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4-PARAMETER TRANSFORMATION OF DIGITIZED ASTRONOMICAL IMAGES

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Abstract. The accuracy of star celestial coordinates determined from astronomical images depends on various parameters. A part of these parameters are the celestial coordinates of the image center and the angle between image axes and the North direction. The 4-parameter transformation (two translations, one rotation and scale) is used to determine the orientation of the digitized astronomical images and the coordinates of their center. The transform parameters are estimated by means of coordinates of identical stars from two images. The influence of the star proper motion uncertainties on the 4-parameter transformation accuracy is investigated by models of digitized astronomical images of one and same part of the sky in different epochs and made from different instruments. The ability of application of the 4-parameter transformation of the digitized astronomical images in determination of star coordinates and proper motions is discussed.

1. INTRODUCTION

The preliminary determination of the precision of the real measurements and observations and their systematic errors and deviations are very important for the quality of scientific research, especially in the cases of observations from classical instruments whose accuracy level is relatively low in comparison with the high-technology modern equipment. When we try to use the digitized astronomical plates, we need to prove that the accuracy of the determined star coordinates from these images and the results of their processing are of the order of 1-2 tenth of arc second, which accuracy is necessary in scientific research of various geodynamic effects connected with the crust deformation, gravity and Earth rotation variations, and etc.

2. 4-PARAMETER HELMERT TRANSFORMATION

The Helmert transformation is used in geodesy. It transforms a set of points into another by rotation, scaling and translation. When both sets of points are given, then least squares can be used to solve the inverse problem of determination of the parameters, or the parameters of the so-called 7 parameter transformation can be obtained by standard methods (Awange et al., 2004). The classic Helmert transformation between two coordinate spatial systems depends on 7 parameters – 3 translations, 3 rotation and a scale parameter. The general form of 7-parameter Helmert transformation between two reference systems *A* and *B* is:

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_B = \begin{bmatrix} C_X \\ C_Y \\ C_Z \end{bmatrix} + (1 + s \times 10^{-6}) \cdot \begin{bmatrix} 1 & -r_Z & r_Y \\ r_Z & 1 & -r_X \\ -r_Y & r_X & 1 \end{bmatrix} \cdot \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_A , \qquad (1)$$

where *X*, *Y* and *Z* are rectangular coordinates, c_X, c_Y, and c_Z are translations, r_X, r_Y, and r_Z are rotations and s is the scale, so that the value of the scale factor $\mu=1+s$ to be around 1. The 4-parameter transformation is obtained from (1) where *Z*, c_Z, r_X and r_Y are zeroes, so the 2-dimension coordinate systems are transformed by the 4-parameter Helmert transformation in the form

$$\begin{aligned} X_B &= c_X + (1 + s \times 10^{-6}) \cdot (X_A - r_Z Y_A), \\ Y_B &= c_Y + (1 + s \times 10^{-6}) \cdot (r_Z X_A + Y_A). \end{aligned} \qquad (2)$$

3. STAR IMAGES AND ERROR MODEL

The model of simulated observations is created by a set of star coordinates from sky image, containing 317 generated by the program StarGazer (author Rumen Bogdanovski, a member of Astroinformatics Project team) with 2arcmin field width (Fig.1). The star image is possible to enlarge and to rescale star coordinates so that the output field width reach 2 degrees. The resulting star coordinates form the first set of points which coordinates correspond to a given star catalog. The second set of identical points is formed from the first set with the assumption that if the simulated star image center is shifted from the true point, small rotation of the image in the focal plane exists and the image scale, which transforms photo coordinates to celestial coordinates is known approximately. The initial values of these systematic errors are chosen as follow: shift of the center coordinates X_0=180 arcsec, Y_0=240 arcsec, rotation around *Z* axis r_Z= –120 arcsec, plate scale uncertainty s = 100 ppm (part per million). The model of random errors of the second set of simulated star coordinates, obtained from the digitized astronomical plate is described in details in (Chapanov et al., 2011). Three different cases of the level of random errors are considered (I) – with standard 0.5 arcsec, which is the best case, (II) – with standard 1.0 arcsec, which corresponds to the case of middle precision, (III) – with standard 5.00 arcsec, a case with poor accuracy. Two more cases are considered: (IV) – with standard 0.5 arcsec and 0.5 arcsec/a uncertainties of the star proper motions on 10-year time spans, and (V) – a case with huge scale error ($s = 10^6$ ppm).

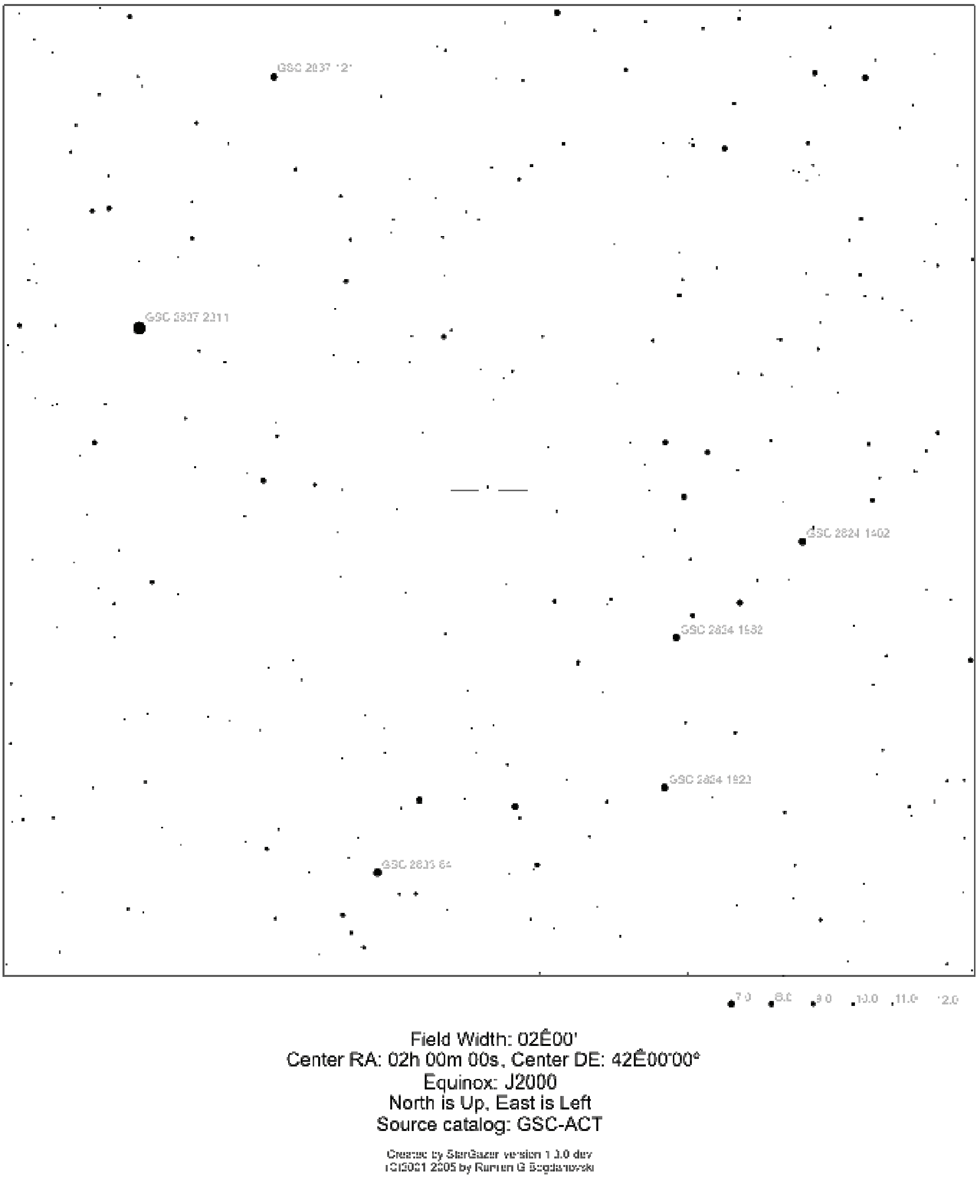

Figure 1: Map of the chosen sky field, containing 317 stars. The map is generated by the program StarGazer, version 1.3.0-dev and source star catalog GSC-ACT.

4. RESULTS

The results for 5 cases of application of 4-parameter transformation to the simulated star coordinates derivation from digitized astronomical plates are given in Tables 1-5.

4.1. Results for error model I, σ=0.5 as

Max. errors <0”.05

Table 1: Results for error model I, σ=0.5 as

Model		Solution	Differences	Errors [as]
Parameters	Initial values			
X [as]	180.0	180.03	–0.03	0.03
Y [as]	240.0	240.02	–0.02	0.02
Rz [as]	–120.0	–119.48	–0.52	
S [ppm]	100.0	92.96	7.04	

4.2. Results for error model II, σ=1.0 as

Table 2: Results for error model II, σ=1.0 as

Model		Solution	Differences	Errors [as]
Parameters	Initial values			
X [as]	180.0	180.03	–0.03	0.03
Y [as]	240.0	240.01	–0.01	0.01
Rz [as]	–120.0	–121.28	1.28	
S [ppm]	100.0	99.20	0.80	

4.3. Results for error model III, σ=5.0 as

Table 3: Results for error model III, σ=5.0 as

Model		Solution	Differences	Errors [as]
Parameters	Initial values			
X [as]	180.0	180.24	–0.24	0.24
Y [as]	240.0	240.15	–0.15	0.15
Rz [as]	–120.0	–121.12	1.12	
S [ppm]	100.0	87.20	12.80	

Max. errors <0”.4

4.4. Results for error model I with proper motion errors

Table 4: Results for error model I with proper motion errors

Model		Solution	Differences	Errors [as]
Parameters	Initial values			
X [as]	180.0	180.22	–0.22	0.22
Y [as]	240.0	240.15	–0.15	0.15
Rz [as]	–120.0	–112.78	–7.22	
S [ppm]	100.0	143.54	0.80	

Max. errors <0".5

4.5. Case of large scale

Table 5: Results for the case of large scale

Model		Solution	Differences	Errors [as]
Parameters	Initial values			
X [as]	360.0	180.22	–0.22	0.22
Y [as]	480.0	240.15	–0.15	0.15
Rz [as]	–120.0	–112.78	–7.22	
S [ppm]	$1x10^6$	1000085.93	–85.93	

Max. errors <0".6

5. CONCLUSIONS

It is possible to estimate the rotation and image center of digitized astronomical plates by the 4-parameter Helmert transformation if the number of identical stars is more than 100. The estimation accuracy is better than 0.4 as at the image borders and better than 0.1as in the center for plates with 100"/mm scale, scan resolution 2400dpi and total RMS errors of star positions below 5 as.

The necessary preprocessing of digitized astronomical images are image convert to small scale, determination of identical stars from the image and star catalog and correction of the atmosphere refraction and distortions due to telescope optics and projection of celestial sphere over the focal plane.

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