

PROPER MOTION ACCURACY OF WFPDB STARS

YAVOR CHAPANOV¹, JAN VONDRÁK², CYRIL RON², VOJTĚCH ŠTEFKA²

¹*National Institute of Geophysics, Geodesy and Geography of Bulgarian Academy of Sciences, Acad. G. Bonchev Bl.3, 1113 Sofia, Bulgaria*

E-mail: chapanov@clg.bas.bg

²*Astronomical Institute, Academy of Sciences of the Czech Rep., v.v.i. Boční II, 141 31 Prague 4, Czech Republic*

E-mail: vondrak@ig.cas.cz, ron@ig.cas.cz, stefka@ig.cas.cz

Abstract. The accuracy of latitude and Universal time determination from the optical astrometry observations, obtained during the last century in programs of monitoring Earth orientation, strongly depends on the quality of the Earth Orientation Catalogs (EOC), where the stars proper motion is critical parameter. The possibility of improving the stars proper motions by WFPDB (Wide-Field Plate Database) is investigated by means of simulated observations. The model includes real star observations from the optical astrometry and simulated observations with epochs taken from the WFPDB, corresponding to existing plates containing the EOC stars. The simulated star coordinate deviations are generated as a sum of given proper motion and random noise, corresponding to the expected standard errors of WFPDB data. Three cases of astrometrical observations with simulated random errors are considered with standard errors 0.5arcsec, 1.0arcsec and 5.0arcsec. It is possible to estimate the star proper motion with sufficient accuracy by means of the corrected plates for image center and distortion with 100⁷/mm scale and scan resolution 2400dpi.

1. INTRODUCTION

The astrometrical observations of geographic latitude and universal time during the most of the last century are a unique source of information of the local plumb-lines and Earth rotation variations and give us unique possibility of searching causality between these variations and global changes of the environment, climate, solar activity, mean sea level arising, earthquakes etc. The realistic maximal level of accuracy of parameters, estimated by optical observations is in positions 1-10mas (equivalent to 3-30cm on Earth surface) and in rates better than 1mas/a (3cm/a). This level of accuracy is achievable after excluding the systematic errors from the raw data by means of the star catalog improvement. The most stars from

the combined Earth Orientation Catalogs (EOC) have accurate coordinates and proper motions. Some EOC stars are double and their periodical terms are determined. Chapanov et al. (2010) points out that some EOC stars are potential candidates for star coordinates and proper motion improvement by means of Wide-Field Plate Database (WFPDB). The possibility of improving the EOC stars proper motions by means of WFPDB data is investigated on the base of simulated observations with real epochs, taken from WFPDB.

2. DATA SELECTION

The WFPDB contains enough information about all stars from the EOC. It is possible to find more than 2000 plates, containing a given EOC star with angular distance to the plate center less than 5 degrees. We may expect significant growth of the plate numbers with a given star when all plates are digitized. The model of simulated observation is based on 3 sequences of plates with the scale less than 100"/mm and scan resolution 2400dpi. These sequences correspond to 3 stars from the EOC-3, whose coordinates are given in Table 1 together with the used box size and plates numbers. The numbers of the plates, containing these stars are between 150 and 2000 (Table 1). These sequences cover time spans longer than 1 century (Fig.1), so they are useful for star proper motion estimation.

Table 1: Plate sequences corresponding to 3 stars from the EOC-3.

| Sequence Number | Right Ascension [h, m, s] | Declination [°, ', "] | Box Size [°] | Plate Number |
|-----------------|---------------------------|-----------------------|--------------|--------------|
| 1 | 11 59 31.80 | +47 45 55.4 | 5 | 2000 |
| 2 | 01 02 54.26 | +41 20 42.6 | 5 | 700 |
| 3 | 11 59 31.80 | -35 18 30.7 | 3 | 150 |

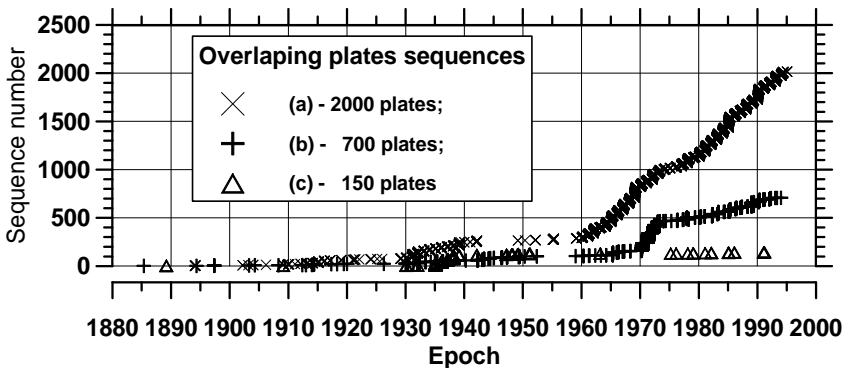


Figure 1. Overlapping plates sequences.

3. NORMALLY DISTRIBUTED RANDOM NUMBERS

The sequence of evenly distributed pseudorandom numbers is calculated by computer program, and used for modeling the observation errors. First, the sequence of evenly distributed pseudorandom numbers is transformed into a sequence of normally distributed numbers $N(0,1)$ by means of Forsythe, Malcolm and Moler algorithm (1977). This algorithm consists of the following steps:

1. Two evenly distributed random numbers U_1 and U_2 from the interval $[0, 1)$ are calculated;
2. Evenly distributed random numbers from the interval $[-1, 1)$ $V_1=2U_1-1$ and $V_2=2U_2-1$ are calculated;
3. $S=V_1^2 + V_2^2$ is calculated. If $S>1$, then V_1 and V_2 are discarded and step 1 is executed.
4. Normally distributed random numbers $X_1 = V_1\sqrt{(-2 \ln S) / S}$, $X_2 = V_2\sqrt{(-2 \ln S) / S}$ are calculated, which belong to $N(0,1)$.

The numbers X_1 and X_2 are chosen independently. It is possible to norm the obtained pseudorandom numbers to a necessary dispersion (Fig.2, 3).

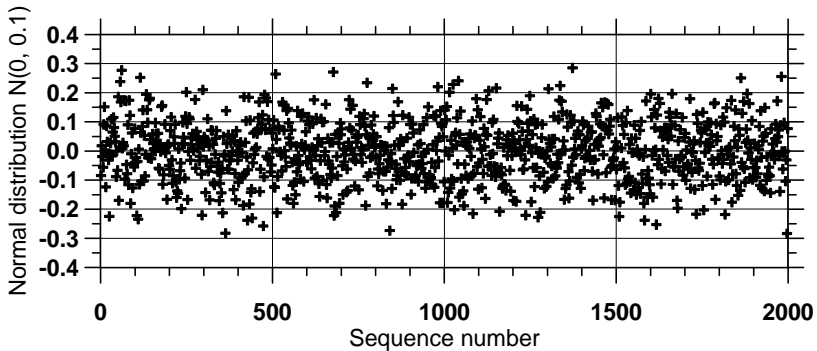


Figure 2. A sample of normally distributed pseudorandom numbers $N(0, 0.1)$.

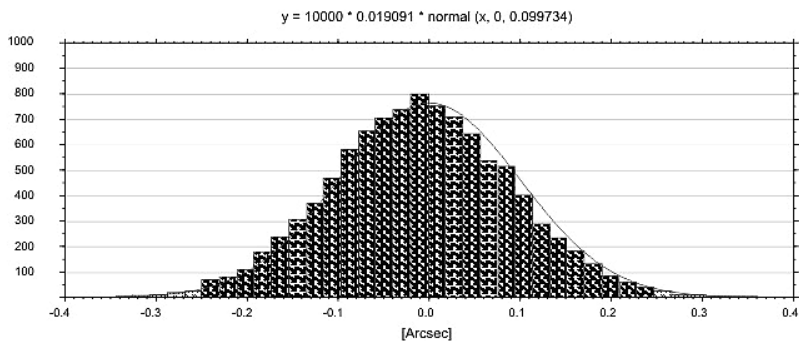


Figure 3. Normally distributed pseudorandom numbers $N(0, 0.1)$.

4. A MODEL OF RANDOM ERRORS OF ASTRONOMICAL OBSERVATIONS

The error distribution of astronomical observations is rather different from the normal distribution, due to different instruments, observers and climatic conditions. Fig.4 shows a typical distribution of astrometrical errors, corresponding to daily residuals of latitude observations at observatory Plana for the period 1987.5-1997.0. The observations with small dispersion form a “hat” above the normal fit, while the less accurate observations form “arms” with significant great errors, but belonging to the confidential interval $(-3\sigma, +3\sigma)$. An empirical model S_i of random errors of astronomical observations, which is close to the real distribution from Fig.4, is composed by 10000 normally distributed numbers $N(0, 0.1)$, where 13% of the numbers are replaced by:

- 1% evenly distributed numbers from intervals $(-0.8, -0.4]$, $[0.4, 0.8)$;
- 6% evenly distributed numbers from intervals $(-0.4, -0.1]$, $[0.1, 0.4)$;
- 6% evenly distributed numbers from intervals $(-0.06, 0.06)$.

The following algorithm is used to create sequence S_i containing 10000 numbers, satisfying the above conditions:

1. A sequence of 10000 normally distributed numbers X_i is computed by means of Forsythe, Malcolm and Moler method (1977) and computer generator of evenly distributed random numbers.

2. New 1300 evenly distributed random numbers U_n belonging to the interval $[0, 1)$ are computed and a new sequence of evenly distributed random numbers $K_n = 9999 U_n + 1$, belonging to the interval $[1, 10000)$.

3. New 1300 evenly distributed random numbers U_m , belonging to the interval $[0, 1)$ are formed, and following evenly distributed random sequences:

- a) 100 numbers $P_m = \pm(0.4 U_m + 0.4)$ belonging to the intervals $(-0.8, -0.4]$, $[0.4, 0.8)$;
- b) 600 numbers $Q_m = \pm(0.3 U_m + 0.1)$ belonging to the intervals $(-0.4, -0.1]$, $[0.1, 0.4)$;
- c) 600 numbers $R_m = 0.12 U_m - 0.06$ belonging to the interval $[-0.06, 0.06)$.

4. The final sequence S_i is created by replacing the elements of sequence X_i by values of P_m, Q_m and R_m , whose places are determined by the sequence K_n .

The distribution of the errors, computed by the above algorithm is shown in Fig. 5.

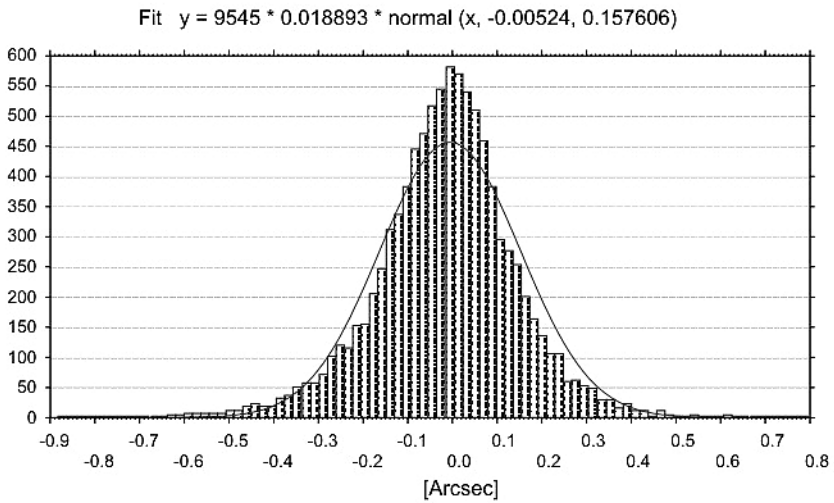


Figure 4. Typical astrometrical errors, corresponding to daily residuals of latitude observations at observatory Plana for the period 1987.5-1997.0.

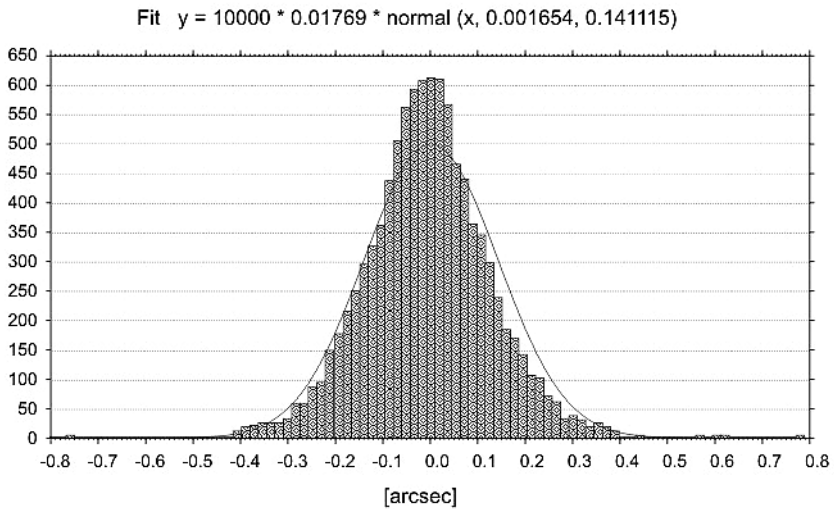


Figure 5. Distribution of errors, according to the model of random errors of astrometrical observations.

Three cases of astrometrical observations with simulated random errors are considered with standards 0.5arcsec, 1.0arcsec and 5.0arcsec. Three different sequences with 2000 random errors with these standards are shown in Figures 6, 7 and 8.

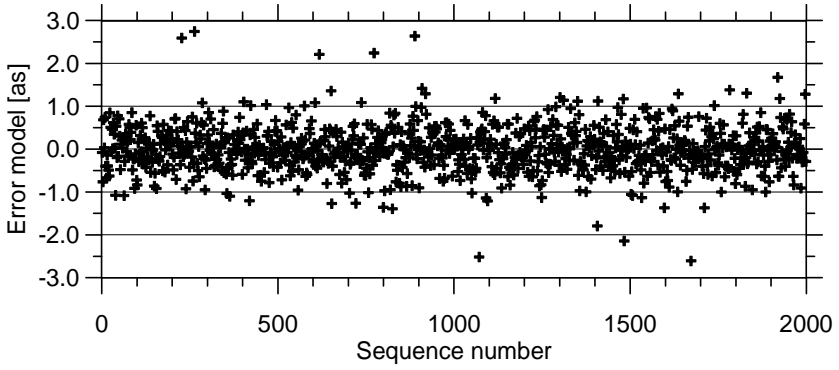


Figure 6. Error model of astrometrical observation from WFPDB with standard 0.5arcsec.

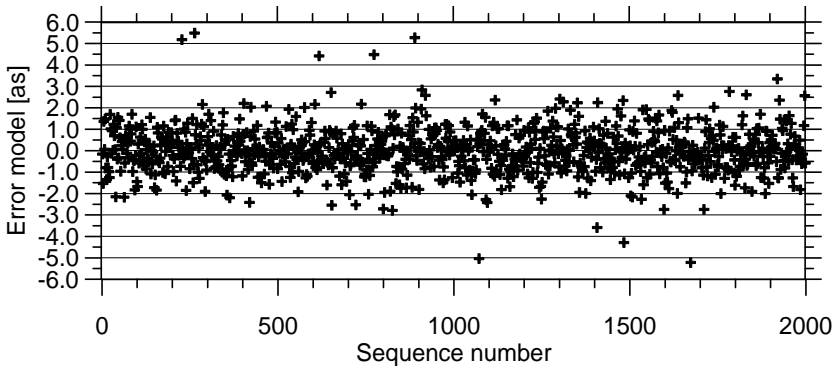


Figure 7. Error model of astrometrical observation from WFPDB with standard 1.0arcsec.

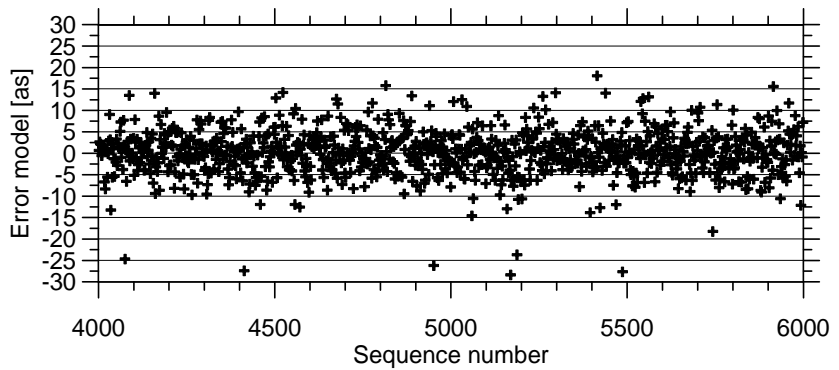


Figure 8. Error model of astrometrical observation from WFPDB with standard 5.0arcsec.

5. ESTIMATION OF THE PROPER MOTION ACCURACY OF WFPDB STARS

The proper motion accuracy of WFPDB stars is estimated by means of three overlapping plates sequences, containing 150, 700 and 2000 plates (Fig.1) and error models with standard 0.5arcsec, 1.0arcsec and 5.0arcsec. The model with standard 0.5arcsec corresponds to the case of corrected star images for the plate center and non-linear distortion. The model with standard 1.0arcsec corresponds to the case of raw plate center and corrected distortion and the last case with standard 5.0arcsec arise when no preliminary corrections of the plate center and distortion are applied. Three values of the modeled proper motion are used - 0.001arcsec/a; 0.01arcsec/a and 0.05arcsec/a. One of the resulting observational series is shown in Fig.9. The unknown parameters are estimated by the Least Squares Method as a linear regression to the data. The results are shown in Table 2. The observational series with standard error 0.5arcsec yield stable solution for stars with proper motion greater than 10mas/a and for slowly moving stars in case of a sufficient number of observations (here greater than 700). The observational series without preliminary corrections for plate center and/or distortion are applicable in the cases of big number of observations and great values ($>10\text{mas/a}$) of the proper motions. It is possible to improve their application by involving robust Danish method for parameter estimation (Kubik, 1982; Juhl, 1984; Kegel, 1987) or Hampel's method with Somogy's modification (Hampel, 1973, 1974; Somogyi, 1987). These methods allow to detect and isolate outliers and to obtain very accurate and reliable solution for the linear trends.

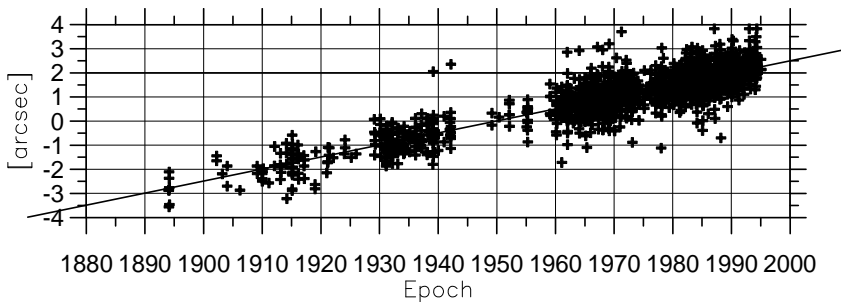


Figure 9. Simulated astrometrical observations with real epochs of a star from WFPDB and error model with standard 0.5arcsec. The number of the observation is 2000 and the modeled proper motion is 0.05arcsec/a.

Table 2: Estimated proper motion of stars from WFPDB with different error models and observation numbers.

| Standard of the error model | Observations number | Initial proper motion [mas/a] | | |
|-----------------------------|---------------------|-------------------------------|----------|----------|
| | | 1 | 10 | 50 |
| | | Estimates | | |
| 0.5as | 2000 | 0.84±0.59 | 9.8±0.6 | 49.8±0.6 |
| | 700 | 1.3±1.1 | 12.1±1.1 | 52.1±1.1 |
| | 150 | - | 11.9±2.5 | 51.9±2.5 |
| 1.0as | 2000 | - | 9.2±1.1 | 49.2±1.1 |
| | 700 | - | - | - |
| | 150 | - | - | - |
| 5.0as | 2000 | - | 7.4±4.9 | 48.4±5.9 |
| | 700 | - | - | - |
| | 150 | - | - | - |

6. CONCLUSIONS

- The WFPDB provide significant number of plates, containing a given star. The plate numbers vary from hundreds to several thousands and they cover a century time span. It is possible to estimate the star proper motion with sufficient accuracy by means of the corrected plates for image center and distortion with 100^μ/mm scale and scan resolution 2400dpi.
- It is possible to use the raw star images for estimation of the great proper motions, if the number of plates is greater than 1000.

Acknowledgments

The present was supported by contract DO 02-275 with the Bulgarian NSF and grant LC506, awarded by the Ministry of Education, Youth and Sports of the Czech Republic.

References

- Chapanov, Ya., Tsvetkova, K., Tsvetkov, M., Vondrák, J., Ron, C., Štefka, V.: 2010, in Proc. of the 7th Bulgarian-Serbian Astronomical Conference (This Issue).
- Forsythe, G. E., Malcolm, M. A., Moler, C. B.: 1977, *Computer Methods for Mathematical Computations*. Englewood Cliffs, New Jersey 07632. Prentice Hall, Inc., 259
- Hampel, F. R.: 1973, *Z. Wahrscheinlichkeitstheorie verw. Geb.*, **27**, 87-104.
- Hampel, F. R.: 1974, *Journal of the American Statistical Association*, **69**, 383-393.
- Juhl, J.: 1984, in XV ISP Congress proc., Comm. III, Rio de Janeiro.
- Kegel, J.: 1987, *Vermessungstechnik*, **35**, №10, Berlin.
- Kubik, K.: 1982, ISP Symposium, Comm. III, Helsinki.
- Somogyi, J.: 1987, DGK, Reihe A.