LONG TERM PRECESSION MODEL

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- ♦ **Solution for different precession parameters;**
- ♦ **Estimation of accuracy, comparison with other models;**
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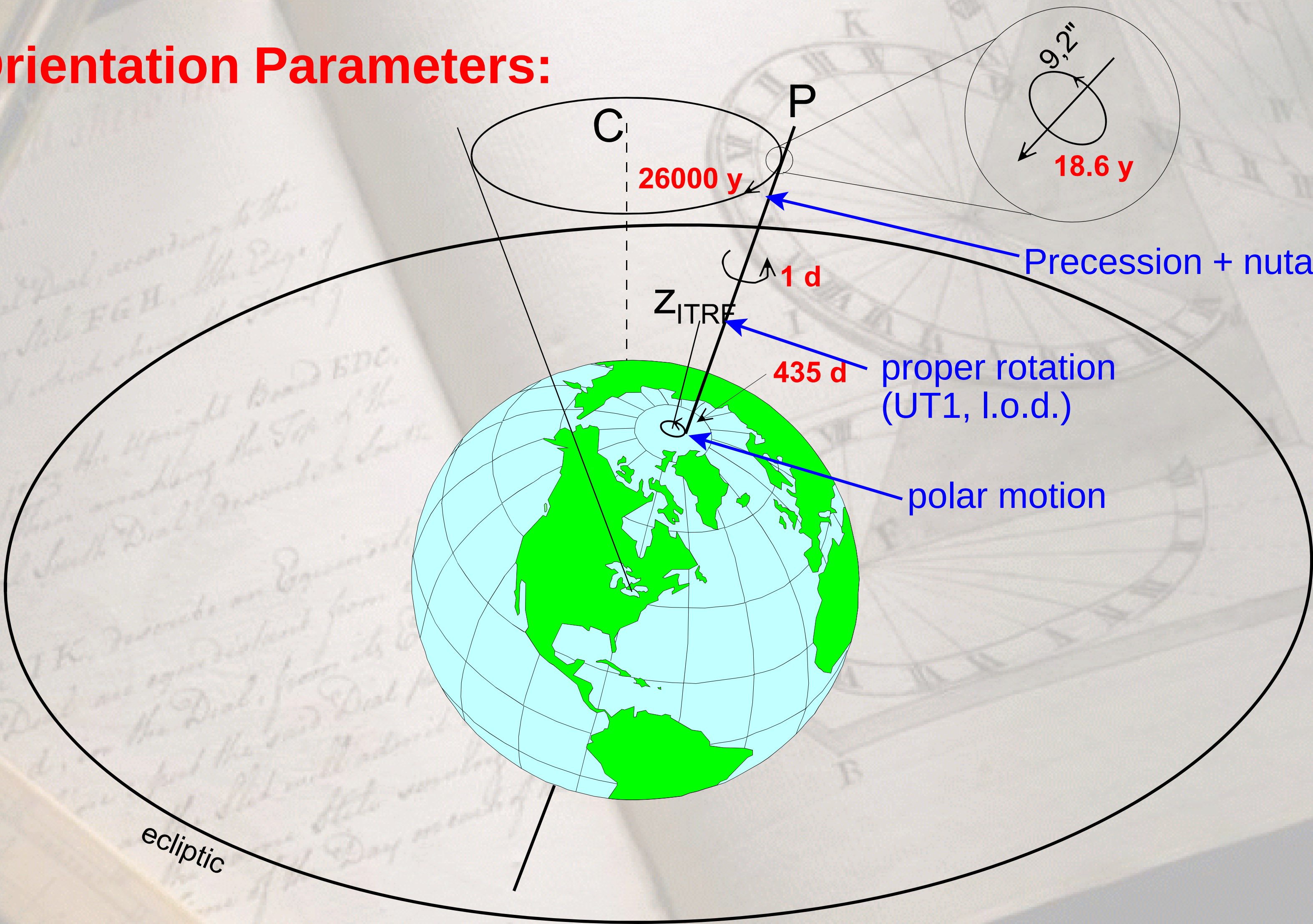
Introduction:

Earth Orientation Parameters:



- The axis of rotation of the Earth is not stable in the inertial reference frame:
  - Under the dominant influence of the Moon and the Sun, it exhibits a rather complicated motion, called precession-nutation.
  - Its very long-periodic part, precession (known already to Hipparchus, app. 150 B.C.), is the slow motion of the pole of Earth’s rotation around the pole of the ecliptic. The angle between the two poles (obliquity) is approximately constant, roughly equal to 23.5°.
  - The axis of rotation of the Earth makes one revolution around the pole of the ecliptic in about 26 thousand years (Platonic year).

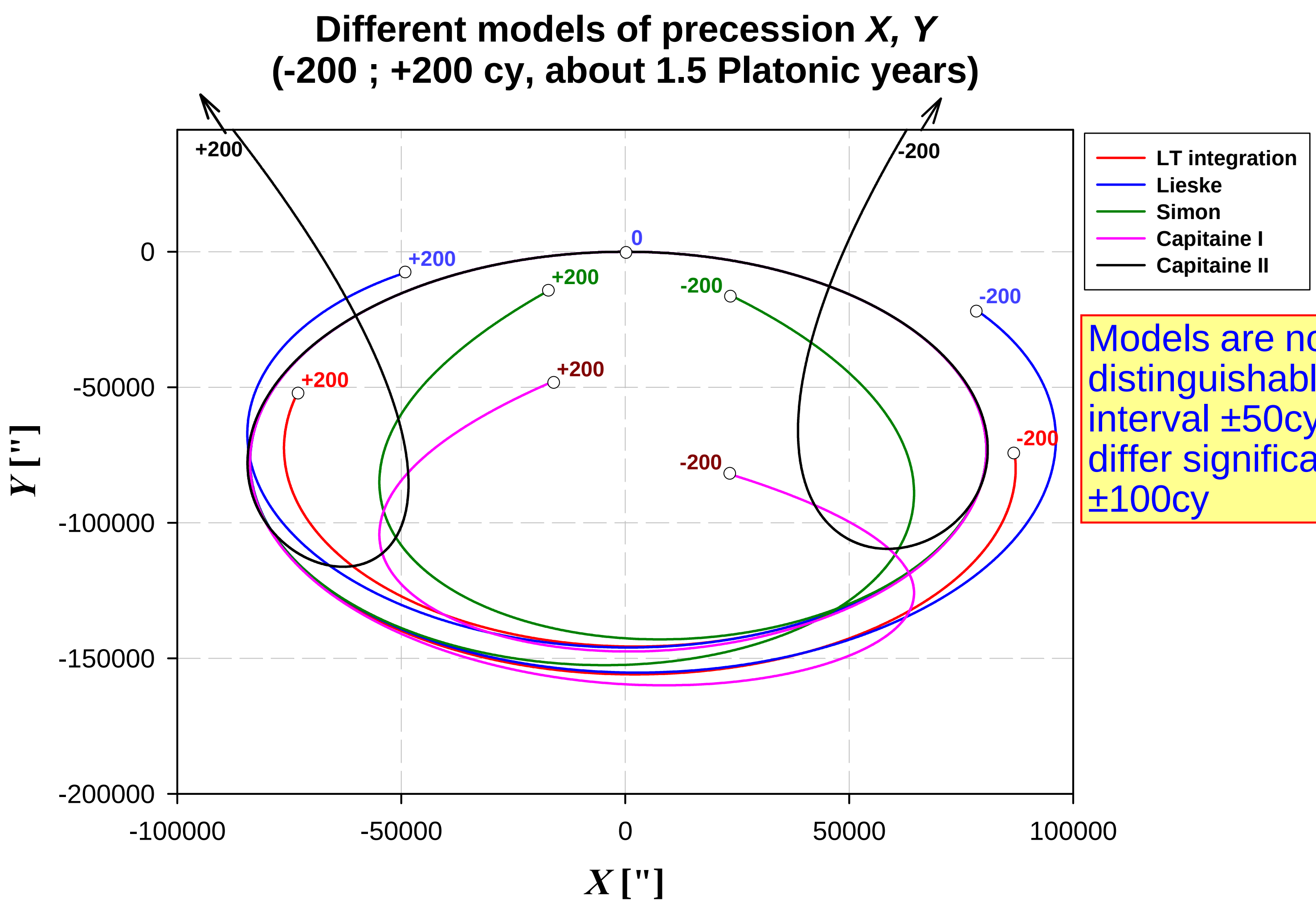
- ♦ Pole of the ecliptic itself is not stable with respect to the stars – it exhibits so called precession of the ecliptic (formerly planetary precession). It is dominantly caused by the attractive forces of all bodies of the solar system.
- ♦ The axis of rotation of the Earth exhibits a motion around the moving pole of ecliptic under the torques exerted by the Moon, Sun, and planets on the rotating oblate Earth, called precession of the equator (formerly luni-solar precession).

♦ To account for precession, precession matrix must be formed:

- ♦ Usually, it consists of three to four rotations, using different sets of precession parameters (angles or direction cosines - see later).

Motivation:

- All precession models used so far are expressed in terms of polynomial developments, no matter which precession parameters are used.
- Most recent model IAU2006 is very accurate, but usable only for a limited time interval (several centuries around the standard epoch J2000):
  - its errors rapidly increase with longer time spans!
- In reality, precession represents a complicated, very long-periodic process, with periods of hundreds of centuries:
  - this can be seen in numerically integrated equations of motion of the Earth in the solar system and its rotation.



Models are not graphically distinguishable in the interval ±50cy, they start to differ significantly outside ±100cy

Goal:

- We assume that precession covers all periods longer than 100 centuries; shorter ones are included in the nutation;
- The goal is to find relatively simple expressions of different precession parameters, with accuracy comparable to the IAU2006 model near the epoch J2000.0, and lower accuracy outside the interval ±1000 years (up to several minutes of arc at the extreme epochs ±200 thousand years);
- New model was published recently:
  - Vondrák J., Capitaine N., Wallace P., A&A 534, A22 (2011).

Methods:

- We use the numerically integrated values (which were shown to be consistent with more recent values):
  - of the precession of the ecliptic P_A, Q_A (Chambers 1999);
  - of the general precession/obliquity p_A, ε_A (Laskar 1993)
- to calculate different precession parameters in the interval ±200 thousand years from J2000.0, with 100-year step;
- Central part (±1000 years from J2000.0) is replaced by IAU2006 values;
- These series are then approximated by a cubic polynomial plus up to 14 long-periodic terms, so that the fit is best around J2000.0.

Precession parameters

a) input (from numerical integration):

$P_A = \sin\pi_A \sin\Pi_A$, $Q_A = \sin\pi_A \cos\Pi_A$, p_A, ε_A

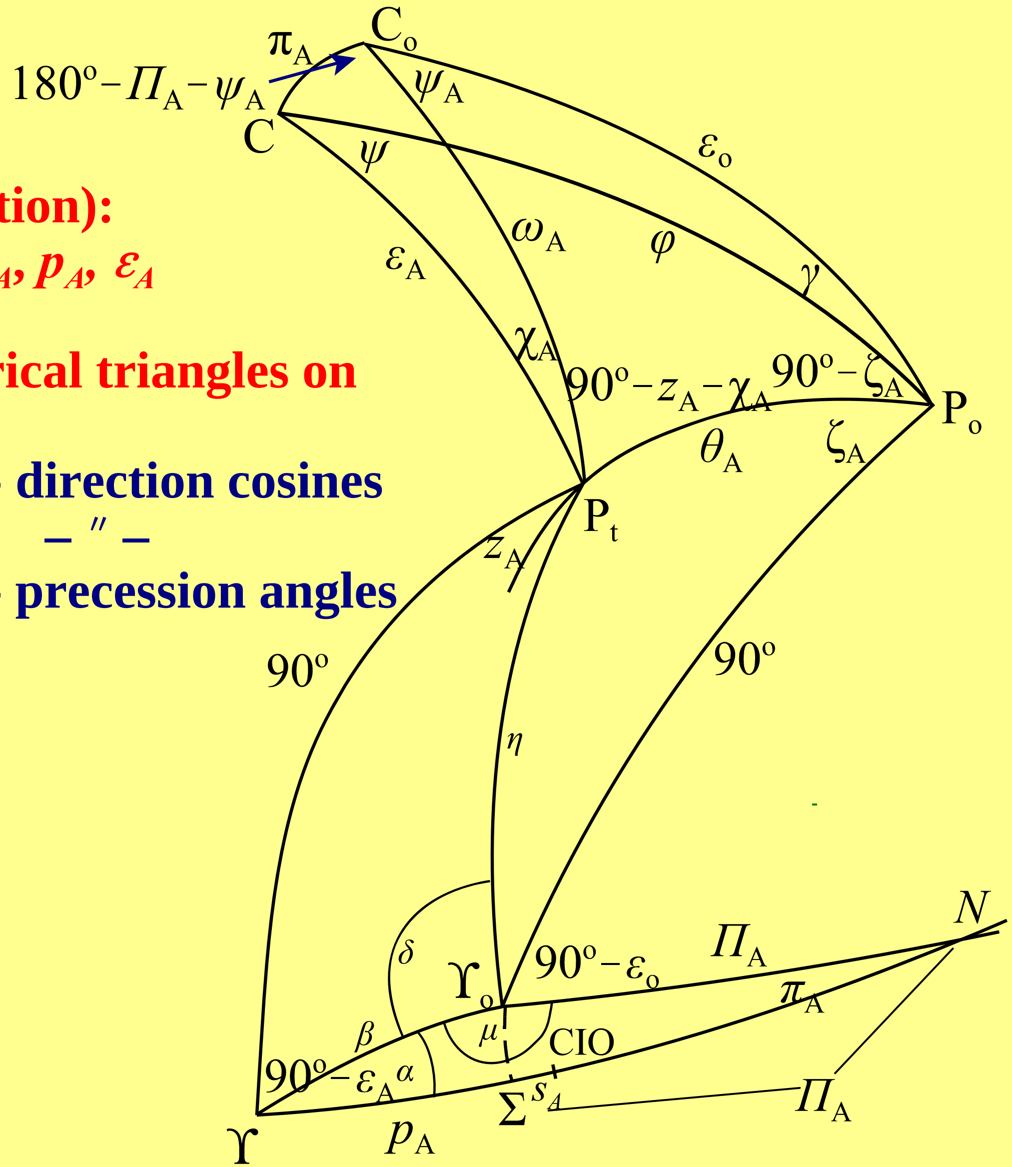
b) derived (computed from spherical triangles on the right):

$X_A = \sin\theta_A \cos\zeta_A$, $Y_A = \sin\theta_A \sin\zeta_A$ - direction cosines

$V_A = \sin\theta_A \sin z_A$, $W_A = \sin\theta_A \cos z_A$ – " –

ω_A, ψ_A, χ_A, φ, γ, ψ - precession angles

CIO locator s_A



Alternative possibilities of expressing precession matrix:

$$\boldsymbol{P} = \boldsymbol{R}_3(-z_A)\boldsymbol{R}_2(\theta_A)\boldsymbol{R}_3(-\zeta_A)$$ "Lieske"

$$\boldsymbol{P} = \boldsymbol{R}_3(\chi_A)\boldsymbol{R}_{\mathbf{1}}(-\omega_A)\boldsymbol{R}_3(-\psi_A)\boldsymbol{R}_{\mathbf{1}}(\varepsilon_{\mathbf{0}})$$ "Capitaine"

$$\boldsymbol{P} = \boldsymbol{R}_{\mathbf{1}}(-\varepsilon_A)\boldsymbol{R}_{\mathbf{3}}(-\psi)\boldsymbol{R}_{\mathbf{1}}(\varphi)\boldsymbol{R}_{\mathbf{3}}(\gamma)$$ "Williams-Fukushima"

All precession parameters expressed as:

$$a + bT + cT^2 + dT^3 + \sum_{i=1}^{n} \left(C_i \cos 2\pi T / P_i + S_i \sin 2\pi T / P_i \right),$$

where T is time in centuries since J2000.0, P_i are the periods and n is the number of periodic terms (8 to 14), coefficients c, d are very small.

All coefficients estimated by least-squares method, with weights equal to 10^4 in the interval ±1000y from J2000.0, and $1/T^2$ outside, to assure the best fit to the time series in the central part.

Two examples:

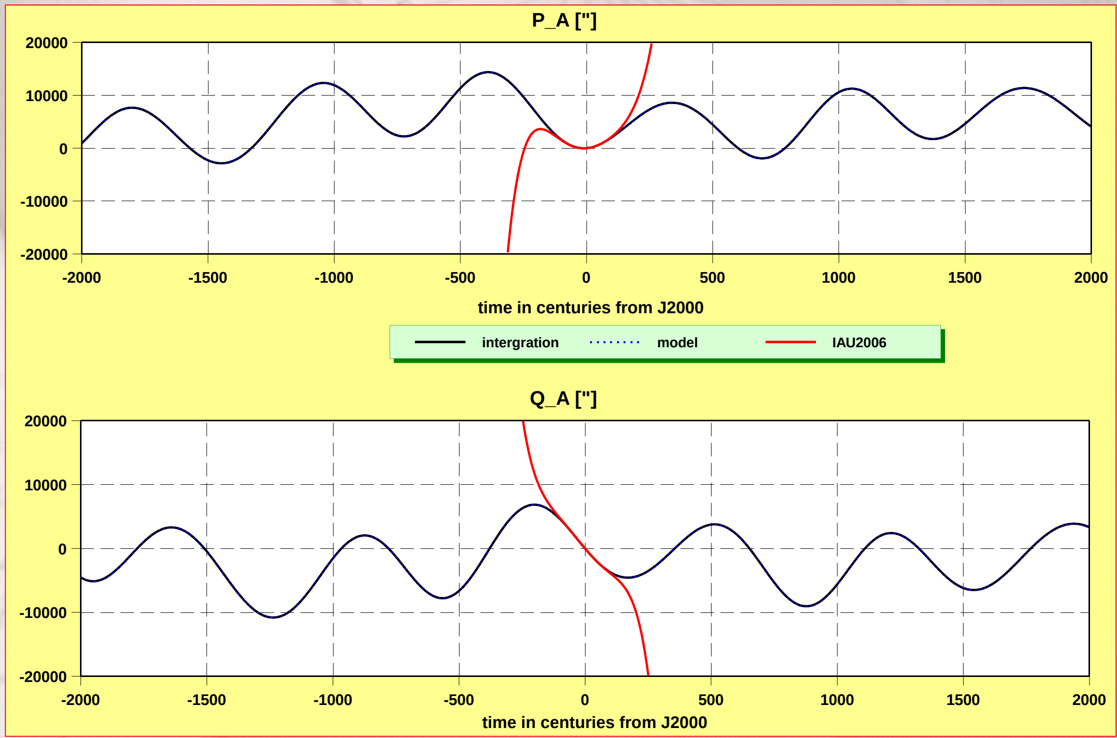
- ♦ Precession of the ecliptic:
  - ♦ Direction cosines of the pole C - P_A, Q_A;
- ♦ Precession of the equator:
  - ♦ Direction cosines of the pole P - X_A, Y_A.

Long-periodic expressions for P_A, Q_A [″]:

$$P_A = 5851.607687 - 0.1189000T - 0.00028913T^2 + 101\times10^{-9}T^3 + \sum_1^8 \left(C_i \cos 2\pi T/P_i + S_i \sin 2\pi T/P_i\right)$$

$$Q_A = -1600.886300 + 1.1689818T - 0.00000020T - 437\times10^{-9}T^3 + \sum_1^8 \left(C_i \cos 2\pi T/P_i + S_i \sin 2\pi T/P_i\right)$$

Term		P_A	Q_A	P [cy]
σ_3	C_1	-5486.751211	-684.661560	708.15
	S_1	667.666730	-5523.863691	
$-s_1$	C_2	-17.127623	2446.283880	2309.00
	S_2	-2354.886252	-549.747450	
	C_3	-617.517403	399.671049	1620.00
	S_3	-428.152441	-310.998056	
$-s_6$	C_4	413.855033	-356.652376	492.20
	S_4	376.202861	421.535876	
	C_5	78.614193	-186.387003	1183.00
	S_5	184.778874	-36.776172	
	C_6	-180.732815	-316.800070	622.00
	S_6	335.321713	-145.278396	
	C_7	-87.676083	198.296701	882.00
	S_7	-185.138669	-34.744450	
	C_8	46.140315	101.135679	547.00
	S_8	-120.972830	22.885731	

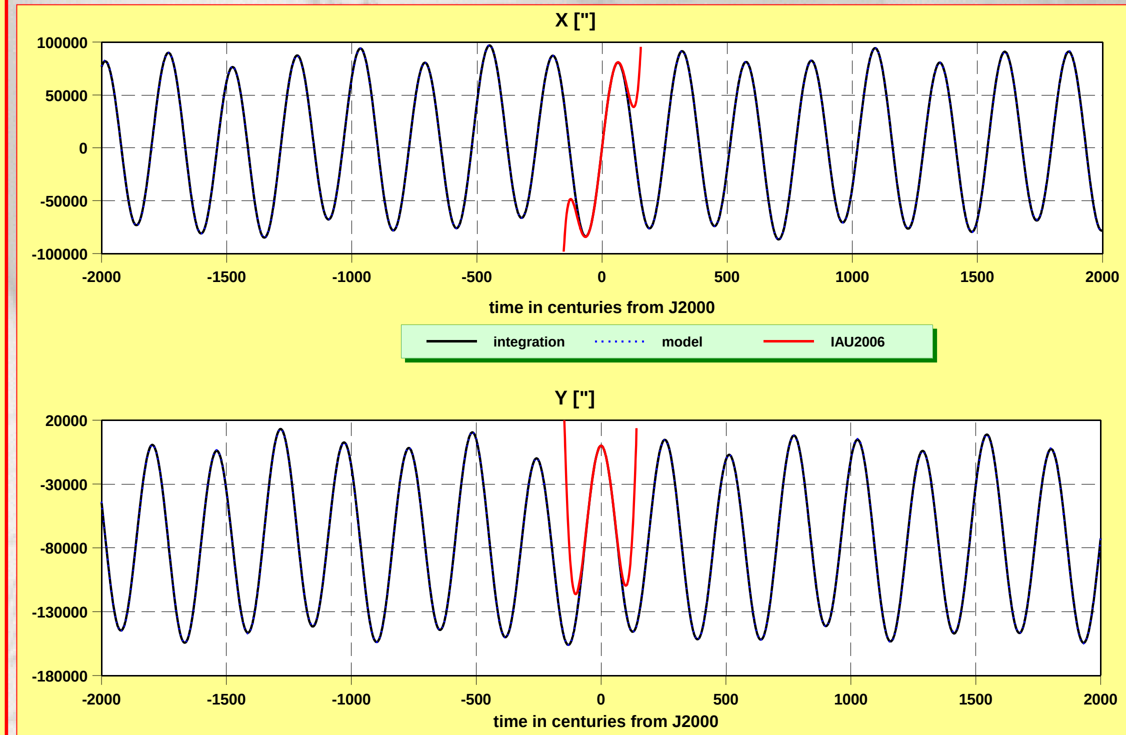


Long-periodic expressions for X_A, Y_A [″]:

$$X_A = 5453.282155 + 0.4252841T - 0.00037173T^2 - 152\times10^{-9}T^3 + \sum_1^{14}\left(C_i\cos2\pi T/P_i + S_i\sin2\pi T/P_i\right)$$

$$Y_A = -73750.930350 - 0.7675452T - 0.00018725T + 231\times10^{-9}T^3 + \sum_1^{14}\left(C_i\cos2\pi T/P_i + S_i\sin2\pi T/P_i\right)$$

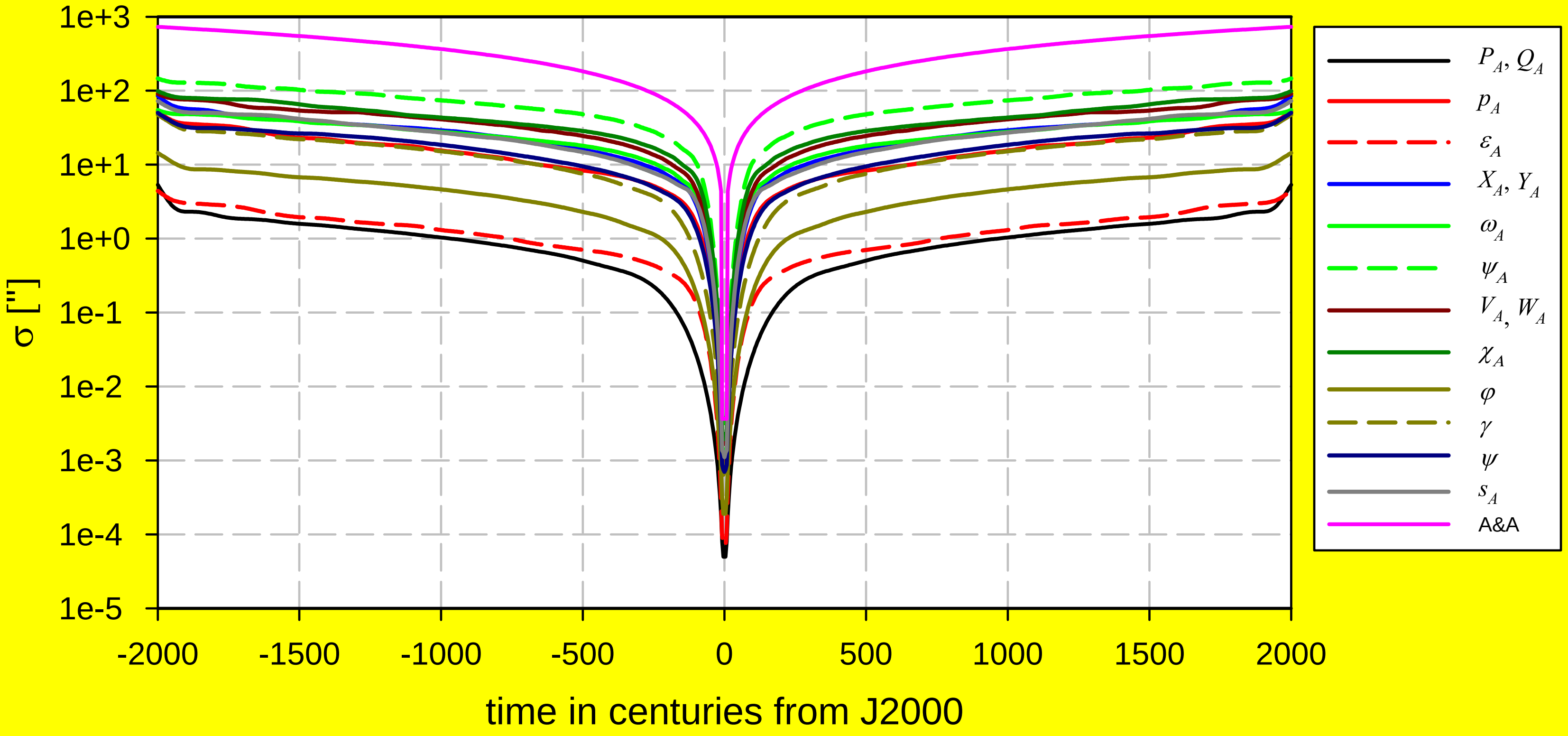
Term		X_A	Y_A	P [cy]
p	C_1	-819.940624	75004.344875	256.75
	S_1	81491.287984	1558.515853	
$-\sigma_3$	C_2	-8444.676815	624.033993	708.15
	S_2	787.163481	7774.939698	
$p-g_2+g_5$	C_3	2600.009459	1251.136893	274.20
	S_3	1251.296102	-2219.534038	
$p+g_2-g_5$	C_4	2755.175630	-1102.212834	241.45
	S_4	-1257.950837	-2523.969396	
$-s_1$	C_5	-167.659835	-2660.664980	2309.00
	S_5	-2966.799730	247.850422	
$-s_6$	C_6	871.855056	699.291817	492.20
	S_6	639.744522	-846.485643	
$p+s_4$	C_7	44.769698	153.167220	396.10
	S_7	131.600209	-1393.124005	
$p+s_1$	C_8	-512.313065	-950.865637	288.90
	S_8	-445.040117	368.526116	
$p-s_1$	C_9	-819.415595	499.754645	231.10
	S_9	584.522874	749.045012	
	C_{10}	-538.071099	-145.188210	1610.00
	S_{10}	-89.756563	444.704518	
	C_{11}	-189.793622	558.116553	620.00
	S_{11}	524.429630	235.934465	
$2p+s_3$	C_{12}	-402.922932	-23.923029	157.87
	S_{12}	-13.549067	374.049623	
	C_{13}	179.516345	-165.405086	220.30
	S_{13}	-210.157124	-171.330180	
	C_{14}	-9.814756	9.344131	1200.00
	S_{14}	-44.919798	-22.899655	



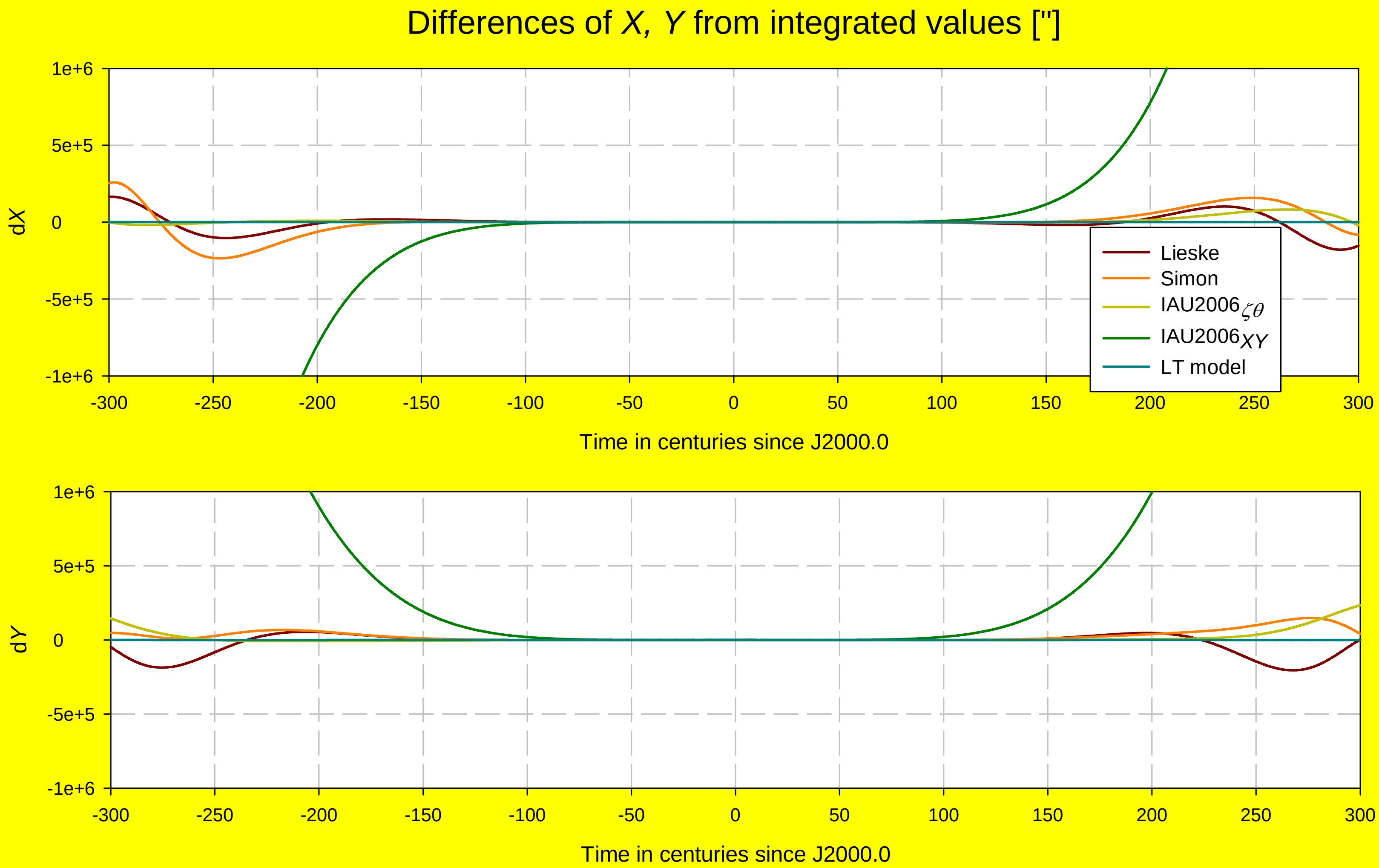
Estimation of accuracy at different epochs from the fit with numerically integrated values:

- In A&A paper, we used a simple expression based on the average sigma and weights at different epochs;
- Here we use a rigorous procedure, based on full variance-covariance matrix, for each parameter separately.

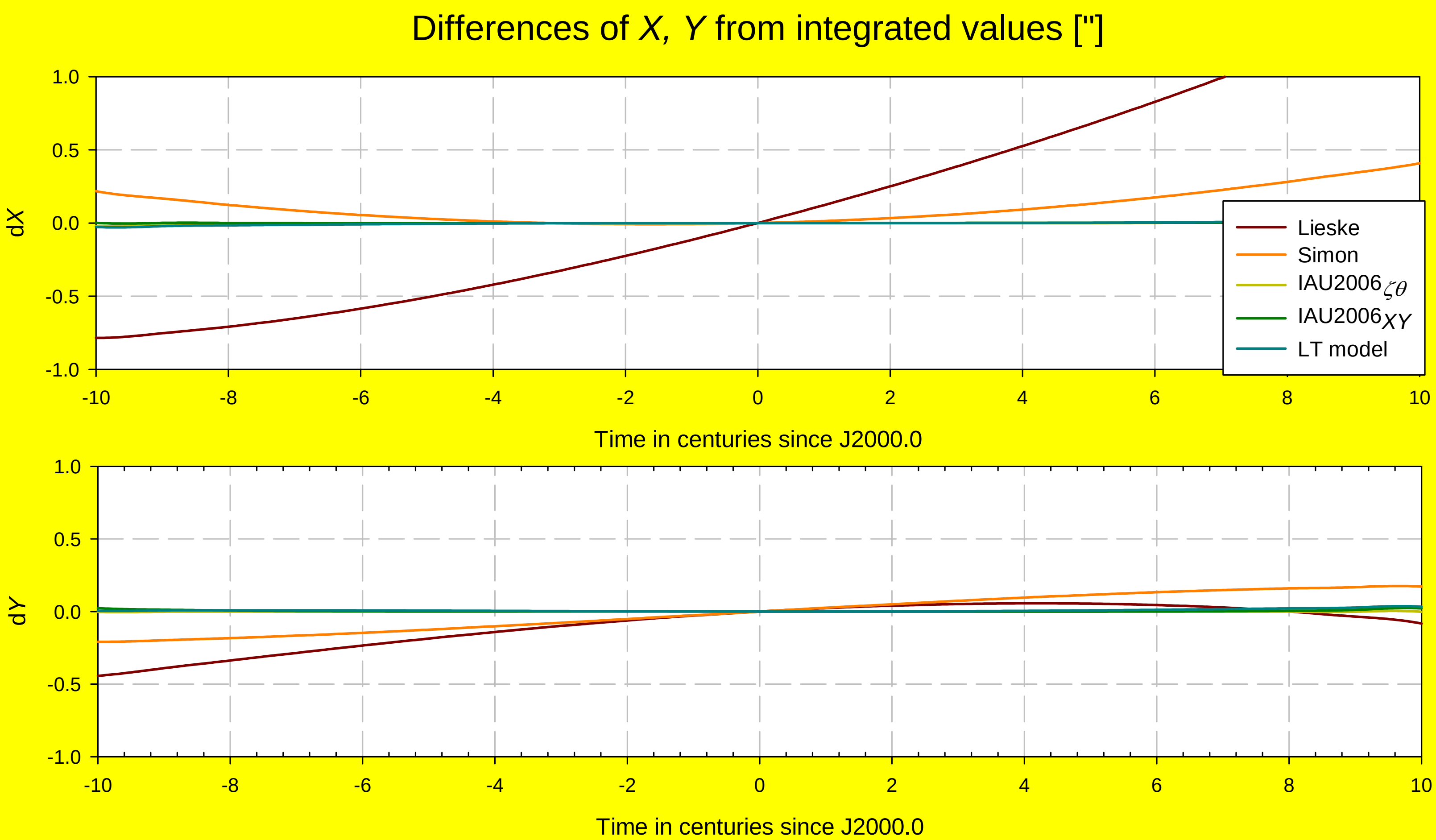
Accuracy of different precession parameters



Comparison of X_A, Y_A for different precession models:



Close-up of the central part:



Conclusions:

- ♦ **IAU2006 model:**
  - ♦ **Analytical expressions of direction cosines X_A, Y_A provide high accuracy over a few centuries;**
  - ♦ **For longer periods, precession angles ζ_A, θ_A are preferable.**
- ♦ **New model of precession, valid over ±200 millenia, is presented:**
  - ♦ **Its accuracy is comparable to IAU2006 model in the interval of several centuries around J2000.0,**
  - ♦ **It fits the numerically integrated position of the pole for longer intervals, with gradually decreasing accuracy (several arcminutes ±200 thousand years away from J2000),**
  - ♦ **The estimation of accuracy in A&A paper is too conservative, it is probably one or two orders of magnoitude better.**

Acknowledgments. This study was supported by the grant No. DO 02-275 awarded by the Bulgarian NSF