems more irrelevant than the Earth.

Though, the gradient, derived by the Terrestrial planets is significantly less, log $P_c \approx 0.3$. (The respective coefficients of near-commensurability of the periods are $P_c \approx 2.5$ and $P_c \approx 1.9$.) Now, in the TBL model for the Terrestrial planets Venus and the Earth receive different numbers, 5 and 6, but dome numbers rest empty.

Figures 3 and 4 show the broken TBL models for the satellite systems of the Jovian planets. There the top placed lines show the down part of the TBL models for the regular satellites (4 of Jupiter, 7 of Saturn, 5 of Uranus and 3 of Neptune). The rotational periods of the planets are not used for the models. The TBL gradients are about 0.3 and the relative standard deviations of these models are about 10 % (Georgiev, 2016, 2018).

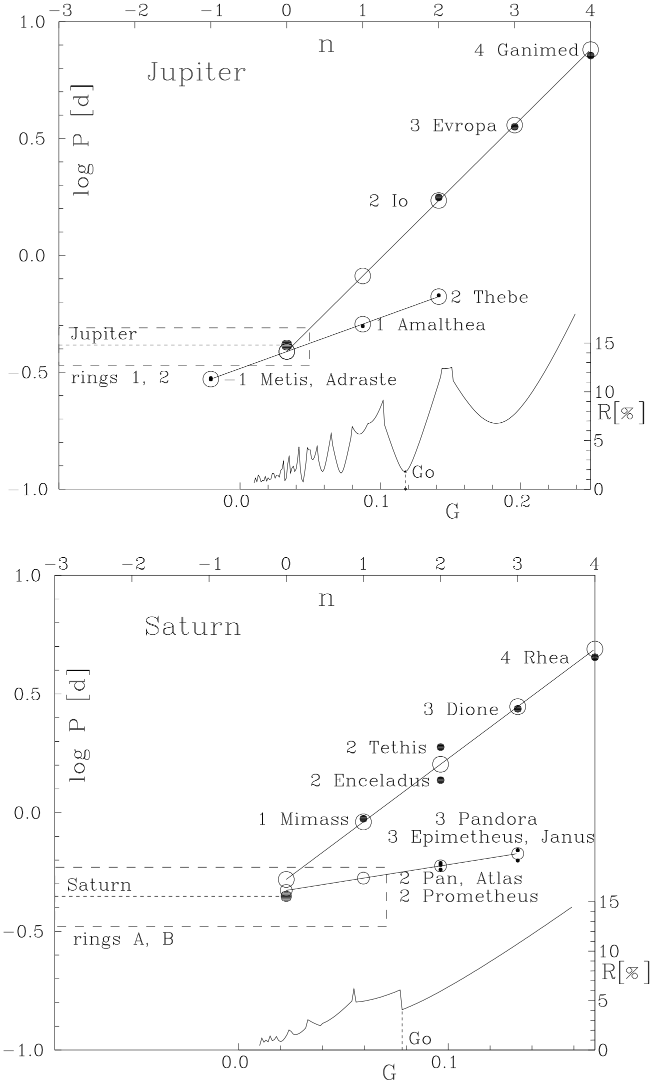
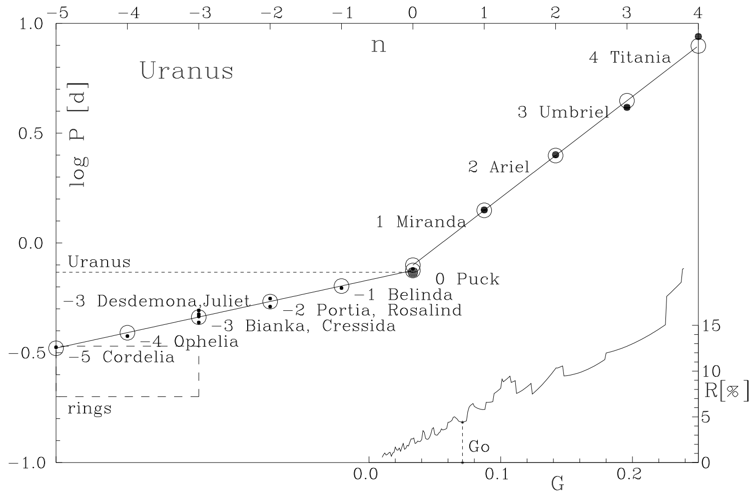


Figure 3: TB relations in the he systems of Jupiter and Saturn. TBL regression modes (solid lines) for the regular planet satellites (large dots), small inner satellites (small dots) and predicted positions of available or missing satellites (circles). The error curves, used for deriving of the optimal gradients G_0 and respective optimal numbering of the inner satellites (Georgiev 2018) are implanted in the right bottom corners. Dashed lines show the approximate bounds of the planetary rings along the axes log P.

The down placed lines in Fig.3 and 4 show the TBL models for the inner satellites with diameters 15-190 km (4 at Jupiter, 6 at Saturn, 10 at Uranus and 6 at Neptune). The places of the minimums of the error curves show the optimal gradients of these TBL models, G_0 = log P_c, about 0.1, The levels of the minimums along the down marked right ordinate axis show that the relative standard deviations of the TBL models for the inner satellites are about 5 %.



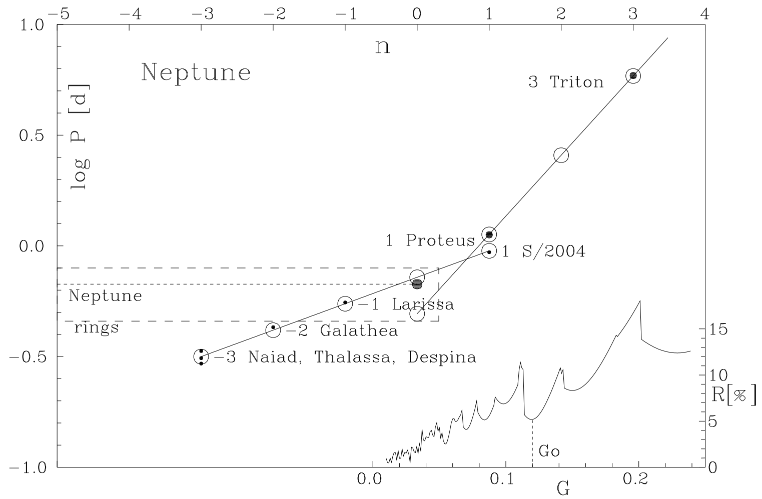


Figure 4: TB relations in the he systems of Uranus and Neptune. See Fig.3.

In Fig.3 and 4 both TBL models of the satellites are shifted horizontally under condition to have common position with the rotational period of the planet under No.0. By this reason some inner satellites receive negative numbers. (The TBL is invariant in respect to addition of a positive or negative number.)

3. RESULTS

The basic results of this work follow.

1. In the cases of Jupiter, Saturn and Uranus the TBL modes, build by the regular satellites predict the rotational period of the planet within error of ± 10 %. However, from this point of view the expected rotational period of the Neptune occur with about 50 % shorter, For the Sun, by the 8 regular planets or for the Jovian planets only, it is longer with about 50 %.

2. The small inner satellites of the Jovian planets obey their own TBL models with low TBL gradient, about 0.1, and small relative standard deviation, about 5 % (Fig.3, 4).

3. The TBL models of the satellite systems of the Jovian planets (Fig.3, 4) seem to be broken like the TBL models for 8 regular planets (Fig.2, left panel).

4. In the cases of Jupiter, Saturn and Neptune the bounds of the planetary rings, marked along the ordinate axes of log *P* (Fig 3 and 4) envelop the rotational periods of the planets and missing small satellites under No. 0. (at Saturn – together with the missing satellite under No.1). However, the rings of Uranus is placed above the known inner satellites. The rings of Uranus may be associated with missing satellite under No.6

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