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RECTANGULAR TORUS DYNAMO MODEL AND MAGNETIC FIELDS IN THE OUTER RINGS OF GALAXIES

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Abstract. Now it is no doubt that some spiral galaxies have regular magnetic fields of several microgauss. Their generation is described by the dynamo mechanism based on joint action of differential rotation and alpha-effect. For most of the galaxies the field generation is described by no-z model, which takes into account that the disc is quite thin. Some of the galaxies have outer rings, where we can suppose the magnetic fields, too. As for these objects, it is more useful to take torus dynamo model, which takes into account the vertical field structure. We describe the field evolution with both of these models for typical values of the parameters. It is quite interesting that the torus dynamo model can give dipolar magnetic fields (no-z model gives the quadrupolar field only).

1. INTRODUCTION

Nowadays it is well-known that several spiral galaxies have the magnetic fields of order of microgauss. Their existence is strongly proved observationally while studying the Faraday rotation measure for the radiowaves. The research of the magnetic fields is done by such modern radio interferometers as LOFAR, VLA, SKA (in future). The magnetic fields are also confirmed by synchrotron emission and another effects (Beck et al 1996; Arshakian et al 2009).

From the theoretical point of view, the magnetic fields are described by the galaxy dynamo mechanism. It is connected with the transition of energy of turbulent motions to the energy of the magnetic fields. It is based on joint action of two different effects. The first one is the differential rotation (characterizng the non-solid rotation of the galaxy). It transforms the radial component of the magnetic field to the angular one. The second part of the dynamo mechanism is alpha-effect, which describes the vorticity of the turbulent motions of the interstellar medium. The alpha-effect transforms the angular magnetic field to the radial one. They compete with the turbulent diffusion which makes the field decay. So the magnetic field generation is a threshold mechanism. If the turbulent motions of the interstellar medium are quite intensive, the magnetic field grows, else the large-scale structures of the field are destroyed.

The dynamo mechanism is usually described by Steenbeck – Krause – Raedler equations (Steenbeck et al 1966). They include the angular velocity of the galaxy and the alpha-effect coefficient. Unfortunately, the equations for the magnetic field are usually quite difficult both for theoretical and numerical analysis, and they are solved using different approximation. One of the most popular models is the no-z approximation which is based on the fact that the galaxy disc is very thin, and the ratio between half-thickness and radius can be assumed as a small parameter (Subramanian & Mestel 1993; Moss 1995; Phillips 2001). So we can assume that the field lies in the equatorial plane. For the derivatives of the field we use algebraic expressions, which allow us to use 2-dimensional model (which can be reduced even to spatially 1-dimensional system for the axisymmetric case).

Some of the galaxies have so-called outer rings, which are situated at some distance from the main disc. The observations show that there is ionized medium in such objects, also they demonstrate the alpha-effect and differential rotation there. So it is quite normal to apologize the magnetic field existence there (Moss et al 2016). As for the models of the magnetic field, we can use no-z approximation, too. However, for the outer ring the typical ratio between the half-thickness of the ring and its half-width cannot be used as a small parameter. So such calculations can give only qualitative results.

Another opportunity is given by the torus dynamo model (Deinzer et al 1993; Brooke & Moss 1992; Mikhailov 2017). It does not assume that the vertical component of the field is negligible, and uses different model assumptions. The field is divided to the angular part and the part which is described by the angular component of the vector potential of the magnetic field. This model describes the field more precisely. Another advantage of the torus dynamo model is that it can describe the field with different structures. The no-z model can give only quadrupolar field configurations and the torus model can describe dipolar magnetic fields, too.

We should also describe non-linear saturation of the field growth. The magnetic field cannot increase infinitely. The field evolution can be restricted from energetic arguments. The field growth is connected with the transition of the energy of the turbulent motions to the field energy. So if the field reaches so-called equipartition value, the growth should stop. The nonlinear effects can cause some important features of the field evolution, mainly for the dipolar one.

Here we present the results of modeling the magnetic fields in the outer rings using both the no-z approximation and the torus dynamo model. We compare the typical field configurations for different cases.

2. BASIC EQUATIONS

The magnetic fields of galaxies contain two different components. The first one is random (or small-scale – it has typical length-scale of several tens parsecs) and the second one is obtained as a result of averaging the field in the regions of 50 – 100 pc. Here we will describe the regular component of the magnetic field, which is described by the Steenbeck – Krause – Raedler equations (Steenbeck et al 1971):

$$\frac{\partial \vec{B}}{\partial t} = \operatorname{curl}\,\left[\vec{V}, \vec{B}\right] + \operatorname{curl}\,\left(\alpha \vec{B}\right) + \eta \Delta \vec{B};$$

where $\vec{B}$ is the large-scale magnetic field, $\vec{V}$ is the velocity of the large-scale motions, α characterizes alpha-effect and η is the turbulent diffusivity coefficient. For the velocity we assume that it is connected with the rotation of the galaxy:

$$\vec{V} = r\Omega\,\vec{e}_{\varphi};$$

where r is the distance from galaxy center and Ω is the angular velocity of the rotation. For the angular velocity we can take the model:

$$\frac{d\Omega}{dr} = -\frac{\Omega}{r}.$$

As for the alpha-effect we can use the following simple model (Arshakian et al 2009):

$$\alpha_0 = \frac{\Omega l^2}{h},$$

where l is the lengthscale of turbulence, and h is the half-thickness of the galaxy disc.

For the turbulent diffusivity we can take that:

$$\eta = \frac{lv}{3},$$

where v is the typical turbulent velocity.

We will assume the field for the following values of the parameters:

$$R - a < r < R + a;$$

$$\frac{a}{k} < z < \frac{a}{k}.$$

This vector equation is a system of three spatially three-dimensional equations. This fact makes quite difficult the possible analysis of the field growth. So we should use some approximations to make the study more simple.

3. NO-Z APPROXIMATION

If we take into account that the field nearly lies in the equatorial plane, we can use the next model of the horizontal components of the field:

$$B_r(r,\varphi,z,t)=B_r(r,\varphi,0,t)\cos\left(\frac{\pi z}{2h}\right);$$

$$B_\varphi(r,\varphi,z,t)=B_\varphi(r,\varphi,0,t)\cos\left(\frac{\pi z}{2h}\right).$$

So the field is assumed quadrupolar. We also shall assume the field axisymmetric (which is quite normal for the outer ring lying in the equatorial plane).

For the alpha-effect we will take the model:

$$\alpha=\alpha_0\frac{z}{h},$$

So we shall have the next equations for the magnetic field (Moss 1995):

$$\frac{\partial B_r}{\partial t}=-\frac{\Omega l^2}{h^2}B_\varphi-\eta\frac{\pi^2k^2B_r}{a^2}+\eta\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(rB_r)\right);$$

$$\frac{\partial B_\varphi}{\partial t}=-\Omega B_\kappa-\eta\frac{\pi^2k^2B_\varphi}{a^2}+\eta\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(rB_\varphi)\right).$$

The field evolution is restricted by the so-called equipartirion field. If the energy density of the magnetic field becomes equal the density of the turbulent motions energy (Arshakian et al 2009):

$$\frac{B_{\max}^2}{8\pi}=\frac{\rho v^2}{2},$$

the field growth should stop. This can be taken into account using the following modification of the equations:

$$\frac{\partial B_r}{\partial t}=-\frac{\Omega l^2}{h^2}B_\varphi\left(1-\frac{B_r^2+B_\varphi^2}{B_{\max}^2}\right)-\eta\frac{\pi^2k^2B_r}{a^2}+\eta\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(rB_r)\right);$$

$$\frac{\partial B_\varphi}{\partial t}=-\Omega B_\kappa-\eta\frac{\pi^2k^2B_\varphi}{a^2}+\eta\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(rB_\varphi)\right).$$

We can measure time in units of a^2/η, distances in R, and the magnetic field in equipartition units. So the equations for the field will be the following:

$$\frac{\partial B_r}{\partial t}=-S_\alpha B_\varphi\left(1-B_r^2-B_\varphi^2\right)-\frac{\pi^2k^2B_r}{4}+\lambda^2\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(rB_r)\right);$$

$$\frac{\partial B_\varphi}{\partial t}=-S_\omega B_\kappa-\frac{\pi^2k^2B_\varphi}{4}+\lambda^2\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(rB_\varphi)\right);$$

where S_α characterizes alpha-effect, S_ω describes differential rotation and $\lambda=\frac{a}{R}$ describes the half-width of the outer ring. For this case we should study the field with following boundary conditions:

$$B_r\big|_{r=1-\lambda}=B_r\big|_{1+\lambda}=B_\varphi\big|_{1-\lambda}=B_\varphi\big|_{1+\lambda}=0.$$

The field evolution will be described by the dimensionless number $Q = S_\alpha S_\omega$.

We describe the results for this case on figure 1. The field for this case is strongly quadrupolar.

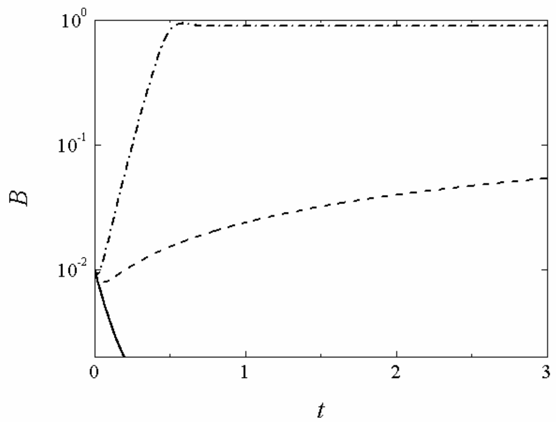

Figure 1: Magnetic field evolution described by no-*z* approximation. Solid line shows *Q=10*, dashed line – *Q=150*, dot-dashed line – *Q=500*.

3. TORUS DYNAMO MODEL

As we have said, it is much more convenient to use the torus dynamo model for the outer ring (Mikhailov 2017; Mikhailov 2018). The field is assumed as a combination of two different components:

$$\vec{B} = B\vec{e}_\varphi + \mathrm{curl}\,(A\vec{e}_\varphi),$$

where B is the angular component of the magnetic field, and A is the angular component of the vector potential.

In the axisymmetric case we shall have the following equations (in nonlinear case):

$$\frac{\partial A}{\partial t} = -\frac{\Omega l^2}{h^2} B\left(1 - \frac{B^2}{B_{\max}^2}\right) + \eta\Delta A;$$

$$\frac{\partial B}{\partial t} = \Omega\frac{\partial A}{\partial z} + \eta\Delta B.$$

Using the same dimensionless numbers, we can obtain the equations:

$$\frac{\partial A}{\partial t} = -S_\alpha B\left(1 - B^2\right) + \eta\Delta A;$$

$$\frac{\partial B}{\partial t} = S_\omega\frac{\partial A}{\partial z} + \eta\Delta B.$$

The field for different Q ($k=2$) is shown on figure 2. It is quite important, that for high D the magnetic field can have quadrupolar symmetry. The field structure for quadrupolar case ($D=150$) is shown on figure 3 – 4. The dipolar field (D=500) is presented on figure 5 – 6.

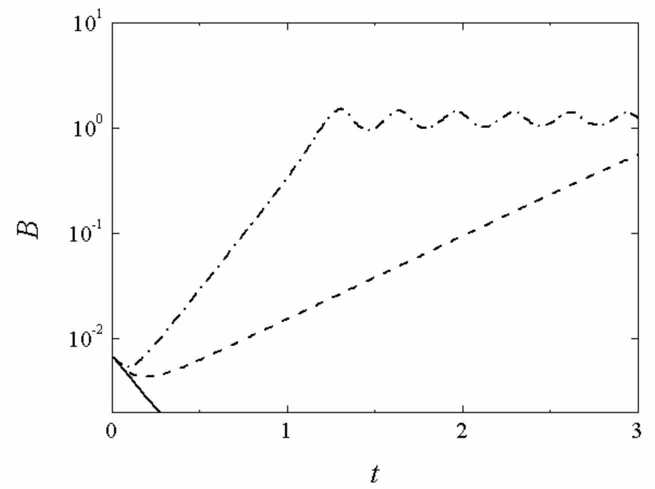

Figure 2: Magnetic field evolution described by torus dynamo model. Solid line shows $Q=10$, dashed line – $Q=150$, dot-dashed line – $Q=500$.

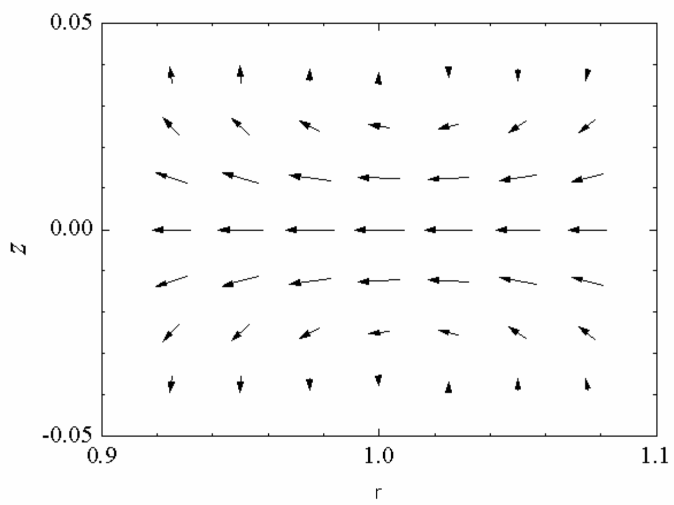

Figure 3: Poloidal magnetic field structure for $Q=150$ (quadrupolar field).

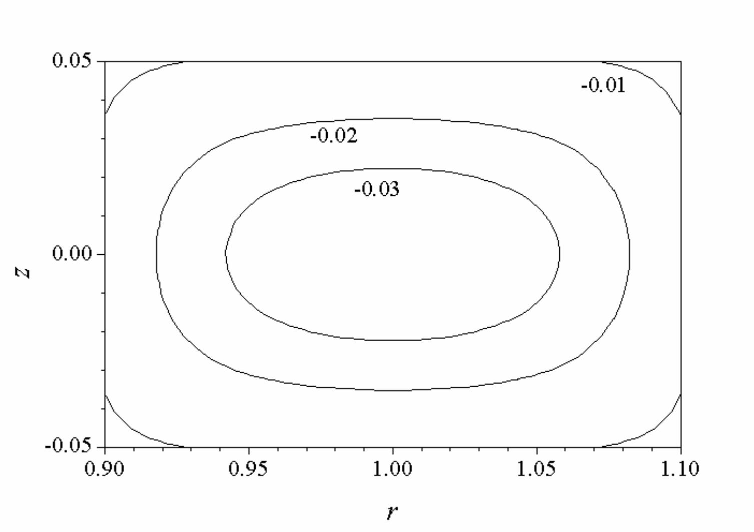

Figure 4: Toroidal magnetic field structure for $Q=150$ (quadrupolar field).

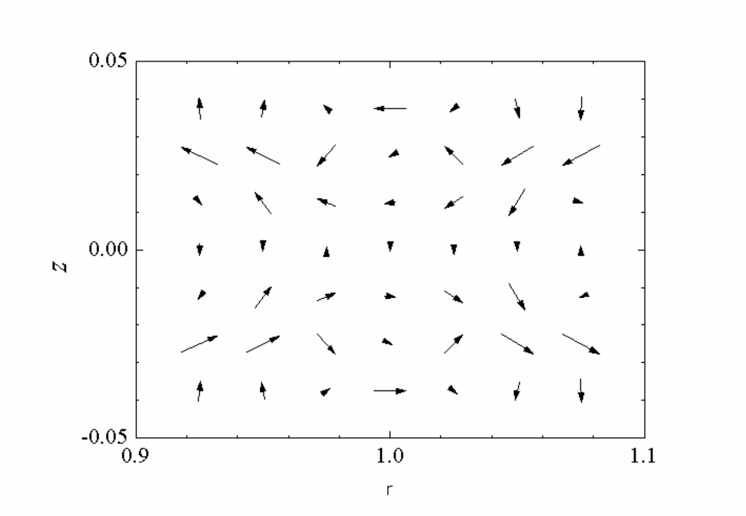

Figure 5: Poloidal magnetic field structure for $Q=500$ (dipolar field).

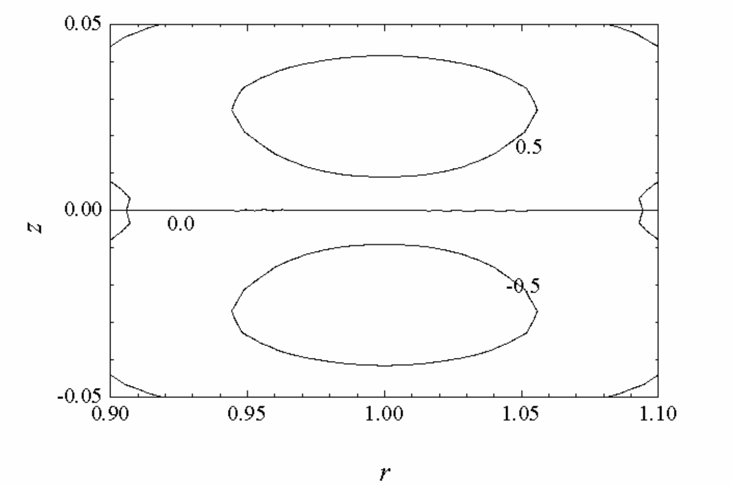

Figure 6: Toroidal magnetic field structure for *Q=500* (dipolar field).

4. CONCLUSIONS

We have studied the magnetic field in the outer rings using both no-*z* approximation and torus dynamo model. The second one is thought to be more realistic. The magnetic field grows slower than for first model. Also the magnetic field can have not only quadrupolar symmetry. If the kinetic energy of the turbulent motion is quite high ($D>153$), the field can become dipolar.

It could be quite important to study the magnetic fields in the outer rings observationally. There are only some observations for the outer ring of NGC4736. In principle, the results are quite correspond to our modelling, but it also would be important to stude some another objects.

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