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STATISTICAL ANALYSIS OF LANGMUIR WAVES ASSOCIATED WITH TYPE III RADIO BURSTS

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Abstract. Interplanetary electron beams, produced by CMEs and flares, are unstable in the solar wind and generate Langmuir waves at the local plasma frequency (f_p) or its harmonic ($2f_p$). Radio observations of the waves in the range 4 - 256 kHz from the WAVES experiment onboard the WIND spacecraft have been statistically analyzed. A subset of 36 events has been selected for this study. The background consisting of thermal noise, type III bursts and Galactic background has been removed and the remaining power spectral density has been fitted by Pearson's system of probability distributions. The coefficients of the probability distributions have been calculated by using two methods: method of moments and maximum likelihood estimation method. We have shown that the probability distributions of the power spectral density of the Langmuir waves belong to three different types of Pearson's probability distributions: type I, type IV and type VI. In order to compare the goodness of the fits, a few statistical tests have been applied, showing for all of the considered events that the Pearson's probability distributions fit the data better than the Gaussian ones. This is in contradiction with the Stochastic Growth Theory which predicts log-normal distribution for the power spectral density of the Langmuir waves. The uncertainty analysis that has been performed also goes in favor of the use of Pearson's system of distributions to fit the data.

1. OBSERVATIONS AND SAMPLE EVENTS SELECTION

We used the measurements obtained by four different experiments on-boarded Wind spacecraft - a laboratory for long-term solar wind measurements, launched on November 1, 1994. We have focused on radio observations obtained by the WAVES experiment (Bougeret et al., 1995). In our study of locally generated Langmuir waves we use data of two multi-channel thermal noise receivers (TNR), which cover the frequency range from 4 kHz to 256 kHz in 5 logarithmically-spaced frequency bands.

Each band covers 2 octaves with one octave overlap. Each of these bands is divided into either 32 or 16 logarithmically-spaced channels. TNR provides rapid measurements of plasma electric field (every 1.5 s or half spacecraft spin). The Langmuir waves that are converted into electromagnetic waves – type III bursts, can then be observed with two radio receivers, RAD1 and RAD2. The RAD1 frequency range, from 20 to 1040 kHz, is divided into 256 linearly spaced channels of 3 kHz bandwidth each. Frequency range of the RAD2 radio receiver, from 1075 to 13825 kHz, is divided in the same number of channels as RAD1, but with 20 kHz bandwidth.

For the selection of a sample events we used: (1) one minute averaged measurements of interplanetary magnetic field vector in Geocentric Solar Ecliptic (GSE) cartesian coordinates from Magnetic field investigation (MFI), (Lepping et al., 1995); (2) for the particles measurements, i.e. for the full three-dimensional distribution of suprathermal electrons and ions, we used 3-D Plasma and Energetic Particle Investigation (3DP) experiment (Lin et al., 1995); (3) for the solar wind velocity we used data from the Solar Wind Experiment (SWE) (Ogilvie, 1995) which provides three-dimensional velocity, density and temperature of the solar wind ions. As the solar wind velocity we used proton velocity averaged over the time interval when our event occurred.

The measurements, taken simultaneously by the four experiments, allow qualitative analysis of the events.

2. ANALYSIS

The stochastic growth theory (SGT) describes situations in which an unstable distribution of particles interacts self-consistently with its driven waves in an inhomogeneous plasma environment and evolves to a state in which the particle distribution fluctuates stochastically about a state close to time and volume averaged marginal stability. These fluctuations drive waves so that the wave gain, $G = 2\ln(E/E_0)$, is a stochastic variable. The wave gain is the time integral of the wave energy density growth rate and it is related to the wave electric field, $E(t)$, by $E^2(t) = E_0^2 \exp[G(t)]$ where E_0 is a constant field. The observed electric field, E, is a consequence of a large number of amplifications and damping: $E = E_0 \prod_{i=1}^{N} e^{G_i}$, $(N \gg 1)$, where gain, G_i, is a stochastic variable. Taking the logarithm of this equation one obtains: $\log E = \log E_0 + \sum_{i=1}^{N} G_i$. The central limit theorem can then be applied to the probability distribution of $\log E$ which is thus a normal distribution (e.g. Robinson, 1992).

In order to see if the Langmuir waves associated with type III solar bursts satisfy predictions of the SGT, we have undertaken the following steps. We have integrated the power spectral density (S_t, index t denotes a certain moment of time) of Langmuir waves through a narrow interval of frequencies (f_1, f_2) around the local plasma frequency (f_p): $P_{LW,t} = \int_{f_1}^{f_2} S_t \, \mathrm{d}f$, $(f_1 < f_p < f_2)$. The integration is done numerically by a trapezium method. In that way we obtain the total power of the Langmuir waves at a given moment of time ($P_{LW,t}$).

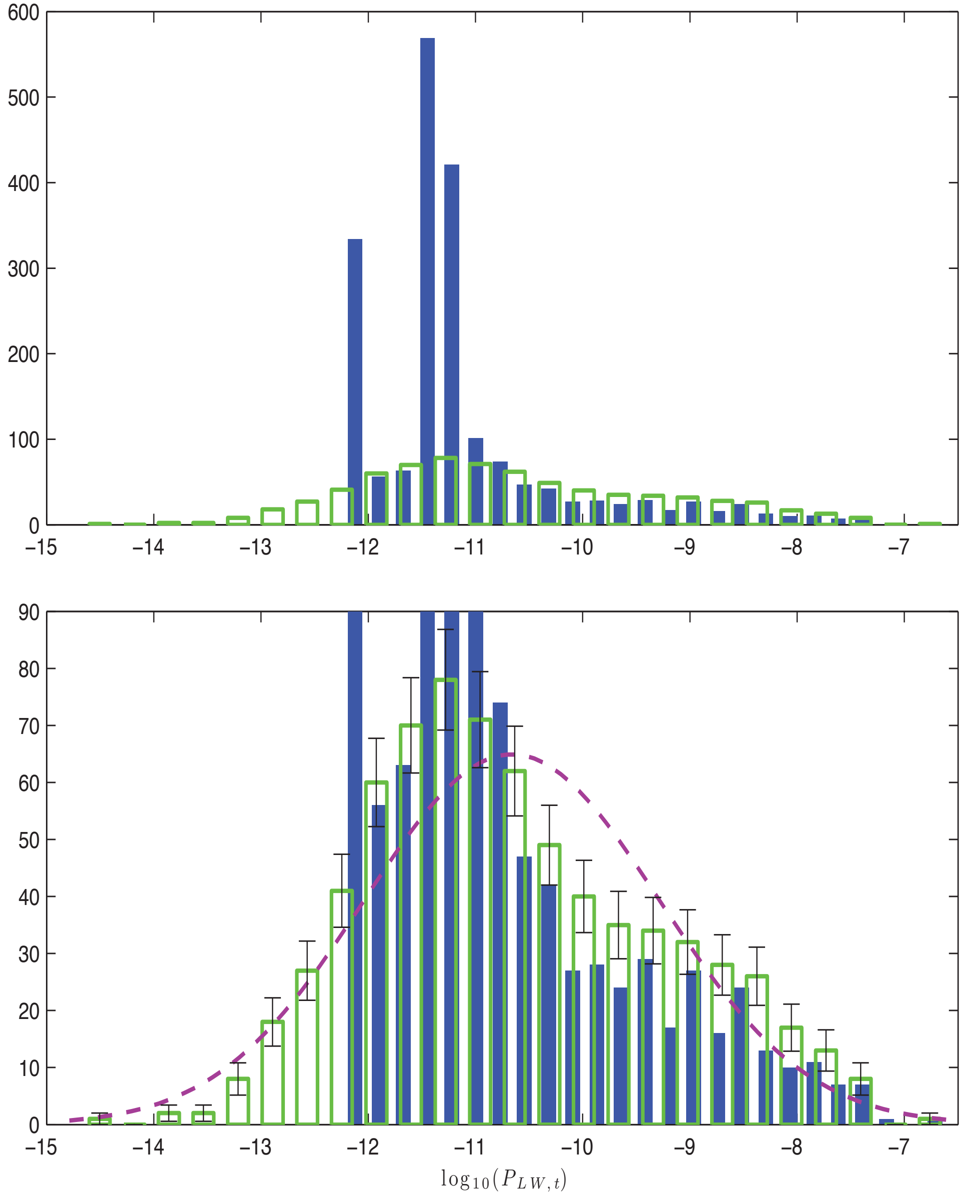

Figure 1. Histograms of Langmuir waves power (2002 October 21st event). Upper panel: Before (filled blue bars) and after (empty green bars) background removing. Lower panel: part of upper panel, dashed line represents Gaussian fit of Langmuir waves power histogram after background removing.

To remove the background consisting of the thermal noise, the type III radio burst and the galactic background, we have developed a heuristic algorithm based on numerical techniques (interpolation and smoothing) with a few parameters. From the remaining data we made new histogram, displayed in Fig. 1 (green empty bars), and fit it with a normal distribution shape function (dashed line in Fig. 1, lower panel). The error bars on the histogram are calculated as standard deviation of counting statistics, i.e. the Poisson distribution.

To find a better approximation for the probability functions we have applied a family of distributions proposed by Pearson (1895).

3. APPLYING pEARSON'S SYSTEM OF DISTRIBUTIONS

When dealing with empirical data with significant skewness and kurtosis, the normal distribution is not the best choice for modeling. The four parameter Pearson's system of distributions is a better choice (see Fig. 2 for an example). It represents a wide class of distributions with a wide variety of shapes and thus provides more accurate representations of the observed data. On the other hand, it includes, as special cases, some well known distributions (normal, beta, gamma, Student's t-distribution etc.).

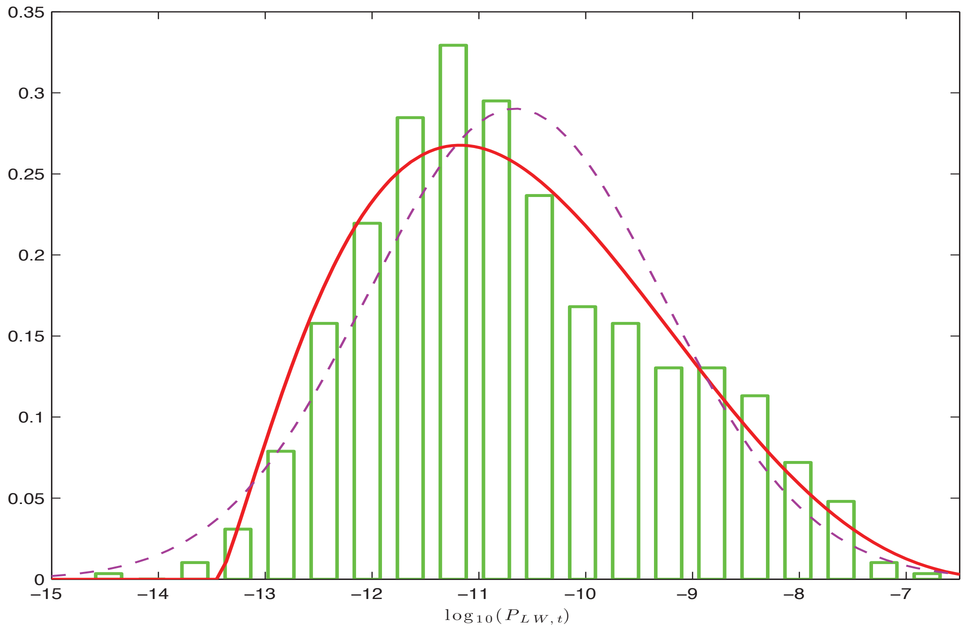

Figure 2. Pearson type I (solid red line) and normal (dashed line) probability density distribution of Langmuir waves power (2002 October 21st event).

In 1895 Pearson (1895) defined this distribution system by the following ordinary first order differential equation for the probability density function $p(x)$:

$$-\frac{p'(x)}{p(x)} = \frac{b_0 + b_1 x}{c_0 + c_1 x + c_2 x},$$

where b_0, b_1, c_0, c_1 and c_2 are five real parameters. After normalizing the fraction with any of them, only four independent parameters remain. The form of the solution of this differential equation depends on the parameter values, resulting in several distribution types. The classification of distributions in the Pearson system is entirely determined by the first moment – mean (μ_1), and the next three central moments (μ_2, μ_3 and μ_4). Pearson proposed two dimensionless parameters, i.e. the two moment ratios square of skewness ($\beta_1 = \mathrm{Sk}^2$) and kurtosis (β_2):

$$\beta_1 = \frac{\mu_3^2}{\mu_2^3}, \quad \beta_2 = \frac{\mu_4}{\mu_2^2}.$$

These two parameters characterize the peakedness and the asymmetry of the distribution and it turns out that the distribution type depends only on two of them. Their values can be estimated from observations (Johnson et al., 1994). The other way of parameter estimation is the Maximal Likelihood Estimation method. For each Pearson distribution type its parameters are determined to maximize the likelihood function of the sample data. The best result over all types is chosen. For optimization we used standard Nelder-Mead and Levenberg-Marquardt methods (Press et al., 2007). Both methods gave very similar results.

We find that our 36 events belong to only 3 types of Pearson's distributions: type I (beta), type IV (not related to any standard distribution) and type VI (beta prime). The positions of all 36 events in the $\beta_1 - \beta_2$ plane are shown in Fig. 3.

Most of the events are close to normal distribution, which is represented by the point $(\beta_1, \beta_2) = (0, 3)$. To see whether they are really different from a normal distribution, i.e. if the point $(0, 3)$ lies within the uncertainty limits of the events, we used two methods to evaluate the error-bars in β_1 and β_2: a Monte Carlo simulation and a method of moments proposed by Karl Pearson (1895). Error bars shown in Fig. 3 are calculated by the method of moments. It is found that the point $(0, 3)$ belongs to only four of the uncertainty ellipses (blue points). Out of 36 events, 32 have no intersection with the normal distribution point, opposite to the predictions of the SGT.

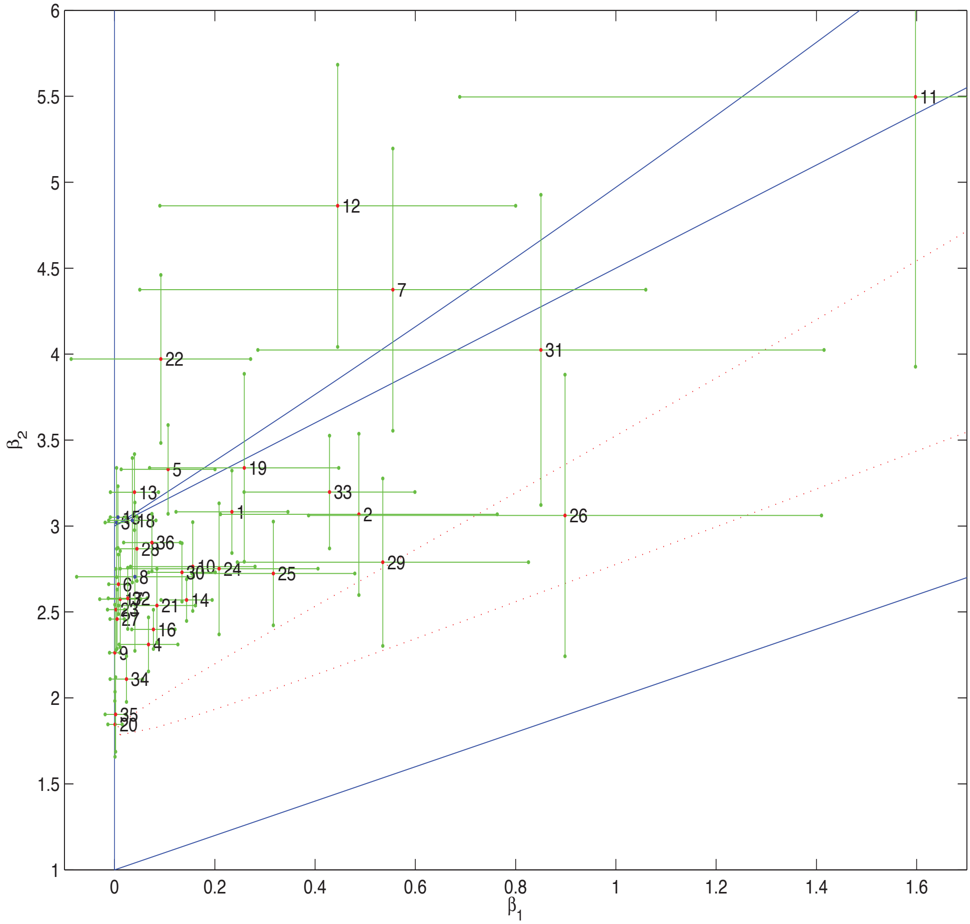

Figure 3. Beta plane. Out of the 36 events: 28 belong to Pearson's type I, 1 to type VI and 7 to type IV probability distribution.

4. CONCLUSIONS

We have examined 36 time intervals containing intense locally formed Langmuir waves that are associated with type III radio bursts. We have shown that the probability distributions of the power of these waves belong to three different types of Pearson's probability distributions: type I, type IV and type VI. The goodness of the fits test (e.g. χ^2) shows that the Pearson's probability distributions fit the data better than Gaussian ones for all of the considered events. This is in contradiction with the SGT which predicts Log-normal distributions for the power of the Langmuir waves. The uncertainty analysis of β_1 and β_2 parameters also goes in favor of the use of Pearson's system of distributions to fit the data.

This result indicates that the SGT possibly requires additional verifications and examinations.

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