

INTERACTIVE COMPUTING OF THE EARTH ROTATION MATRIX ACCORDING TO IERS CONVENTIONS

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Abstract. The IAU 1976 Precession Model and the IAU 1980 Theory of Nutation are replaced by the precession-nutation model IAU 2000 A (MHB 2000), based on the transfer functions of (Mathews et al., 2002) at the beginning of 2003. In the proposed work the basic steps for computing of the rotation matrix of the Earth crust (the International Terrestrial Reference System) with respect to a geocentric set of axes tied to the quasars (the International Celestial Reference System) are described. IERS Conventions 1996 (July, 1996, <http://maia.usno.navy.mil/conventions.html>) and IERS Conventions 2003 (November, 2003, <http://maia.usno.navy.mil/conv2000.html>) are used in the basic modules of the interactive program, written using IDL Version 6.0 Win32 (x86). It enables users to obtain the components of the coordinate transformation matrix – the transformation matrices arising from the motion of the celestial pole in the celestial system, from the rotation of the Earth around the axis of the pole, and from polar motion. The program allows taking into account the effect of:

- 1) The Earth Orientation Parameters (EOP), <http://hpiers.obspm.fr/eop-pc/>
 - The celestial pole offsets ($\delta\psi$, $\delta\varepsilon$) or (δX , δY);
 - The rotation angle around the celestial intermediate pole, UT1-UTC;
 - The polar motion of the celestial intermediate pole with respect to the terrestrial crust;
- 2) Tidal gravitational forcing;
- 3) Oceanic forcing;
- 4) Atmospheric forcing.

The results are compared with the interactive site of IERS EOP Product Center.

1. INTRODUCTION

The IERS Technical Note 21 (McCarthy, 1996) revises the most chapters of IERS Technical Note 13. The JPL DE 403 ephemeris (Standish et al., 1995) replaces the DE 200 model of IERS Technical Note 13. The NUVEL NNR-1A Model (DeMets et al., 1994) for plate motion has replaced the Nuvel NNR-1 Model of IERS Technical note 13. An empirical model, used to predict the difference in the celestial pole coordinates between those published by the IERS and those given by the IAU model is added.

The JGM3 model replaces the GEM-T3 and Love Numbers are revised. The subdaily and daily tidal variations in Earth orientation due to the effect of ocean tides are also added. The formulation has been modified to be consistent with IAU/IUGG resolutions.

The IERS Conventions 2003 were finalized in October, 2003, and the printed version will be available as IERS Technical Note 32. It is a continuation of the series of documents begun with the Project MERIT Standards (Melbourne et al., 1983) and continued with the IERS Standards (McCarthy, 1989; McCarthy, 1992) and IERS Conventions (McCarthy, 1996). The most significant changes from previous IERS standards and conventions are due to the incorporation of the recommendations of the 24th IAU General Assembly held in 2000. These recommendations clarify and extend the concepts of the reference systems in use by the IERS and introduce a major revision of the procedures used to transform between them. A new theory of precession-nutation and new Terrestrial Reference Frame (ITRF2000) (Altamini et al., 2002) have been adopted.

In the proposed work are described the basic steps for calculation of the earth rotation matrix that transforms a coordinate \mathbf{M} from the international terrestrial reference frame (ITRF) to the international celestial reference frame (ICRF). The matrix is composed according to IERS Conventions 1996/2003.

An interactive program module on the basis of the FORTRAN routines, provided from IERS web page is performed, using IDL Version 6.0 Win32 (x86). The program allows taking into consideration the effects of the celestial pole offsets, the rotation angle around the celestial intermediate pole, the polar motion of the celestial intermediate pole with respect to the terrestrial crust and the tidal gravitational forcing and the atmospheric forcing,

The results are compared with the matrix, on-line computed in the interactive web site of the IERS EOP Product Center, <http://hpiers.obspm.fr/eop-pc/>. The accuracy is $\sim 10^{-9}$ radians. The Cartesian or spherical point coordinates input/output are possible.

2. A COORDINATE TRANSFORMATION BETWEEN THE CELESTIAL AND TERRESTRIAL REFERENCE SYSTEM ACCORDING TO IERS CONVENTIONS

The matrix performing a transformation of the point coordinates from terrestrial to celestial reference system in the common case is

$$\mathfrak{R}_{ITRS \rightarrow ICRS} = Q(t) \cdot R(t) \cdot W(t) \quad (1)$$

where $Q(t)$ - transformation matrix arising from the motion of the celestial pole in the celestial system, $R(t)$ - transformation matrix arising from the rotation of the Earth around the axis of the pole, $W(t)$ - transformation matrix arising from the polar motion

Table 1. describes the basic steps for computation of the transformation matrix.

The parameters are described in more details in *Appendix A*.

Table 1: Computing the terrestrial to celestial transformation matrix – brief description.

$\mathfrak{R}_{\text{ITRS} \rightarrow \text{ICRS}} = \underline{Q}(t) \cdot R(t) \cdot W(t)$	The polar motion $W(t)$	The rotation of the Earth around the axis of the pole $R(t)$	The motion of the celestial pole in the celestial system $\underline{Q}(t)$
IERS Conventions 1996 Coordinate transformation referred to the Equinox (McCarthy 1996, pp.20-33) http://maia.usno.navy.mil/conventions.html	$W_{\text{Eq1996}}(t) = \mathfrak{R}_1(y_p) \cdot \mathfrak{R}_2(x_p)$	$R_{\text{Eq1996}}(t) = \mathfrak{R}_3(-GAST)$	$Q_{\text{Eq1996}}(t) = P_{\text{Eq1996}}(t) \cdot N_{\text{Eq1996}}(t)$ $N_{\text{Eq1996}}(t) = \mathfrak{R}_1(-\varepsilon_A) \cdot \mathfrak{R}_3(\Delta\psi) \cdot \mathfrak{R}_1(\varepsilon_A + \Delta\varepsilon)$ IAU 1980 Theory of Nutation (Seidelmann 1982, Wahr 1981) $P_{\text{Eq1996}}(t) = \mathfrak{R}_3(\xi_A) \cdot \mathfrak{R}_2(-\theta_A) \cdot \mathfrak{R}_3(z_A)$ (Lieske et al. 1977)
IERS Conventions 1996 Coordinate transformation referred to the Nonrotating Origin (McCarthy 1996, pp.34-37) http://maia.usno.navy.mil/conventions.html	$W_{\text{NR01996}}(t) = \mathfrak{R}_3(-s')$ $\cdot \mathfrak{R}_1(y_p) \cdot \mathfrak{R}_2(x_p)$	$R_{\text{NR01996}}(t) = \mathfrak{R}_3(-\theta)$	$Q_{\text{NR01996}}(t) = \mathfrak{R}_3(-E) \cdot \mathfrak{R}_2(-d) \cdot \mathfrak{R}_3(E) \cdot \mathfrak{R}_3(s)$ $Q_{\text{NR01996}}(t) = Q_{\text{XT}} \cdot \mathfrak{R}_3(s)$ IERS 1996 Theory of Precession/Nutation, ICRS (IERS Conventions 2003, Chapter 5., p.4)
IERS Conventions 2003 CEO-based coordinate transformation (http://maia.usno.navy.mil/conv2000.html)	$W_{\text{CEO2003}}(t) = \mathfrak{R}_3(-s')$ $\cdot \mathfrak{R}_1(y_p) \cdot \mathfrak{R}_2(x_p)$	$R_{\text{CEO1996}}(t) = \mathfrak{R}_3(-\theta)$	$Q_{\text{CEO2003}}(t) = \mathfrak{R}_3(-E) \cdot \mathfrak{R}_2(-d) \cdot \mathfrak{R}_3(E) \cdot \mathfrak{R}_3(s)$ CEO-based bias-precession-nutation matrix consistent with IAU 2000A precession-nutation
IERS Conventions 2003 Equinox-based coordinate transformation, (http://maia.usno.navy.mil/conv2000.html)	$W_{\text{Eq2003}}(t) = \mathfrak{R}_3(-s')$ $\cdot \mathfrak{R}_1(y_p) \cdot \mathfrak{R}_2(x_p)$	$R_{\text{Eq2003}}(t) = \mathfrak{R}_3(-GAST)$	$Q_{\text{Eq2003}}(t) = B_{\text{Eq2003}}(t) \cdot P_{\text{Eq2003}}(t) \cdot N_{\text{Eq2003}}(t)$ $N_{\text{Eq2003}}(t) = \mathfrak{R}_1(-\varepsilon_A) \cdot \mathfrak{R}_3(\Delta\psi) \cdot \mathfrak{R}_1(\varepsilon_A + \Delta\varepsilon)$ $P_{\text{Eq2003}}(t) = \mathfrak{R}_1(-\varepsilon_0) \cdot \mathfrak{R}_3(\psi_A) \cdot \mathfrak{R}_1(\omega_A) \cdot \mathfrak{R}_3(\chi_A)$ $B_{\text{Eq2003}}(t) = \mathfrak{R}_3(-\delta\alpha_0) \cdot \mathfrak{R}_2(-\delta\psi_0 \cdot \sin \varepsilon_0) \cdot \mathfrak{R}_1(\delta\varepsilon)$ Classical bias-precession-nutation matrix using IAU 2000A precession-nutation

The parameter t , used in the expressions below, is defined to be consistent with IAU Resolution C7 (1994), which recommends that the epoch J2000.0 be defined at the geocenter and at the date 2000 January 1.5 TT = Julian Date 2451545.0TT, by

$$t = \frac{JD(TT) - JD(2000, \text{January}, 1, 12^h TT)}{36525} \quad (2)$$

$\mathfrak{R}_{1,2,3}$ means a rotation around axis OX, OY and OZ respectively.

3. COMPUTATION – BASIC STEPS

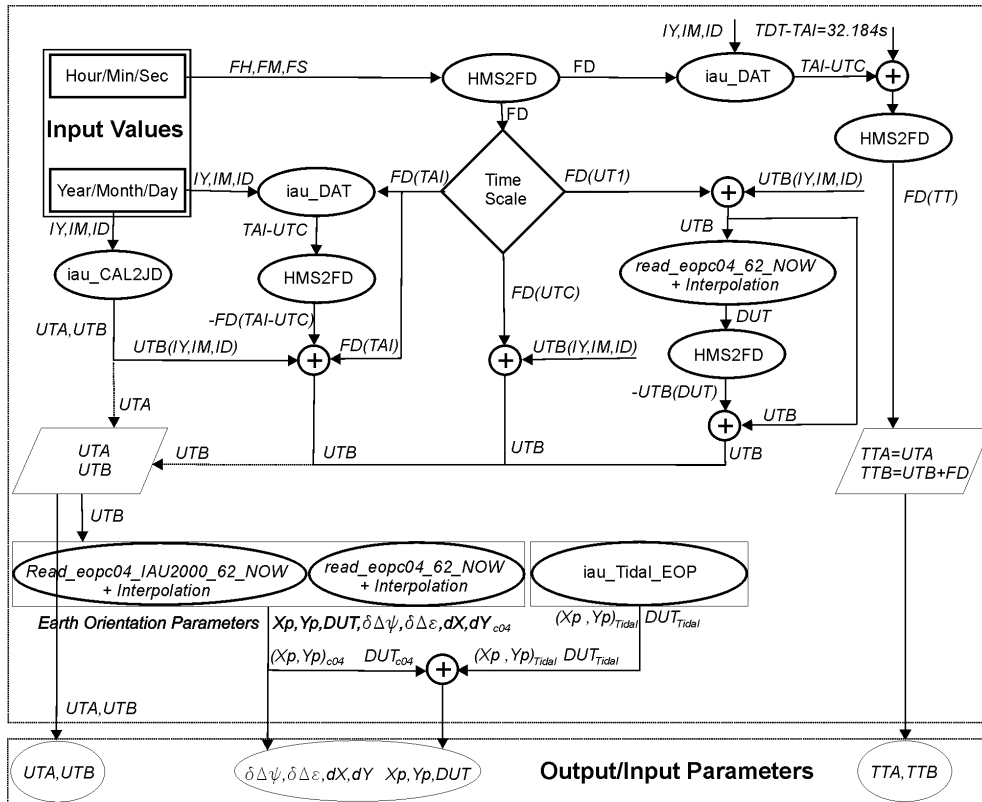


Figure 1: The basic steps of time scales calculation and obtaining Earth Orientation Parameters.

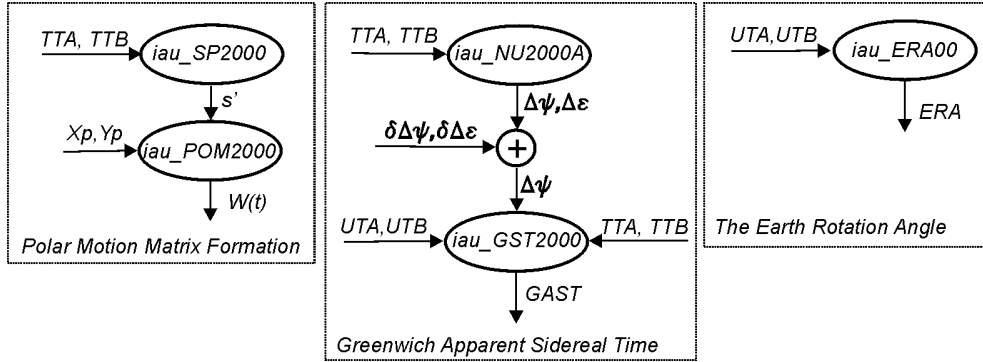


Figure 2: Illustration of the obtaining the transformation matrices, arising from the polar motion $W(t)$ and the rotation of the Earth around the axis of the pole $R(t)$.

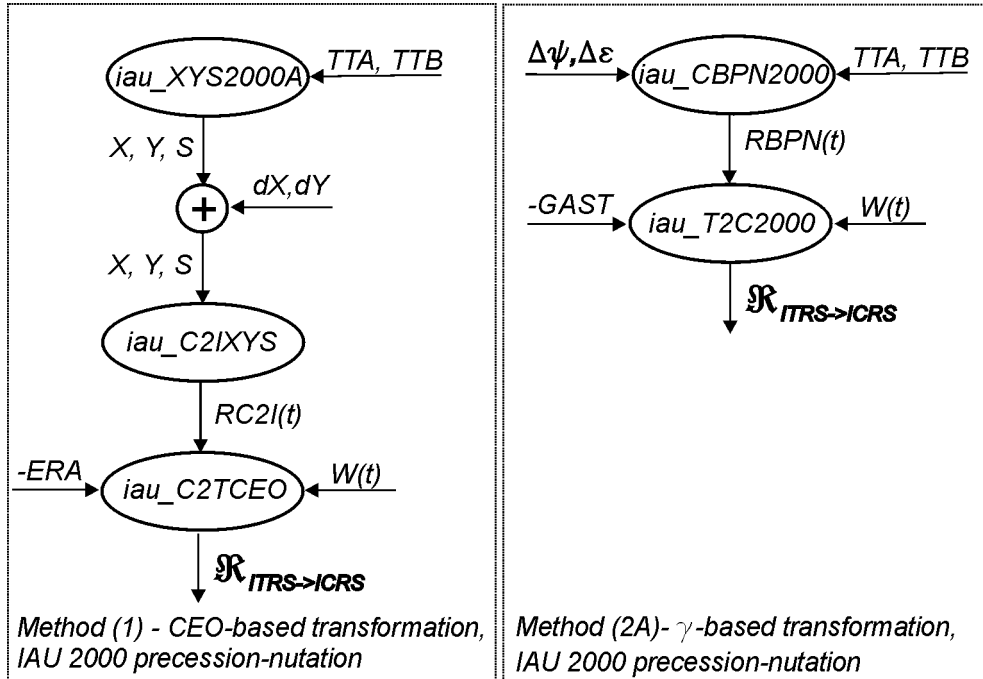
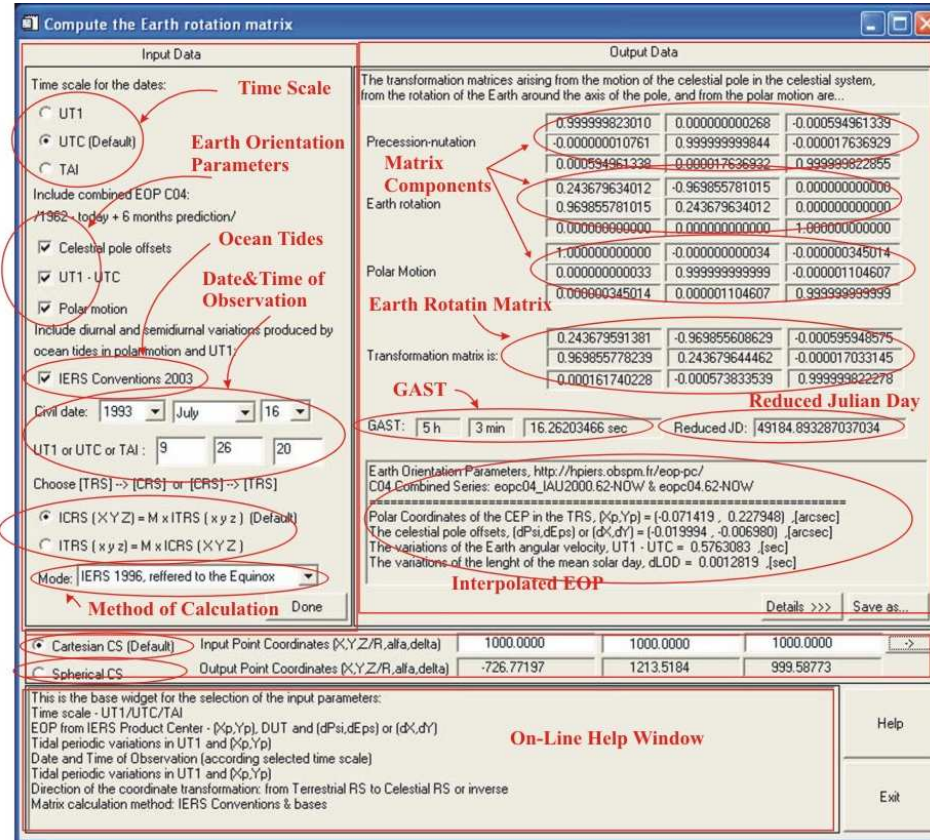


Figure 3: Computing transformation matrix $\mathcal{R}_{ITRS \rightarrow ICRS}$ according to IERS Conventions 2003 – basic steps for Method (1) and Method (2A).

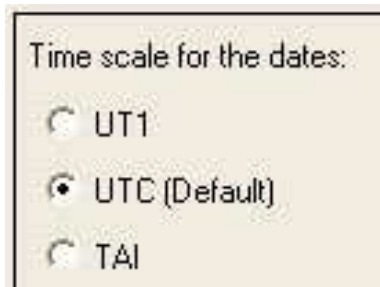
4. PROGRAM MODULE FOR COMPUTING THE EARTH ROTATION MATRIX

The software begins by displaying the following main menu screen:



The main screen consists from four windows – Input Window, Output Window, Point Coordinate Transformation Window and On-line Help Window.

This input window allows the user to select such things as - input time scale (UT1/UTC/TAI);



- including EOP series (celestial pole offset $(\delta\Delta\psi, \delta\Delta\varepsilon) / (dX, dY)$, $UT1 - UTC$ and polar coordinates (Xp, Yp));

- including diurnal and semidiurnal variations, produced by ocean tides;

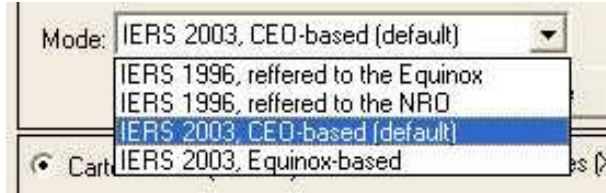
The user can enter the input date and time of observation in selected above time scale.

The "radio buttons" allows user to select the "direction" of the coordinate transformation – from terrestrial reference system to celestial reference system (default option) or inverse.

Four methods for computing the transformation matrix are available – computation according to

- *IERS Conventions 1996* – referred to the Equinox (Option 1) and referred to the Nonrotating origin (Option 2)

- *IERS Conventions 2003* – CEO based transformation consistent with IAU 2000A precession-nutation (default option) and Equinox –based transformation, using IAU 2000A precession-nutation.



Output Window consists of:

- A window that displays transformation matrix and its parts – Polar motion matrix, Earth rotation matrix and Precession-Nutation matrix;

Precession-nutation	0.999999823010	0.000000000647	-0.000594961454
	-0.000000011141	0.999999999844	-0.000017636755
	0.000594961454	0.000017636758	0.999999822855
Earth rotation	0.243679634234	-0.969855780959	0.000000000000
	0.969855780959	0.243679634234	0.000000000000
	0.000000000000	0.000000000000	1.000000000000
Polar Motion	1.000000000000	0.000000000014	-0.000000345014
	-0.000000000015	0.999999999999	-0.000001104607
	0.000000345014	0.000001104607	0.999999999999
Transformation matrix is:	0.243679591542	-0.969855609800	-0.000593974218
	0.969855778084	0.243679644995	-0.000018240538
	0.000162430115	-0.000571624480	0.999999823431

- Window that displays the interpolated EOP at the time of observation;

Earth Orientation Parameters, <http://hpiers.obspm.fr/eop-pc/>
 C04 Combined Series: eopc04_IAU2000.62-NOW & eopc04.62-NOW

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Polar Coordinates of the CEP in the TRS, (Xp,Yp) = (-0.071419 , 0.227948) ,[arcsec]
 The celestial pole offsets, (dPsi,dEps) or (dX,dY) = (0.000395 , -0.000119) ,[arcsec]
 The variations of the Earth angular velocity, UT1 - UTC = 0.5763083 ,[sec]
 The variations of the lenght of the mean solar day, dLOD = 0.0012819 ,[sec]

- The Greenwich Apparent Sidereal Time and Reduced Julian Date are displayed also

GAST: Reduced JD:

The software makes it possible to calculate point coordinate from terrestrial reference system to celestial reference system or inverse in Cartesian or Spherical coordinate system.

<input checked="" type="radio"/> Cartesian CS (Default)	Input Point Coordinates (X,Y,Z/R,alfa,delta)	1000.0000	1000.0000	10000.000	>
<input type="radio"/> Spherical CS	Output Point Coordinates (X,Y,Z/R,alfa,delta)	-732.11576	1213.3530	9999.5890	

In on-line help window moving the mouse across windows and fields we can have instant information that supports our work.

5. CONCLUSION

The software, described above, is written in the Interactive Data Language (IDL), Version 5.6 Win 32 (x86). The basic modules are translated from the FORTRAN routines, provided from <ftp://maia.usno.navy.mil/conv2000/chapter5> Fortran Code and some routines from the IAU Standards of Fundamental Astronomy software collection.

The accuracy, achieved at calculating the transformation matrix is assessed in comparison to the on-line calculated transformation matrix on <http://hpiers.obspm.fr/eop-pc/>. An accuracy of less than 10⁻⁹ radians was achieved.

The sources of the software described are available on <ftp://aquila.skyarchive.org>.

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Appendix A: Description and expressions for transformation matrix quantities.

Parameter	Description	Expression
x_p, y_p	The "pole coordinates" of the CEP in ITRS, Earth Orientation Parameters	IERS value: ftp://hpiers.obspm.fr/eop-pc/eopc04.62-NOW or ftp://hpiers.obspm.fr/eop-pc/eopc04_IAU2000.62-NOW
s'	The accumulated displacement of the terrestrial origin on the equator due to polar motion.	$s' = -47 \mu\text{as} \cdot t$
$GAST$	Greenwich Apparent Sidereal Time at date t , including both the effect of Earth rotation and precession and nutation in right ascension.	$GAST = GMST + \Delta\psi \cdot \cos \varepsilon_A + 0'' \cdot 00264 \cdot \sin \Omega + 0'' \cdot 000063 \cdot \sin 2\Omega$ $GMST = GMST_{0UT1} + r \cdot [(UT1 - UTC) + UTC]$ $GMST_{0UT1} = 6^h 41^m 50^s \cdot 54841 + 8640184^s \cdot 812866 \cdot T_u$ $+ 0^s \cdot 093104 \cdot T_u^2 - 6^s \cdot 2 \cdot 10^{-6} \cdot T_u^3$ $r = 1.002737909350795 + 5.9006 \cdot 10^{-11} \cdot T_u + 5.9 \cdot 10^{-15} \cdot T_u^2$ <i>(Aoki et al. 1982)</i> IERS value: ftp://hpiers.obspm.fr/eop-pc/eopc04.62-NOW or ftp://hpiers.obspm.fr/eop-pc/eopc04_IAU2000.62-NOW
$DUT = UT1 - UTC$	Earth Orientation Parameter	
θ	Earth Rotation Angle - the stellar angle at date t due to the Earth angle of rotation (Capitaine et al. 1986)	$\theta(T_u) = 2\pi \cdot (0.779057273264 + 1.00273781191135448 \cdot T_u \cdot x \cdot 36525)$
ξ_A, θ_A, z_A	The precession quantities as functions of two parameters (t, T) . When the arbitrary epoch is chosen to be J2000.0, $T=0$ (Lieske et al. 1977).	$\xi_A = 2306'' \cdot 2181 \cdot t + 0'' \cdot 30188 \cdot t^2 + 0'' \cdot 017998 \cdot t^3$ $\theta_A = 2004'' \cdot 3109 \cdot t - 0'' \cdot 42665 \cdot t^2 - 0'' \cdot 041833 \cdot t^3$ $z_A = 2306'' \cdot 2181 \cdot t + 1'' \cdot 09468 \cdot t^2 + 0'' \cdot 018203 \cdot t^3$
$\Delta\psi$	The nutation in longitude	$\Delta\psi = \Delta\psi(IAU1980) + \delta\Delta\psi$ $\Delta\psi(IAU1980) = \sum_{i=1}^{106} (A_i + A'_i \cdot t) \cdot \sin(ARG)$ IAU 1980 - IAU 1980 Theory of Nutation in longitude and obliquity $\Delta\psi(IERS1996) = \sum_{i=1}^{112} A_i \cdot \sin(ARG) + A'_i \cdot \cos(ARG)$ IERS 1996 - IERS 1996 Theory of Precession/Nutation

$\Delta\varepsilon$	The nutation in obliquity	$\Delta\varepsilon = \Delta\varepsilon(\text{IAU1980}) + \delta\Delta\varepsilon$ $\Delta\varepsilon(\text{IAU1980}) = \sum_{i=1}^{106} (B_i + B'_i \cdot t) \cdot \cos(\text{ARG})$ <p><i>IAU1980 - IAU 1980 Theory of Nutation in longitude and obliquity</i></p> $\Delta\varepsilon = \sum_{i=1}^{112} B_i \cdot \sin(\text{ARG}) + B'_i \cdot \cos(\text{ARG})$ <p><i>IERS1996 - IERS 1996 Theory of Precession/Nutation</i></p>
E, d IERS Conventions 1996	The coordinates (X, Y, Z) of the CEP in ICRS (McCarthy 1996, p. 34)	$E = \arctg(Y / X)$ $d = \arctg\left(\sqrt{\frac{X^2 + Y^2}{1 - (X^2 + Y^2)}}\right)$
X IERS Conventions 1996	The coordinate X of the CEP in ICRS (McCarthy 1996, p. 36)	$X = X(\text{IAU 2000}) + \delta X$ $X = \text{PolynomialPart} + \text{NonPolynomialPart}$ $X(\text{PolynomialPart}) = 2004'' \cdot 3109 \cdot t - 0'' \cdot 442665 \cdot t^2 - 0'' \cdot 198656 \cdot t^3$ $+ 0'' \cdot 0000140 \cdot t^4 + 0'' \cdot 000006 \cdot t^2 \cdot \cos\Omega$ $+ 0'' \cdot 00204 \cdot t^2 \cdot \sin\Omega + 0'' \cdot 00016 \cdot t^2 \cdot \sin 2 \cdot (F - D + \Omega)$ $X(\text{NonPolynomialPart}) = \sin \varepsilon_0 \cdot \left\{ \sum [(A_i + A'_i \cdot t) \cdot \sin(\text{ARG}) + A''_i \cdot t \cdot \cos(\text{ARG})] \right\}$
Y IERS Conventions 1996	The coordinate Y of the CEP in ICRS (McCarthy 1996, p. 36)	$Y = Y(\text{IAU 2000}) + \delta Y$ $Y = \text{PolynomialPart} + \text{NonPolynomialPart}$ $Y(\text{PolynomialPart}) = -0'' \cdot 00013 - 22'' \cdot 40992 \cdot t^2 + 0'' \cdot 001836 \cdot t^3 + 0'' \cdot 000014 \cdot t^4$ $- 0'' \cdot 00231 \cdot t^2 \cdot \cos\Omega - 0'' \cdot 00014 \cdot t^2 \cdot \cos[2 \cdot (F - D + \Omega)]$ $Y(\text{NonPolynomialPart}) = \left\{ \sum [(B_i + B'_i \cdot t) \cdot \cos(\text{ARG}) + B''_i \cdot t \cdot \sin(\text{ARG})] \right\}$
S IERS Conventions 1996	The accumulated rotation of the celestial CEO on the true equator due to the celestial motion of the CEP, IAU 1980 Theory of Nutation (Lieske <i>et al.</i> 1977)	$S = -XY / 2 + 0'' \cdot 00385 \cdot t - 0'' \cdot 07259 \cdot t^3 - 0'' \cdot 00264 \cdot \sin\Omega + 0'' \cdot 00006 \cdot \sin 2\Omega$ $+ 0'' \cdot 00074 \cdot t^2 \cdot \sin\Omega + 0'' \cdot 00006 \cdot t^2 \cdot \sin 2 \cdot (F - D + \Omega)$

<p>E, d IAU 2000A Precession- Nutation Model</p>	<p>The coordinates (X, Y, Z) of the CEP in ICRS (IERS Conventions 2003, Chapter 5., p.4)</p>	$E = \arctg(Y / X)$ $d = \arctg\left(\sqrt{\frac{X^2 + Y^2}{1 - (X^2 + Y^2)}}\right)$
<p>X IAU 2000A Precession- Nutation Model</p>	<p>The coordinate X of the CEP in ICRS (IERS Conventions 2003, Chapter 5., p.7)</p>	$X = X(\text{IAU 2000}) + \delta X$
<p>$X(\text{IAU 2000})$ IAU 2000A Precession- Nutation Model</p>	<p>The coordinate X of the CEP in ICRS (IERS Conventions 2003, Chapter 5., p.7)</p>	$X(\text{IAU 2000}) = X(\text{PolynomialPart}) + X(\text{NonPolynomialPart})$ $X(\text{NonPolynomialPart}) = -0''.01661699 + 2004''.19174288 \cdot t$ $- 0''.42721905 \cdot t^2 - 0''.19862054 \cdot t^3$ $- 0''.00004605 \cdot t^4 + 0''.000000598 \cdot t^5$ $X(\text{PolynomialPart}) = \sum_{i=1}^{1306} [(a_{s,0})_i \cdot \sin(ARG) + (a_{c,0})_i \cdot \cos(ARG)]$ $+ \sum_{i=1}^{353} [(a_{s,1})_i \cdot t \cdot \sin(ARG) + (a_{c,1})_i \cdot t \cdot \cos(ARG)]$ $+ \sum_{i=1}^{36} [(a_{s,2})_i \cdot t^2 \cdot \sin(ARG) + (a_{c,2})_i \cdot t^2 \cdot \cos(ARG)]$
<p>Y IAU 2000A Precession- Nutation Model</p>	<p>The coordinate Y of the CEP in ICRS (IERS Conventions 2003, Chapter 5., p.7)</p>	$Y = Y(\text{IAU 2000}) + \delta Y$
<p>$Y(\text{IAU 2000})$ IAU 2000A Precession- Nutation Model</p>	<p>The coordinate Y of the CEP in ICRS (IERS Conventions 2003, Chapter 5., p.7)</p>	$Y(\text{IAU 2000}) = Y(\text{PolynomialPart}) + Y(\text{NonPolynomialPart})$ $Y(\text{PolynomialPart}) = -0''.00695078 - 0''.02538199 \cdot t$ $- 22''.40725099 \cdot t^2 + 0''.00184228 \cdot t^3$ $- 0''.00111306 \cdot t^4 + 0''.000000099 \cdot t^5$

		$Y(\text{NonPolynomialPart}) = \sum_{i=1}^{962} [(b_{c,0})_i \cdot \cos(\text{ARG}) + (b_{s,0})_i \cdot \sin(\text{ARG})]$ $+ \sum_{i=1}^{277} [(b_{e,1})_i \cdot t \cdot \cos(\text{ARG}) + (b_{s,1})_i \cdot t \cdot \sin(\text{ARG})]$ $+ \sum_{i=1}^{30} [(b_{e,2})_i \cdot t^2 \cdot \cos(\text{ARG}) + (b_{s,2})_i \cdot t^2 \cdot \sin(\text{ARG})]$
s IAU 2000A Precession- Nutation Model	The accumulated rotation (between the epoch J2000.0 and the date t) of the celestial CEO on the true equator due to the celestial motion of the CEP, IAU 2000 Precession-Nutation Model (Capitaine <i>et al.</i> 2003)	$s(t) = -(XY / 2) + \text{PolynomialPart} + \text{NonPolynomialPart}$ $\text{PolynomialPart} = +94.0 + 3808.35 \cdot t - 119.94 \cdot t^2$ $- 72574.09 \cdot t^3 + 27.70 \cdot t^4 + 15.61 \cdot t^5$ $\text{NonPolynomialPart} = \sum_{i=1}^{33} [(c_{s,0})_i \cdot \sin(\text{ARG}) + (c_{e,0})_i \cdot \cos(\text{ARG})]$ $+ \sum_{i=1}^3 [(c_{s,1})_i \cdot t \cdot \sin(\text{ARG}) + (c_{e,1})_i \cdot \cos(\text{ARG})]$ $+ \sum_{i=1}^{25} [(c_{s,2})_i \cdot t^2 \cdot \sin(\text{ARG}) + (c_{e,2})_i \cdot \cos(\text{ARG})]$ $+ \sum_{i=1}^4 [(c_{s,3})_i \cdot t^3 \cdot \sin(\text{ARG}) + (c_{e,3})_i \cdot \cos(\text{ARG})]$ $+ \sum_{i=1}^1 [(c_{s,4})_i \cdot t^4 \cdot \sin(\text{ARG}) + (c_{e,4})_i \cdot \cos(\text{ARG})]$
$\Delta\psi$ The IAU 2000 Nutation Model	The nutation in longitude	$\Delta\psi = \Delta\psi(\text{IAU 2000}) + \delta\Delta\psi$ $\Delta\psi(\text{IAU 2000}) = \sum_{i=1}^{106} (A_i + A'_i \cdot t) \cdot \sin(\text{ARG}) + (A''_i + A'''_i \cdot t) \cdot \cos(\text{ARG})$
$\Delta\varepsilon$ The IAU 2000 Nutation Model	The nutation in obliquity	$\Delta\varepsilon = \Delta\varepsilon(\text{IAU 1980}) + \delta\Delta\varepsilon$ $\Delta\varepsilon(\text{IAU 2000}) = \sum_{i=1}^{106} (B_i + B'_i \cdot t) \cdot \cos(\text{ARG}) + (B''_i + B'''_i \cdot t) \cdot \sin(\text{ARG})$

ψ_A, ω_A, χ_A	The precession quantities, compatible with the IAU 2000 Precession-Nutation Model (Lieske <i>et al.</i> 1977).	$\psi_A = \varepsilon_0 + 0''.05127 \cdot t^2 - 0''.007726 \cdot t^3$ $\omega_A = \varepsilon_0 - 46''.8150 \cdot t - 0''.00059 \cdot t^2 - 0''.001813 \cdot t^3$ $\chi_A = 10''.5526 \cdot t - 2''.38064 \cdot t^2 + 0''.018203 \cdot t^3$
ε_A	The mean obliquity at epoch t (Lieske <i>et al.</i> 1977).	$\varepsilon_A = \varepsilon_0 - 46''.8150 \cdot t - 0''.00059 \cdot t^2 + 0''.001813 \cdot t^3$
ε_0	The J2000 obliquity (Lieske <i>et al.</i> 1977)	$\varepsilon_0 = 84381.448''$
$\delta\alpha_0$	The ICRS RA of the J2000 equinox (Chapront <i>et al.</i> 2002)	$\delta\alpha_0 = -0''.0146$
$\delta\psi_0$	The frame bias corrections in longitude	$\delta\psi_0 = -0''.0417750$
$\delta\varepsilon_0$	The frame bias corrections in obliquity	$\delta\varepsilon_0 = -0''.0068192$
$\delta\Delta\psi, \delta\Delta\varepsilon$	Earth Orientation Parameters	IERS values: ftp://hpiers.obspm.fr/eop-pc/eopc04.62-NOW
$\delta X, \delta Y$	Earth Orientation Parameters	IERS values: ftp://hpiers.obspm.fr/eop-pc/eopc04_IAU2000.62-NOW