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IMPROVED KOVAL'SKIJ METHOD AND ITS NEW POSSIBILITIES

D. OLEVIĆ and Z. CVETKOVIĆ
Astronomical Observatory, Volgina 7, 11160 Belgrade 74, Serbia
E–mail dolevic@aob.bg.ac.yu
E–mail zcvetkovic@aob.bg.ac.yu

Abstract. We improved the Koval'skij analytical method for binary star orbital elements determination, in order to enlarge its possibilities. The improved method is applicable to the determination of orbital elements for all cases of observation distributions on the apparent arc of the orbit, and always gives the elliptical solution.

1. INTRODUCTION

The duplicity of star Zeta UMa-Misara was noticed by Ricciolli as early as in 1650. Afterwards the duplicity was discovered for a constantly enlarging number of stars more or less accidentally. By 1775 the number of discovered binaries attained a few tens. In the late XVIII century H. Meier published a list containing 89 pairs for which he wrote that he had noticed the motion of a satellite around the parent star. These "observations" made by him were derided by his contemporaries. With regard to the equipment he used and large measuring errors this claim was bold. The year 1803 is considered as the year when binaries were discovered. In that year Herschel published a paper on the changes of mutual positions for binaries where he explained the causes of these changes. The action of the gravitation law was through this obviously noticed also beyond the Solar System confirming thus its universal validity.

At that time the campaign of discovering binaries was initiated: their equatorial coordinates were determined, micrometric measurements of the separations and position angles were done. V. Ya. Struve formulated this as his main task.

When a sufficiently large observational material became available, the first methods for the calculation of the orbital elements were proposed. The first orbital elements for visual binaries were determined by using the graphical method (Zwiers, 1896). The first analytical method was proposed by Koval'skij (see Subbotin, 1968), to be a little bit later improved by Glazenap. Further on many new methods for the purpose of solving some concrete problems were proposed: how to calculate orbital elements on the basis of a low number of measurements, how to obtain qualitative results in the presence of large observational errors and how to avoid a too long calculation. The tables for some analytic expressions were formed (Thiele-Innes (TI), TI- van den Boss (TIvdB) etc).

Table 1: Observations and residuals for WDS 07143-2621 = FIN 323

t	θ°	ρ''	$\Delta\theta^\circ_{KO}$	$\Delta\rho''_{KO}$	$\Delta\theta^\circ_{K}$	$\Delta\rho''_{K}$
1955.2600	136.6	.100	3.4	.000	61.9	−.221
1955.3100	135.5	.102	1.7	.002	60.2	−.217
1956.2100	135.5	.105	0.2	.004	57.0	−.178
1958.2400	139.6	.102	1.1	−.001	49.6	−.093
1959.2500	140.8	.101	0.7	−.003	40.0	−.051
1960.2400	137.8	.098	−3.8	−.006	19.1	−.019
1961.2900	141.8	.098	−1.4	−.006	−7.0	.000
1962.2700	142.7	.100	−2.1	−.003	−36.1	−.010
1963.2460	145.9	.102	−0.4	.000	−53.3	−.039
1964.2830	148.4	.098	0.4	−.002	−63.5	−.087
1965.2730	145.8	.098	−3.9	.000	−73.5	−.131
1966.2780	150.5	.099	−1.0	.004	−73.8	−.175
1967.2820	151.0	.099	−2.4	.008	−77.0	−.219
1968.3210	157.6	.098	2.0	.012	−73.2	−.263
1989.3083	141.6	.129	1.1	.010	65.3	−.314
1989.9336	142.0	.129	1.1	.010	64.7	−.296
1990.9137	142.6	.132	0.7	.006	63.5	−.265
1991.2500	144.0	.131	1.7	.002	64.2	−.256
1993.0953	144.6	.133	0.6	−.008	60.2	−.195
1996.1752	123.4	.150	−22.9	−.009	25.8	−.076

Table 2: Observations and residuals for WDS 13320-6519 = FIN 369

t	θ°	ρ''	$\Delta\theta^\circ_{KO}$	$\Delta\rho''_{KO}$	$\Delta\theta^\circ_{K}$	$\Delta\rho''_{K}$
1961.5600	117.3	.121	7.7	.020	−18.9	−.003
1962.5400	115.5	.126	−10.6	.023	−26.0	.001
1963.5590	143.5	.124	1.7	.013	−3.2	−.003
1964.5430	161.4	.130	7.0	.007	9.7	.001
1965.5630	164.2	.134	−0.7	−.004	7.6	.003
1966.5390	171.7	.140	−1.3	−.013	10.5	.007
1967.5800	185.5	.143	5.4	−.026	19.6	.007
1989.3036	344.5	.099	10.8	−.006	−3.1	−.008
1990.3437	355.2	.121	6.5	.005	0.9	.002
1991.2500	358.0	.131	−1.5	.004	−1.2	.003
1993.0984	8.2	.144	−8.5	−.003	0.8	−.002

With appearance of computers Koval'skij's method became more practical with regard that the "geometric" elements (first Kepler's law) are calculated by using the least-square method. A disadvantage of this method is that the dynamical elements P and T are calculated on the basis of these geometric elements without satisfying second Kepler's law (law of areas) simultaneously.

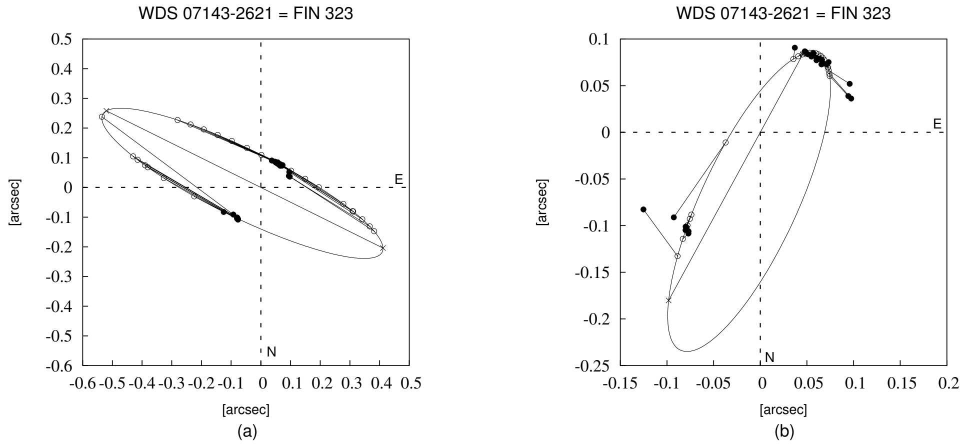

Figure 1: Orbits of FIN 323. (a) obtained by applying Koval'skij's method; (b) obtained by applying KOVOLE.

The TIvdB method uses among its input data the area constant determined through plots being a significantly crude and uncertain approach. This method, as most frequently used, has been improved by Docobo (1985). The main progress of Docobo's imrovement is that it became unnecessary to know the area constant.

Among other methods we shall also mention that proposed by Eichhorn (1985).

Koval'skij's method has been improved by one of us (Olević). In this way one obtains a new method (KOVOLE) usable for all cases. Unlike Koval'skij's method KOVOLE always yields an elliptical solution. Its more detailed description can be found in Olević and Cvetković (2004). By varying the amount for a number of fictive separations corresponding to given position angles the elements are calculated through an iterative procedure and the solution yielding the best fit to both first and second Kepler's laws is adopted.

The advantages of KOVOLE will be demonstrated through two examples.

2. EXAMPLES DEMONSTRATING THE SUCCESSFULNESS OF KOVOLE

In Fig. 1 one presents orbits of star WDS 07143-2621 = FIN 323: a) obtained by applying Koval'skij's method; b) obtained by applying KOVOLE.

In Fig. 2 one presents orbits of star WDS 13320-6519 = FIN 369: a) obtained by applying Koval'skij's method; b) obtained by applying KOVOLE.

The filled circles are for the measured positions, the empty ones are for the corresponding ephemeris values.

In Tables 1 and 2 one presents the observations and residuals. The designations are: t - observational epoch; θ and ρ - polar coordinates; $\Delta\theta_{KO}$ and $\Delta\rho_{KO}$ - residuals obtained by applying KOVOLE; $\Delta\theta_K$ and $\Delta\rho_K$ - residuals obtained by applying Koval'skij's method.

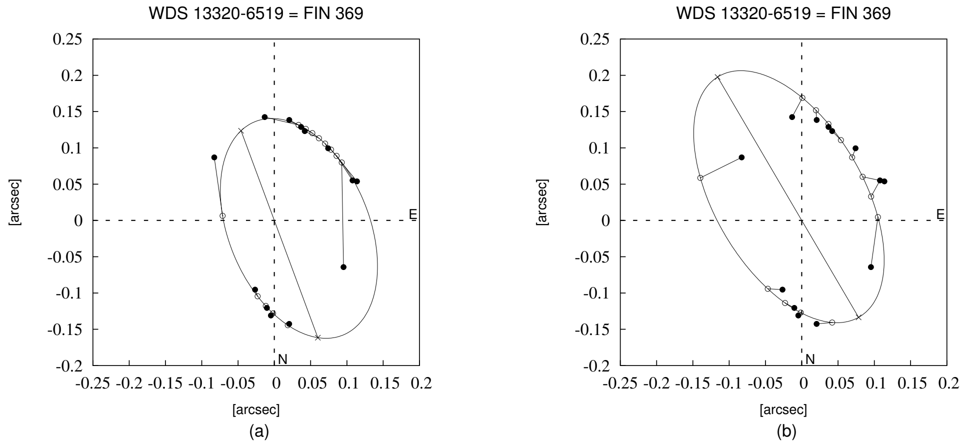

Figure 2: Orbits of FIN 369. (a) obtained by applying Koval'skij's method; (b) obtained by applying KOVOLE.

The $\Delta\theta_{KO}$ and $\Delta\rho_{KO}$ residuals are significantly smaller than the $\Delta\theta_K$ and $\Delta\rho_K$ ones. This is a clear confirmation of KOVOLE advantages. In favour are also the rms errors for the residuals of the observed values from the calculated ones.

The rms errors $(O - C)$ for star FIN 323 are: 0.2280 arc seconds, i.e. 0.0134 arc seconds, when one applies Koval'skij's method, i.e. KOVOLE, respectively. In the case of FIN 369 the rms errors $(O - C)$ are: 0.0174 arc seconds and 0.0138 arc seconds for the use of Koval'skij's method and KOVOLE, respectively.

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