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INFLUENCE OF THE PHASE OF THE SPHERICAL PLANET ON THE POSITION OF ITS PHOTOCENTER

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Abstract. The analysis of the reasons influencing a position of photocenter of a spherical planet under various conditions of its illumination intensity and various conditions of its observations is carried out. According to the indicated reasons various methods of determination of the position of photocenter of a planet in which the photocenter of a planet is considered as center of an illuminated part of its visible disk or as light center of its disk are offered. Some regularity of the position of photocenter and positions of the basic photometric points on an illuminated part of diameter of intensity of the planet are established. The example of determination of a position of photocenter of Mercury is given.

1. INTRODUCTION

At photometrical and positional observations of planets of the Solar system and their natural satellites there is a problem of determination of the position of photocenter on images of their visible disks. For terrestrial planets which shape can be counted as a first approximation spherical, this problem becomes complicated through influence of the phase. It is observed when the geometric planetary disk is illuminated by the Sun not completely, i.e. the phase angle Φ is distinct from zero. Influence of the phase produces a noticeable defect on the planet's visible disk and causes the photocenter of planet to be shifted relative to center of its geometric disk. Thus the phase of the planet is considered in an orthographic approximation, i.e. it is supposed, that the Sun represents the point source of light infinitely removed from the planet, hence, the orthographic terminator is the boundary of the illuminated part of the visible planetary disk.

At ground-based and near-earth observations it is possible to neglect radius of the planet in comparison with its geocentric distance. In this case we suppose, that the Earth is infinitely removed from the planet, therefore the initial image of the visible planetary disk represents an orthographic projection of its surface to the plane of the sky.

The position of photocenter of the planet depends on the phase angle and from allocation of the brightness over the visible planetary disk. On the position of photocenter are influenced also the conditions of observation and the resolving power of the telescope.

For determination of the position of photocenter of planets are devoted the contributions (Mikhalchuk 2001a, Sveshnikov 1978, 1985, Safronov 1978, Lindegren 1977, Chollet 1984, Standish 1990, Toulmonde and Chollet 1994), of their satellites – contributions (Devyatkin and Bobylev 1988, 1991) and of asteroids – contributions (Sitarski 1984, Hestroffer 1998, Lupishko *et al.* 2002) in which various formulae are applied to an evaluation of angular distance between photocenter and geometric center of the visible disk of planet. These formulae are connected with different distribution laws of the brightness over visible planetary disk. However in these contributions there are no precise substantiations and physical criterions for concrete application of this or that method of determination of a position of photocenter. Therefore there is a problem in development of criterions of application of the known methods based on physical distribution laws of the brightness over the visible planetary disk for various conditions of its illumination.

The purpose of the present contribution is to obtain the criterions, which are allowing carrying out a select of existing methods of determination of the position of photocenter at various distribution laws of the brightness over the visible planetary disk with account of conditions of observation and the resolving power of the telescope.

2. THE POSITION OF THE PHOTOCENTER OF THE PLANET

The position of photocenter of the planet matters in the correction for phase for the reduction to geometric center of the visible disk in positional observations of planets, and also at their photometrical observations.

The photocenter represents a point in which there is the center of the visible image of the planet. At a reduction his position is always determined concerning geometric center of the visible planetary disk and measured by angular distance σ between these points. Thus, the reduction of equatorial coordinates α_0 and δ_0 of the observable photocenter of the planet to its true equatorial coordinates α and δ concerning of the geometric center of the visible planetary disk is carried out.

The position of photocenter of the planet in general case depends on the phase angle and from the local albedo of each point of its surface. These characteristics determine the distribution law of the brightness on the visible planetary disk.

The displacement σ of the photocenter of the planet concerning geometric center of its visible disk at the same value of the phase angle depends on following reasons:

- allocation of the brightness over the visible planetary disk;
- conditions of observations (the state of an Earth's atmosphere, the sky background brightness);
- resolving power of the telescope.

Let's consider the spherical planet, which is not having the atmosphere. We shall assume, that a surface of a planet is smooth, without a relief (like that of lunar maria). As the local albedo of all points of the surface of the planet equally, its photocenter is always located on an illuminated part of diameter of intensity. In this case, as shown in (Mikhalchuk 2001a), the reduction formulas for the determination of equatorial coordinates of geometric center of the planetary disk have a following aspect:

$$\left.\begin{aligned} \alpha &= \alpha_0 + \sigma \frac{\sin Q}{\cos \delta_0} \\ \delta &= \delta_0 + \sigma \cos Q \end{aligned}\right\}, \tag{1}$$

where Q is the angle of position of the point of the least illumination of the disk.

If the surface of the planet is covered with a regolith, then the reflection of light from it is diffuse. For ideally smooth surface the reflection of light is mirror. In general case at the diffuse reflection of light the brightness is distributed over the illuminated part of the visible disk nonuniformly – its decrease to terminator is observed. Analogically, is observed and in general case the mirror reflection, but decrease of the brightness to terminator is more sharp. Thus, the solution of a problem of determination of the position of photocenter of the planet becomes complicated through the distribution law of the brightness by an illuminated part of its visible disk, and the angular distance σ in formulas (1) is function of the phase angle $\sigma = p(\Phi)$. The laws of variation of $p(\Phi)$ are given in (Sveshnikov 1978, 1985, Safronov 1978).

For simplification of the solution of the task as a first approximation we shall consider two models of allocation of the brightness over the illuminated part of the visible planetary disk: at the diffuse reflection of light we shall accept, that the brightness is distributed uniformly, and at the mirror reflection we shall accept, that the reflection of light from the surface of the planet is absolutely mirror – all the brightness is concentrated in the mirror point.

Let's consider the diffuse reflection of light from the surface of the planet. As it is accepted as a first approximation, that the brightness is uniformly distributed over the illuminated part of its visible disk, then the position of photocenter is determined by conditions of observations and the resolving power of the telescope.

Let's assume, that observations of the planet will be carried out at a non-turbulent atmosphere and at ideally dark sky. If the observable image of the visible planetary disk is resolvable (his apparent angular radius is of much more radius of the diffraction disk of Airy), then the photocenter of the planet coincides with the center of an illuminated part of its visible disk, limited by the limb on the one side and by the terminator on the other side. On Fig. 1 the basic photometric points of the visible disk of the spherical planet are shown: O – geometric center of the planetary disk (the subterral point), E – the subsolar point (the pole of illumination), C – center of the illuminated part of the visible planetary disk, L – the pole of phase, F – the point of the least illumination of the disk, T – the visible center of orthographic terminator, M – the mirror point, points A and B – orthographic cusps of the disk.

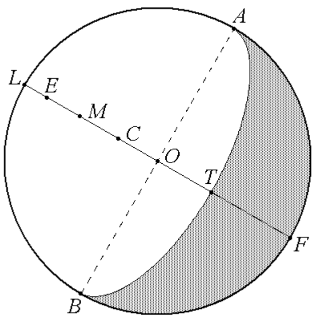

Figure 1: The basic points of an illuminated part of the visible disk of the spherical planet.

In the considered case the angular displacement of photocenter can be calculated from the formula

$$\sigma = r \sin \gamma \,, \tag{2}$$

where r is the visible angular radius of the planet; γ is the phase shift of its center of the disk that is determined from equation:

$$\sin \gamma = \sin^2 \frac{\Phi}{2} \,. \tag{3}$$

From expressions (2) and (3) we shall receive the formula linking an angular displacement of photocenter of the planet with the phase angle

$$\sigma = r \sin^2 \frac{\Phi}{2} \,. \tag{4}$$

If the observable image of the visible disk is nonresolvable (in case of the planetary satellite or the spherical asteroid when the visible angular radius of the object is less than radius of the diffraction disk of Airy) then the photocenter of this object is considered as light center of its visible disk. By the light center of the disk, we mean the center of a photographic density of the object image on the photoplate, or the center of a visually observed diffraction disk. In this case the position of light center of the disk of the spherical object (the point R) coincides with a barycentre of the flat figure representing a projection of the illuminated part of his disk on the plane of the sky (Fig. 2). Having accepted as a first approximation, that the brightness is uniformly distributed on the surface of this projection, we shall choose the system of rectangular coordinates (x, y) lying in the plane of the sky with the center in the point O. Axis Y is directed toward northern orthographic cusp of the object, axis X is directed along diameter of intensity toward the east.

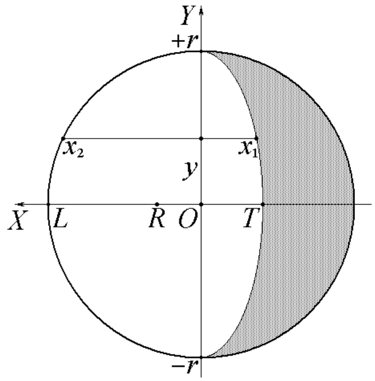

Figure 2: The position of the photocenter at the nonresolvable image of the visible planetary disk.

According to (Chollet 1984), the position of the photocenter is determined by the following formula:

$$\sigma = \frac{\iint\limits_S x\,dx\,dy}{\iint\limits_S dx\,dy}. \tag{5}$$

Taking into account, that at the uniform distribution of the brightness on the surface of the projection of an illuminated part of the disk of spherical object in an orthographic approximation, after transformations we shall receive

$$\iint\limits_S x\,dx\,dy = \int\limits_{-r}^{+r} dy \int\limits_{x_1}^{x_2} x\,dx = \frac{2}{3} r^3 \sin^2 \Phi\,, \tag{6}$$

where $x_1 = -r\cos\Phi\sqrt{1-\frac{y^2}{r^2}}$ and $x_2 = r\sqrt{1-\frac{y^2}{r^2}}$ is the coordinates of the points of the orthographic terminator and the light orthographic limb accordingly.
The area of the projection of an illuminated part of the disk of the object on the plane of the sky in an orthographic approximation is expressed by the formula

$$\iint\limits_S dx\,dy = \int\limits_{-r}^{+r} dy \int\limits_{x_1}^{x_2} dx = \pi r^2 \cos^2 \frac{\Phi}{2}. \tag{7}$$

We then substitute expressions (6) and (7) in the formula (5), and taking into account, that $\sin\Phi = 2\sin\frac{\Phi}{2}\cos\frac{\Phi}{2}$, to derive the final formula linking an angular displacement of light center with a phase angle:

$$\sigma = \frac{8r}{3\pi}\sin^2\frac{\Phi}{2}. \tag{8}$$

If observations of the planet the apparent radius is of much more exceed the radius of the diffraction disk of Airy, will be carried out at the dark sky, but a turbulent atmosphere, then its photocenter also coincides with light center of the visible disk and his angular displacement is expressed by the formula (8).

For absolute mirror reflection of light from the surface of the planet the photocenter coincides with the mirror point (the point *M*), shown on Fig. 1. Then in any case, both at the resolvable image of the visible planetary disk, and at nonresolvable, the angular displacement of photocenter is determined from the formula

$$\sigma = r\sin\frac{\Phi}{2}\,. \qquad (9)$$

At daytime observations of planets the decrease of the brightness to terminator becomes so noticeable, that the illuminated part of the visible disk becomes limited isophote on which the threshold of perception lies. Therefore allocation of the brightness over the illuminated part of the visible planetary disk should be counted such, as at a mirror reflection of light from its surface. In this case it is necessary to apply the formula (9) to an evaluation of an angular displacement of photocenter.

3. THE DEPENDENCE OF THE POSITION OF PHOTOCENTER OF THE PLANET ON PHASE ANGLE

Let's consider the resolvable image of the visible disk of the spherical planet. To spot, as far as positions of points *C* and *M*, lying on an illuminated part of diameter of intensity differ, we shall consider associations of their angular distances σ from the phase angle. Let us denote the angular distance between a mirror point and center of an illuminated part of the visible planetary disk through Δσ. Then from formulae (2), (3) and (9) we shall receive

$$\Delta\sigma = r\sin\frac{\Phi}{2}\left(1-\sin\frac{\Phi}{2}\right). \qquad (10)$$

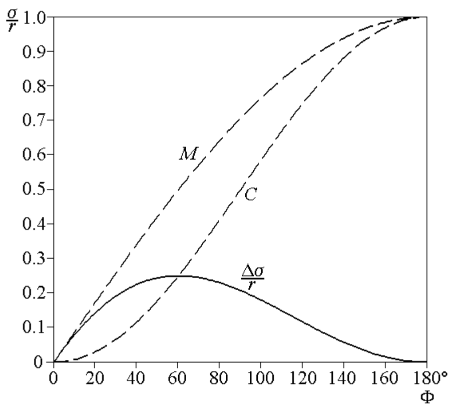

Figure 3: The dependence of angular distance σ for points *C* and *M*, and also angular distance Δσ between these points, are given in the shares of the visible radius of the planet, on the phase angle.

The curves of dependence of angular distance σ for points *C* and *M*, and also of angular distance Δσ between them, are given in the shares of the visible radius of the planet, on the phase angle are shown on Fig. 3.

From Fig. 3 it is visible, that the curve $\frac{\Delta\sigma}{r}(\Phi)$ has the maximum. It means, that angular distance Δσ between the mirror point and center of the illuminated part of the visible planetary disk, are given in the shares of its visible radius, reaches a maximum value at the particular phase angle. If to differentiate the equation (10) on the phase angle and to equate the derivative to zero, then it is possible to establish that the maximum value $\Delta\sigma = 0.250r$ is observed at $\Phi = 60°$. As $\Phi < 90°$, then the subsolar point is on the visible planetary disk. The value of the phase of the planet we shall find by means of the formula

$$k = \cos^2 \frac{\Phi}{2}. \qquad (11)$$

From the formula (11) follows, that in this case the phase of the planet in exactitude is equal $k = \frac{3}{4}$. The center of the illuminated part of the visible planetary disk is located on angular distance $\sigma = 0.25r$, and the mirror point is on angular distance $\sigma = 0.50r$ from geometric center of its visible disk (Fig. 4).

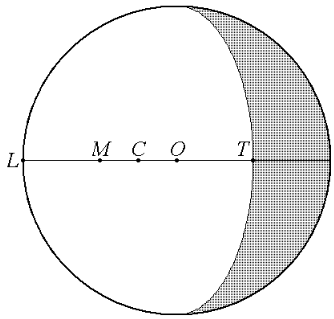

Figure 4: The visible planetary disk at the phase *k*=3/4.

The analysis of conditions of illumination of the visible planetary disk, are shown on Fig. 4, allows to find following regularity in the mutual positioning of the basic points on diameter of intensity at the maximum of dependence $\frac{\Delta\sigma}{r}(\Phi)$: *OM*=*OT*=*LM*, i.e. the point *M* is located symmetrically to point *T* relative the point *O* and is precisely in the middle between points *L* and *O*, and *MC*=*OC*, i.e. the point *C* is precisely in the middle between points *M* and *O*. From here follows, that $OC = \frac{OT}{2}$. Thus, in a considered case the visible center of the orthographic terminator is located on angular distance $\sigma = 0.50r$ from geometric center of the planetary disk.

For the nonresolvable image of the planet the angular distance σ, calculated from the formula (8) at defined values of the phase angle can appear such that the photocenter of the planet (the point *R*) falls in nonilluminated part of its visible disk. We shall term as the critical phase angle such value Φ_0 at which the photocenter of the planet is at visible center of its orthographic terminator, i.e. the points *R* and *T* coincide. Then at $\Phi < \Phi_0$, the photocenter of the planet will be on an illuminated part of its visible disk, and otherwise – on a nonilluminated. From the formula (8) follows, that the critical phase angle Φ_0 can be found by means of the relation:

$$\frac{8r}{3\pi}\sin^2\frac{\Phi_0}{2} = -r\cos\Phi_0 .$$

Having decided this equation, we shall receive $\Phi_0 = 137°.51$. We then substitute the obtained value Φ_0 in the formula (11), to derive the phase accordant to the critical phase angle: $k = 0.1313$. According to the formula (8), the angular displacement of light center at the critical phase angle is equal $\sigma = 0.737r$.

Thus, at $\Phi > \Phi_0$ there should be a paradoxical situation: the photocenter of the planet is on a nonilluminated part of its visible disk. As the image of the visible planetary disk in the considered case is nonresolvable then the given paradox in practice cannot be implemented. Therefore the concept of the critical phase angle has only theoretical meaning.

4. DETERMINATION OF A POSITION OF PHOTOCENTER OF MERCURY

Let's consider an example of definition of a position of photocenter of Mercury on 0^h UT on January 8, 2004. According to physical ephemeredes (Glebova 2003), for the time instant $r = 4''.08$, $\Phi = 105°.2$, $k = 0.369$, $Q = 279°.03$.

For Mercury $Q > 180°$, hence the east part of his visible disk is illuminated. As $\Phi > 90°$, then the pole of illumination of Mercury (the point *E*) is located on an invisible side of the planet.

The computations were conducted using programs of a software package (Mikhalchuk 2001b). For the given time instant the phase shift of center of the disk of Mercury and a position of his photocenter are obtained at various conditions of observations. Were determined the reduction differences (*O-C*) $\Delta\alpha = \alpha_0 - \alpha$ and $\Delta\delta = \delta_0 - \delta$ by means of the formulae (1). The results of computations are submitted in Table 1.

Table 1. The position of photocenter of Mercury and the reduction differences $\Delta\alpha$ and $\Delta\delta$ on 0^h UT on January 8, 2004

The conditions of observations	The image of the visible disk	The point	The formula	σ	$\Delta\alpha$	$\Delta\delta$
Under ideal dark-sky	Resolvable	*C*	(4)	2″.57	+2″.687	-0″.404
	Nonresolvable	*R*	(8)	2.19	+2.305	-0.343
During daytime observations	Resolvable or nonresolvable	*M*	(9)	3.24	+3.419	-0.509

From Table 1 follows, that for the resolvable image of the visible disk of Mercury the angular distance between points *C* and *M* is equal $\Delta\sigma = 0''.67$. It means that at the resolving power of the telescope 1″ by the difference in a position of photocenter, measured at ideally dark sky and at daytime observations, it is possible to neglect.

5. CONCLUSION

The main results obtained in this contribution allow us to make the following conclusions:
1. For two simplified models of reflection of light from the surface of the spherical planet with account of the resolving power of the telescope the formulae of determination of a position of photocenter of the planet conterminous with one of the basic photometric points of the illuminated part of its visible disk are offered.
2. From the result of the analysis of conditions of illumination of planets is established, that the angular distance between the mirror point and center of the illuminated part of the visible planetary disk, expressed in the shares of the visible radius of the planet that is maximum at the phase angle 60°, accordant to the value of the phase which is exactly equal 3/4.
3. It is shown, that the obtained extremums of the disposition of the mirror point on an illuminated part of diameter of intensity of the planet allow discovering out some regularities of the mutual positioning of other basic points on diameter of intensity of its visible disk.

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