

# Constraining theories of gravity by velocity distribution of elliptical galaxies

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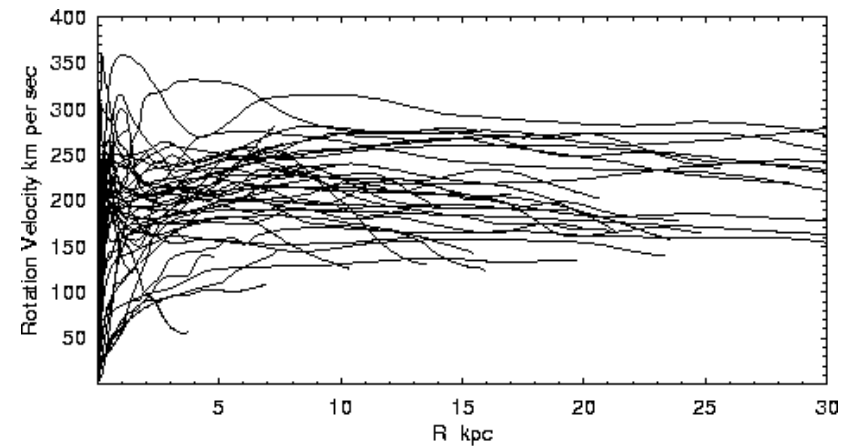
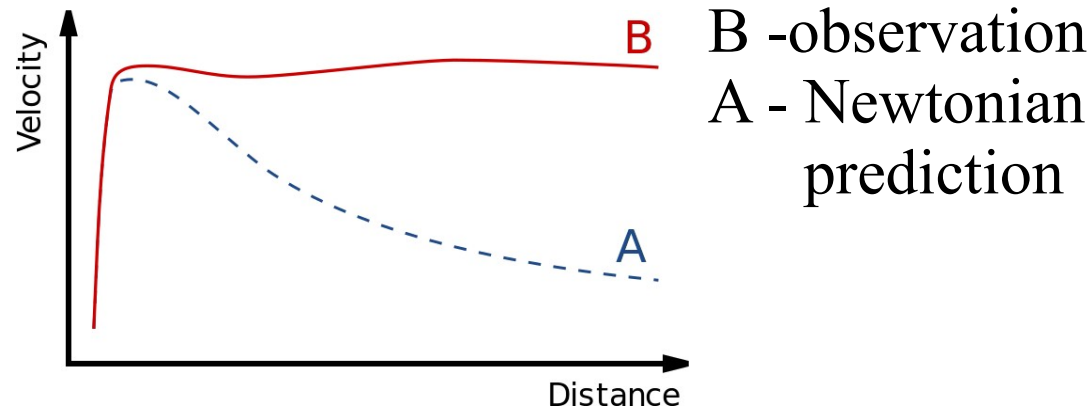
# Outline of the talk

- **Motivation**
- **Fundamental plane of elliptical galaxies**
- **The singular isothermal sphere model**
- **Extended Theories of Gravity (ETGs)**
- **Observational data**
- **Results**
- **Conclusions**
- **References**

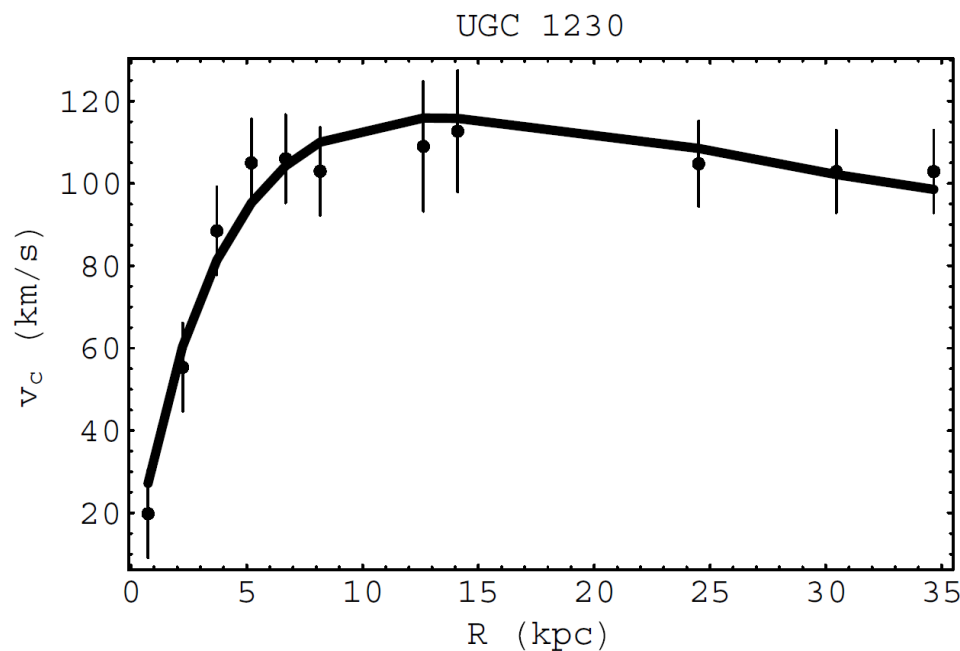
# Motivation I

- Generally, our main aim is to explain the observed astrophysical phenomena on different astrophysical scales: the Solar System, binary pulsars, galaxies, clusters of galaxies, and the large-scale structure of the Universe using gravitational potentials derived from Extended Gravity Theories (ETGs) without considering the presence of dark matter (DM).
- We believe that that astrophysical structures could be naturally explained without DM since no final evidence of this ingredient has been found, up to now, also with very high precision experiments as those running at CERN.
- In this study our motivation is to explain theoretically stellar kinematics and photometry of elliptical galaxies using gravitational potentials derived from Extended Theories of Gravity, without DM hypothesis.

# Flat Rotation Curves of Spiral Galaxies



Sofue Y. and Rubin V., Rotation Curves of Spiral Galaxies, Annual Review of Astronomy and Astrophysics, Vol. 39, p. 137-174 (2001).



V. F. Cardone and S. Capozziello, Systematic biases on galaxy haloes parameters from Yukawa-like gravitational potentials, Mon. Not. R. Astron. Soc. 414, 1301 (2011).

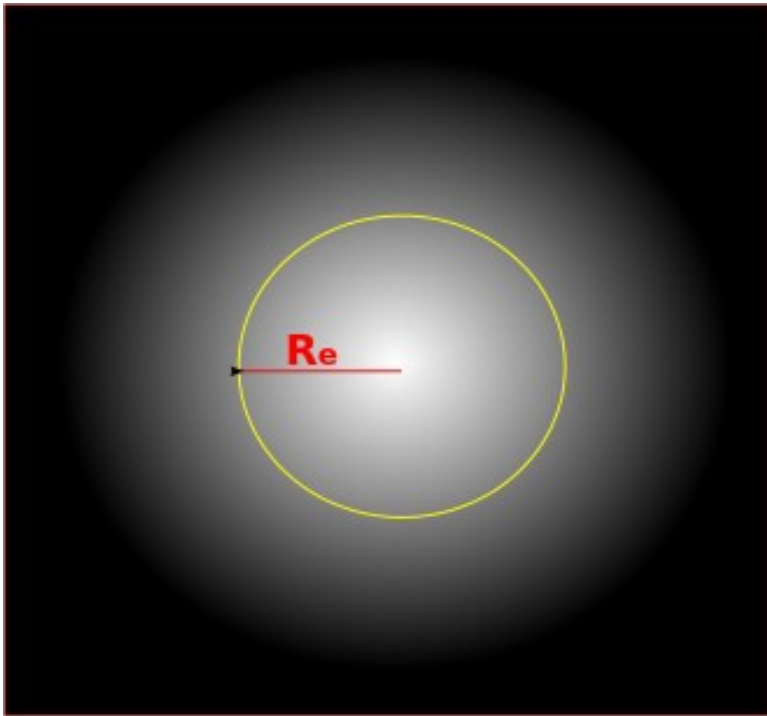
# Motivation II

- Also, our aim is to constrain parameters of the gravitational potentials derived from modified gravity models in absence of dark matter. In our investigations, from available sample of observational data, we use surface brightnesses, effective radius and velocity dispersion in order to constrain the different gravity parameters.
- In particular, we used the power-law version  $f(R) \propto R^n$ , Yukawa-like, Hybrid and Non-local Gravity model in the weak field limit.
- It is well known that besides the spiral galaxies, elliptical galaxies could also have so called missing mass problem, where an extra mass is required to explain the observed differences between their dynamical masses and luminosities.

# Fundamental plane of elliptical galaxies I

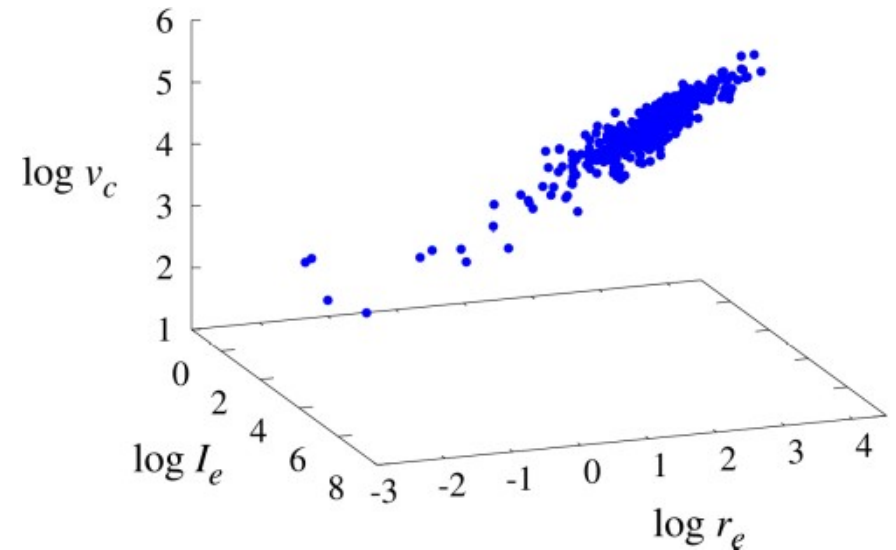
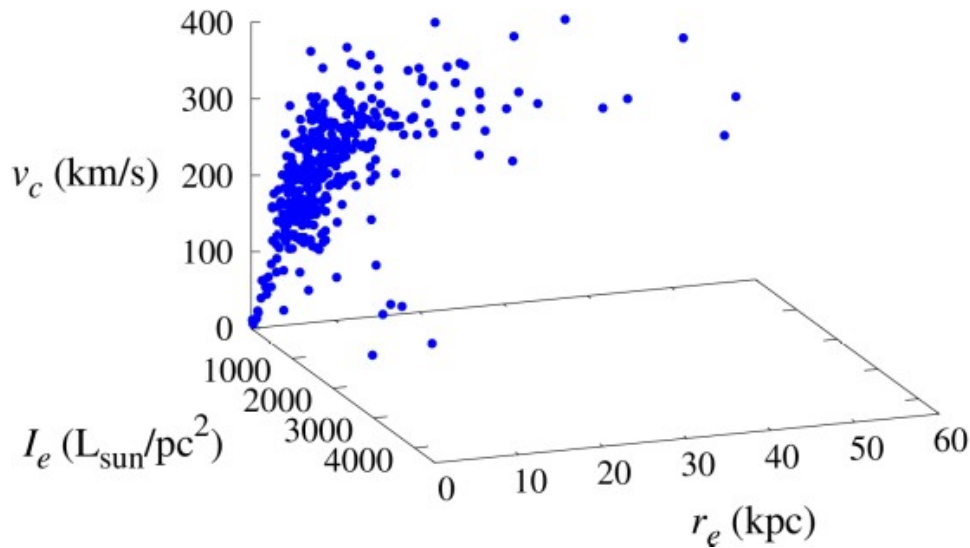
- the global properties of elliptical galaxies are connected, and empirical relation which shows this connection is called **fundamental plane**:

$$\log(r_e) = a \log(v_c) + b \log(I_e) + c$$



- $r_e$  - effective radius: encloses half of the total luminosity emitted by a galaxy
  - $v_c$  - central velocity dispersion
  - $I_e$  - mean surface brightness within  $r_e$
  - $a, b, c$  - coefficients
- some object can be represented as a point in the parameter space  $r_e, v_c, I_e$
  - when presented in logarithmic form, we obtain a plane

# Fundamental plane of elliptical galaxies II



- The three FP parameters: surface brightness  $I_e$ , effective radius  $r_e$  and circular velocity  $v_c$  (**linear** scale), for a sample of elliptical galaxies listed in Table 1 from Burstein et al. 1997.
- The FP parameter space presented by **logarithms** of the three parameters:  $\log I_e$ ,  $\log r_e$  and  $\log v_c$ .



# Fundamental plane of elliptical galaxies III

By writing the VT in terms of a set of the above mentioned quantities (e.g the effective radius  $r_e$ , the central velocity dispersion  $\sigma_0$ , and the mean surface brightness within the effective radius  $I_e$ ; hereafter we shall refer to them as the ‘observables’), one obtains:

$$r_e = C \sigma_0^2 I_e^{-1} \left( \frac{M}{L} \right)^{-1}, \quad (1)$$

where  $(M/L)$  is the mass-to-light ratio, and  $C$  is a combination of terms linking the observables to the corresponding physical quantities (namely, the central velocity dispersion to the kinetic energy, the effective radius to the ‘gravitational’ radius, and the effective surface brightness to the overall light profile, see e.g. DS93).

On the other hand, the FP writes:

$$r_e = c \sigma^a I_e^b, \quad (2)$$

where  $a$  and  $b$  are constants which depend on the sample, on the

# Fundamental plane of elliptical galaxies IV

- There is so-called "**tilt**" of FP, with respect to the expected virial plane:
  - prediction of the Virial Theorem (VT):  $a = 2$ ,  $b = -1$
- The (scalar) virial theorem connects the potential energy  $V$  and the kinetic energy  $T$  of a stationary self-gravitating system by the relation  $2T + V = 0$  (e.g. Landau & Lifshits 1976).
- - empirical result (Bender et al. 1992):  $a = 1.4$ ,  $b = -0.85$  (using the Virgo Cluster elliptical galaxies as a sample)
- When presented in logarithmic form, the two planes appear to be tilted by an angle of  $\sim 15^\circ$ .

(see G. Busarello, M. Capaccioli, S. Capozziello, G. Longo, E. Puddu, *The relation between the virial theorem and the fundamental plane of elliptical galaxies*, Astron. Astrophys. 320, 415-420 (1997))

# Fundamental plane of elliptical galaxies V:

## Recovering fundamental plane

- to recover fundamental plane (FP) using  $R^n$  gravity means to find connection between values of FP equation and values of  $R^n$  potential. In that sense, every of the three addends we connect with certain parameters:

1. addend with  $\mathbf{r_e}$ : correlation between  $r_e$  and  $r_c$  ( $r_c$  - parameter of the  $R^n$  gravity)
2. addend with  $\sigma_0$ : correlation between  $\sigma_0$  and  $v_{\text{vir}}$  ( $v_{\text{vir}}$  - virial velocity in  $R^n$ )
3. addend with  $\mathbf{I_e}$ : correlation between  $I_e$  and  $r_e$  (through the  $r_c/r_e$  ratio)

- correction term in gravitational potential for spherically symmetric extended systems (Capozziello et al., MNRAS 375, 1423, 2007, eqs. (22)-(24))

- for the mass distribution we take Hernquist profile:

$$\rho(r) = a M / (2 \pi r (r + a)^3), \text{ where } a = r_e / (1 + \sqrt{2})$$

(L. Hernquist, ApJ 356, 359 (1990))

# The singular isothermal sphere model

- For modeling the stellar kinematics, i.e. in order to describe the mass distribution in elliptical galaxies, we assume that the mass distribution within them is described by the singular isothermal sphere (SIS) model:

$M(r) = 2\sigma_{SIS}^2 G^{-1} r$ , like in our previous paper (Capozziello et al., 2020a). The density profile has the form:  $\rho_{SIS}(r) = \frac{\sigma_{SIS}^2}{2\pi G r^2}$ , and the corresponding mass within a radius  $r$  is:

$$M_{SIS}(r) = \frac{2\sigma_{SIS}^2}{G} \cdot r. \quad (5)$$

We can see that mass grows linearly with  $r$ . Taking into account:

$$v_N^2(r) = \frac{GM(r)}{r} \quad (6)$$

and

$$v_N(r_e) = \sigma_0, \quad (7)$$

where  $v_N(r_e)$  is the Newtonian circular velocity at the effective radius, and  $\sigma_0$  is the observed velocity dispersion (Burstein et al., 1997), we can obtain:

$$v_N^2(r) = 2\sigma_{SIS}^2.$$

Therefore, for  $r = r_e$  it stands:

$$\sqrt{2}\sigma_{SIS} = \sigma_0,$$

and furthermore:

$$v_N(r_e)^2 = \sigma_0^2.$$

We are using following substitutions:

$$v_N^2(r) = \frac{GM(r)}{r},$$

$$\Phi_N(r) = -\frac{GM(r)}{r},$$

$$\Phi_N(r)' = \left(-\frac{GM(r)}{r}\right)',$$

$$v_N^2(r) = -\Phi_N(r)'. \quad (8)$$

# Extended Theories of Gravity (ETGs)

- Extended Theories of Gravity have been proposed like alternative approaches to Newtonian gravity in order to explain galactic and extragalactic dynamics without introducing dark matter.
- In the case of  $f(R)$  gravity, one assumes a generic function  $f$  of the Ricci scalar  $R$  (in particular, analytic functions) and searches for a theory of gravity having suitable behavior at small and large scale lengths.
- We adopt  $f(R)$  gravity which is the straightforward generalization of Einstein's General Relativity as soon as the function is  $f(R) \neq R$ .
- We start from the action: 
$$A = \int d^4x \sqrt{-g} [f(R) + \mathcal{L}_m]$$

See review in: S. Capozziello, M. De Laurentis, *Extended Theories of Gravity*, Phys. Rep. 509, 167-321 (2011).

# Velocity distribution of elliptical galaxies in the framework of $R^n$ Gravity model

## 2. The gravitational potential in $f(R)$ gravity

Let us now shortly review the  $f(R)$  gravity theory that can be considered a straightforward extension of Einstein's General Relativity. The action is :

$$\mathcal{A} = \int d^4x \sqrt{-g} [f(R) + \mathcal{L}_m] \quad (1)$$

where  $f(R)$  is an analytic function of the Ricci curvature scalar  $R$  and  $\mathcal{L}_m$  is the standard matter contribution. Clearly  $f(R) = R$  implies the recovering of General Relativity. The variation of the action with respect to the metric  $g_{\mu\nu}$  gives the field equations [see [28]] :

$$G_{\mu\nu} = \frac{1}{f'(R)} \left\{ \frac{1}{2} g_{\mu\nu} [f(R) - Rf'(R)] + f'(R)_{;\mu\nu} - g_{\mu\nu} \square f'(R) \right\} + \frac{T_{\mu\nu}^{(m)}}{f'(R)} \quad (2)$$

here  $G_{\mu\nu} = R_{\mu\nu} - (R/2)g_{\mu\nu}$  is the Einstein tensor; the prime denotes derivative with respect to  $R$ . The terms  $f'(R)_{;\mu\nu}$  and  $\square f'(R)$  are of fourth order in derivatives with respect the metric  $g_{\mu\nu}$ . The case

$$f(R) = f_0 R^n \quad (3)$$

Let us now take into account the gravitational field generated by a pointlike source and solve the field equations (2). Under the hypothesis of weak gravitational fields and slow motions (the same holding for standard self-gravitating systems), we can write the spacetime metric in spherical symmetry as :

$$ds^2 = A(r)dt^2 - B(r)dr^2 - r^2 d\Omega^2 \quad (4)$$

where  $d\Omega^2 = d\theta^2 + \sin^2\theta d\varphi^2$  is the line element on the unit sphere. A physically motivated hypothesis to search for solutions is (see the discussion in [23])

$$A(r) = \frac{1}{B(r)} = 1 + \frac{2\Phi(r)}{c^2} \quad (5)$$

where  $\Phi(r)$  is the gravitational potential generated by a pointlike mass  $m$  at the distance  $r$ . It is possible to show that a solution is

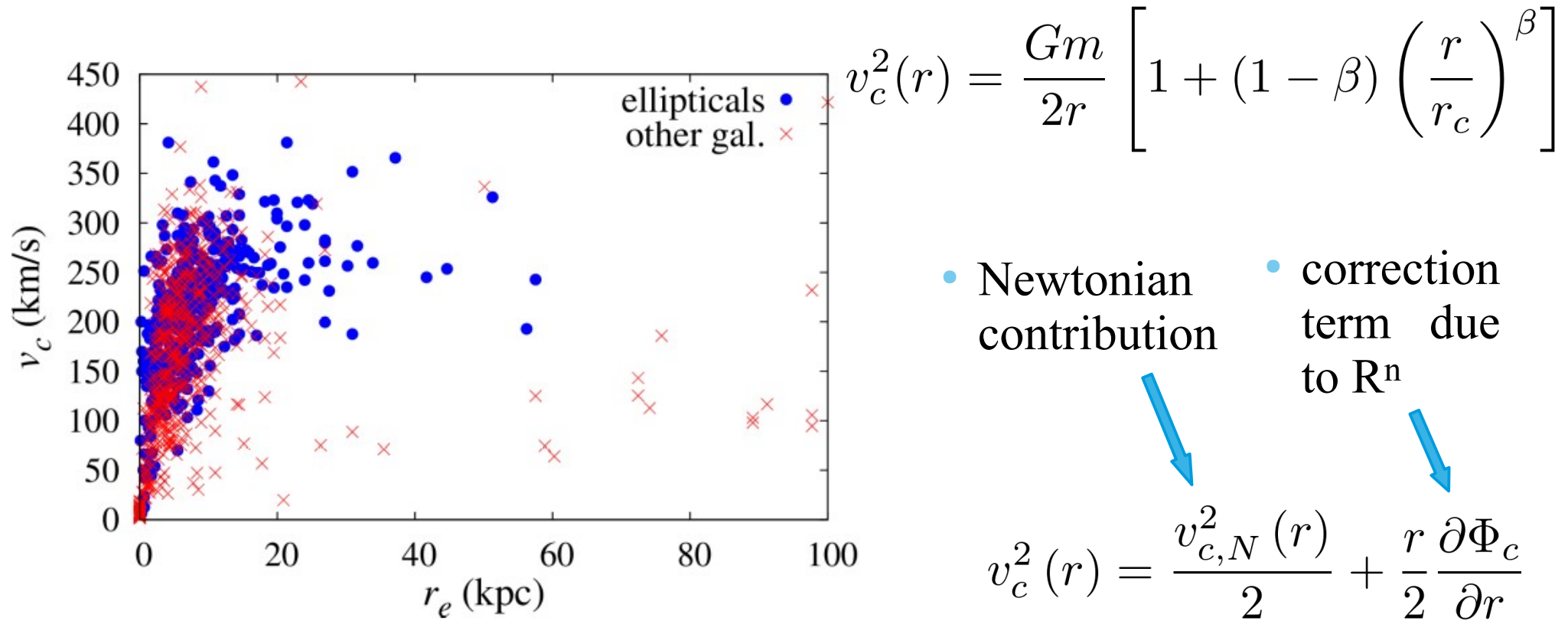
$$\Phi(r) = -\frac{Gm}{2r} \left[ 1 + \left( \frac{r}{r_c} \right)^\beta \right] \quad (6)$$

where the gravitational potential deviates from the Newtonian due to the correction induced by  $f(R)$  gravity. Here

$$\beta = \frac{12n^2 - 7n - 1 - \sqrt{36n^4 + 12n^3 - 83n^2 + 50n + 1}}{6n^2 - 4n + 2} \quad (7)$$

and  $r_c$  is the gravitational radius induced by the Noether symmetry [19].

# Velocity distribution of elliptical galaxies in the framework of $R^n$ Gravity model II



Circular velocity  $v_c$  as a function of effective radius  $r_e$  for a sample of galaxies listed in Table 1 from Burstein et al. 1997.



# Velocity distribution of elliptical galaxies in the framework of Yukawa-like Gravity model

$$\Phi(r) = -\frac{GM(r)}{r} [1 + \alpha \exp(-\lambda r)] . \quad (13)$$

Again  $\lambda = L^{-1}$  is a scale length and  $\alpha$  gives the strength of the correction [30,45,49]. It is worth noticing that  $L = \lambda^{-1} = \frac{\hbar}{m_R c}$  is a Compton length which can be related to an effective mass  $m_R$  coming from curvature. In early universe cosmology, it is the so-called *Starobinsky scalaron* [50].

In the case of Newtonian potential, it is:  $\Phi_N(r) = -\frac{GM(r)}{r}$  and circular velocity  $v_N^2(r) = r \cdot \Phi'_N(r)$  [30]. In the case of modified potential, supposing the spherically distributed mass in elliptical galaxies, we have  $v_c^2(r) = r \cdot \Phi'(r)$  [51]. Here we start from the Yukawa-like gravitational potential (13) and derive the connection between  $v_c^2(r)$  and parameters of this potential. Let us start from Eq. (13) and its derivative. We have:

$$\begin{aligned} \Phi(r) &= -\frac{GM(r)}{r} - \alpha \frac{GM(r)}{r} e^{-\lambda r}; \\ \Phi'(r) &= \left( -\frac{GM(r)}{r} \right)' + \alpha \left( -\frac{GM(r)}{r} \right)' e^{-\lambda r} \\ &\quad + \alpha \left( -\frac{GM(r)}{r} \right) (e^{-\lambda r})' \\ &= \Phi'_N(r) + \alpha e^{-\lambda r} \Phi'_N(r) + \alpha \Phi_N(r) (-\lambda) e^{-\lambda r} \\ r\Phi'(r) &= r\Phi'_N(r) + \alpha e^{-\lambda r} r\Phi'_N(r) - \alpha \lambda e^{-\lambda r} r\Phi_N(r) \\ v_c^2(r) &= v_N^2(r) + \alpha e^{-\lambda r} v_N^2(r) + \alpha \lambda r e^{-\lambda r} v_N^2(r) \\ &= \frac{GM(r)}{r} + \frac{GM(r)}{r} \alpha (1 + \lambda r) e^{-\lambda r}. \end{aligned} \quad (14)$$

Therefore the squared circular velocity  $v_c^2(r) = r \cdot \Phi'(r)$  takes the form:  $v_c^2(r) = \frac{GM(r)}{r} (1 + \alpha (1 + \lambda r) e^{-\lambda r})$ . Now, we can write this expression as a sum of the Newtonian contribution  $v_N^2(r)$  and the correction term due to modified gravity  $v_{corr}^2(r)$ :

$$v_c^2(r) = v_N^2(r) + v_{corr}^2(r), \quad (15)$$

where:

$$\begin{aligned} v_N^2(r) &= \frac{GM(r)}{r} \\ v_{corr}^2(r) &= \frac{\alpha GM(r) \cdot (1 + \lambda r)}{r} e^{-\lambda r}. \end{aligned} \quad (16)$$

For the considered sample of elliptical galaxies, the Newtonian circular velocity at the effective radius is [see the explanation of Table 1 in Ref. 48]  $v_N(r_e) = \sigma_0$ , where  $\sigma_0$  is the observed velocity dispersion. Therefore,

$$\sqrt{2}\sigma_{SIS} = \sigma_0. \quad (20)$$

From Eqs. (18) and (20), we have:

$$v_c^2(r) = \sigma_0^2 (1 + \alpha (1 + \lambda r) e^{-\lambda r}), \quad (21)$$

and, furthermore,

$$v_N^2 = \sigma_0^2$$



# Velocity distribution of elliptical galaxies in the framework of Hybrid Gravity model

## 2 A summary of Hybrid Gravity

Let us present here the basic formalism for Hybrid Metric-Palatini Gravity within the equivalent scalar-tensor representation (we refer the reader to [12,22–25] for more details). The action for Hybrid Gravity is given by

$$S = \int d^4x \sqrt{-g} [R + f(\mathcal{R}) + 2\kappa^2 \mathcal{L}_m] . \quad (2)$$

where  $\kappa^2 \equiv 8\pi G$ ,  $R$  is the Einstein-Hilbert term defined with the Levi-Civita connection,  $\mathcal{R} \equiv g^{\mu\nu} \mathcal{R}_{\mu\nu}$  is the Palatini curvature with the connection  $\hat{\Gamma}_{\mu\nu}^\alpha$  independent of the metric  $g_{\mu\nu}$ . As discussed in [12], it is possible to recast such an action as

$$S = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} [R + \phi \mathcal{R} - V(\phi)] + S_m, \quad (3)$$

where the scalar field  $\phi$  is derived from  $f'(\mathcal{R})$ , the first derivative in  $\mathcal{R}$  of  $f(\mathcal{R})$ .

In the weak gravitational field limit, it is possible to demonstrate that Newtonian potential for Hybrid Gravity is given as [25]:

$$\Phi(r) = -\frac{G}{1 + \phi_0} \left[ 1 - \frac{\phi_0}{3} e^{-m_\phi r} \right] \frac{M(r)}{r}, \quad (4)$$

where the main parameters of Hybrid Gravity are  $m_\phi$  and  $\phi_0$ , derived from the self-interaction potential  $V(\phi)$ .

For modeling the stellar kinematics in the elliptical galaxies, we assumed that the mass distribution within them may be described by the singular isothermal sphere (SIS) model:  $M(r) = 2\sigma_{SIS}^2 G^{-1} r$ , like in our previous paper [26].

Having in mind that the Newtonian circular velocity at the effective radius, i.e. for  $r = r_e$ , is  $v_N(r_e) = \sigma_0$ , where  $\sigma_0$  is the observed velocity dispersion see Table 1 in Ref. [27] in relation to our considered sample of elliptical galaxies), the circular velocity at the  $r_e$  can be written in the following form:

$$v_c^2(r_e) = \frac{\sigma_0^2}{1 + \phi_0} \left[ 1 - \frac{\phi_0}{3} (m_\phi r_e + 1) e^{-m_\phi r_e} \right]. \quad (7)$$

Furthermore, let us introduce the following variable:  $w = m_\phi r_e$ .

Then, one can obtain the following  $v_c/\sigma$  relation at  $r_e$  for the ellipticals in Hybrid Gravity:

$$\frac{v_c}{\sigma} = \frac{v_c(r_e)}{\sigma_0} = \sqrt{\frac{1}{1 + \phi_0} \left[ 1 - \frac{\phi_0}{3} (w + 1) e^{-w} \right]}. \quad (8)$$

# Velocity distribution of elliptical galaxies in the framework of Non-local Gravity model

A possible action describing Non-local Gravity was suggested in Deser & Woodard (2007). The authors proposed a non-local modification of Einstein-Hilbert action with the following form:

$$\mathcal{S} = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} [R(1 + f(\square^{-1}R))] \quad (2)$$

If we set  $f(\square^{-1}R) = 0$ , the Einstein theory is immediately recovered. It is possible to show that the weak-field limit of this Non-local Gravity model has the free parameters,  $\phi_c$ ,  $r_\phi$  and  $r_\xi$  (see Dialektopoulos et al. (2019) for more details). We can reasonably take specific values for  $\phi_c$  around 1 (from 0.9 to 1.1), and constrain the parameter space of  $r_\phi$  and  $r_\xi$  parameters. The related weak field potential reads:

$$\begin{aligned} \Phi(r) = & - \frac{GM}{r} \phi_c + \frac{G^2 M^2}{2c^2 r^2} \left[ \frac{14}{9} \phi_c^2 + \frac{18r_\xi - 11r_\phi}{6r_\xi r_\phi} r \right] + \\ & + \frac{G^3 M^3}{2c^4 r^3} \left[ \frac{7r_\phi - 50r_\xi}{12r_\xi r_\phi} \phi_c r - \frac{16\phi_c^3}{27} + \frac{2r_\xi^2 - r_\phi^2}{r_\xi^2 r_\phi^2} r^2 \right]. \end{aligned} \quad (4)$$

In order to derive expression for velocity distribution, we start from the non-local gravitational potential and derive the connection between  $v_c^2(r)$  and parameters of this potential, we obtain:

$$\begin{aligned} v_c^2(r) = & \phi_c v_N^2(r) - \frac{v_N^4(r)}{c^2} \left[ \frac{r}{2} \cdot \frac{18r_\xi - 11r_\phi}{6r_\xi r_\phi} + \frac{14}{9} \phi_c^2 \right] + \\ & + \frac{v_N^6(r)}{2c^4} \left[ -\frac{\phi_c r}{6} \cdot \frac{7r_\phi - 50r_\xi}{r_\xi r_\phi} + \frac{16}{9} \phi_c^3 - r^2 \frac{2r_\xi^2 - r_\phi^2}{r_\xi^2 r_\phi^2} \right]. \end{aligned} \quad (12)$$

If we take into account that the Newtonian circular velocity at the effective radius for elliptical galaxies, i.e. for  $r = r_e$ , is  $v_N(r_e) = \sigma_0$ , where  $\sigma_0$  is the observed velocity dispersion (Burstein et al. 1997), the velocity dispersion becomes:

$$\sigma^{theor}(r_e) = v_c(r_e) = \sigma_0. \quad (13)$$

This is not the case for non-ellipticals. After combining Eq.(12) and Eq.(13) we obtained:

# Observational data I

- - we use some physical properties of stellar systems: among 1150 observed galaxies, there is a sample of 401 ellipticals
  - - observational data publicly available (as ASCII format) among the [source files](#) of the arxiv version of the paper Burstein et al., AJ 114, 1365 (1997):
  - <https://arxiv.org/format/astro-ph/9707037>, see 'metaplanetab1'
- 
- |   |  |
|---|--|
| • column (5): $\log v_c$ (km/s)                           | • $v_c$ - circular velocity (observed)         |
| • column (6): $\log \sigma_0$ (km/s)                      | • $\sigma_0$ - central velocity dispersion     |
| • column (7): $\log r_e$ (kpc)                            | • $r_e$ - effective or half-light radius       |
| • column (8): $\log I_e$ ( $L_{\text{sun}}/\text{pc}^2$ ) | • $I_e$ - mean surface brightness within $r_e$ |
- 
- $\sigma_0$  - derived (consistent values for all stellar systems)
  - **Note:** for elliptical galaxies, the circular velocity inside effective radius is  $v_c(r_e) = \sigma_0$ , while for other stellar systems it is  $v_c \neq \sigma_0$

**Burstein, D., Bender, R., Faber, S., Nolthenius, R., 1997. Global Relationships Among the Physical Properties of Stellar Systems. Astron. J. 114, 1365–1392.**

# Observational data II

| Obj<br>Name<br>(1) | ID #<br>(2) | Obj<br>Code<br>(3) | Dist<br>(Mpc)<br>(4) | $\log V_{\text{char}}$<br>Obs<br>(5) | $\log \sigma_c$<br>Used<br>(6) | $\log r_e$<br>(kpc)<br>(7) | $\log I_e$<br>$L_{\odot} \text{ pc}^{-2}$<br>(8) |
|--------------------|-------------|--------------------|----------------------|--------------------------------------|--------------------------------|----------------------------|--|
| NGC 221            | 8           | 1                  | 0.7                  | 1.903                                | 1.903                          | -0.95                      | 3.47   |
| NGC 315            | 14          | 1                  | 107.2                | 2.546                                | 2.546                          | 1.49                       | 1.86   |
| NGC 720            | 56          | 1                  | 35.8                 | 2.392                                | 2.392                          | 0.84                       | 2.34   |
| NGC 777            | 64          | 1                  | 99.4                 | 2.542                                | 2.542                          | 1.13                       | 2.16   |
| NGC 821            | 67          | 1                  | 37.7                 | 2.208                                | 2.208                          | 0.97                       | 2.06   |

Table 1 in [Burstein et al. \(1997\)](#) lists the data (the first page only is printed for the journal). The table contains effective radii, effective luminosities and characteristic velocities which have been derived for gravitationally-bound stellar systems, including galaxies, galaxy groups, galaxy clusters and globular clusters. Also, related data, including stellar population information, are included, as summarized in the table explanation [Burstein et al. \(1997\)](#).

The full table is electronically available in a convenient ASCII format given in ‘metaplanetab1’ among ‘source’ files of its arXiv version: <https://arxiv.org/e-print/astro-ph/9707037>. The data are organized in 19 columns there, and for our investigation the most important were the following ones:

- Column 1 gives the name for the object.
- Column 2 gives an internal identifying number.
- Column 3 gives a numerical code for the type of object.
- Column 4 gives the distance in units of Mpc.
- Column 5 gives the logarithm of the original characteristic internal velocity.
- Column 6 gives either the observed value of  $\log r_e$  or the transformed value  $\log \sigma_0$ .
- Column 7 gives the values of  $\log r_e$  in kpc.
- Column 8 gives the values of  $\log I_e$  in  $L_{\odot} \text{ pc}^{-2}$ .
- Columns 15 and 16 give effective mass  $\log M_e$  and effective B-band luminosity  $\log L_e$ , in solar units.

# Observational data III

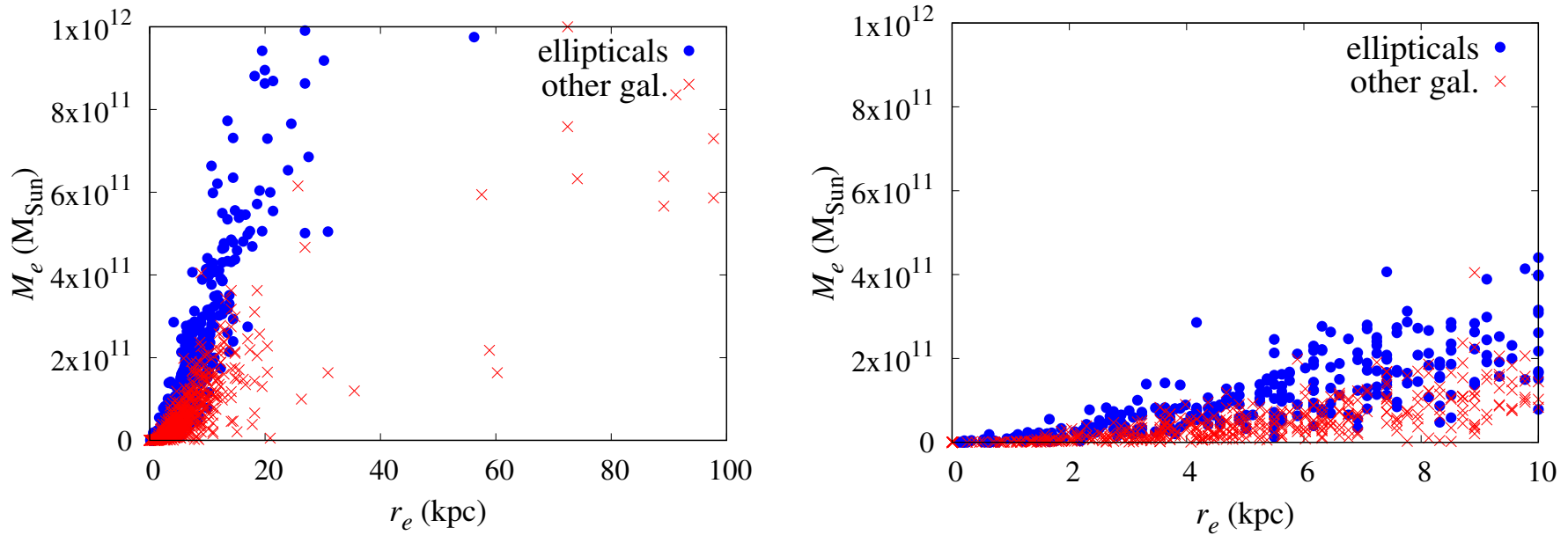


Fig. 1. (left): Galaxy masses  $M_e$  as a function of effective radius  $r_e$ , for elliptical and other types of galaxies (for the sample of galaxies listed in Table 1 from Burstein et al. (1997)). (right): Right panel shows a zoomed part of the figure, for  $r_e$  less than 10 kpc.

# Observational data IV

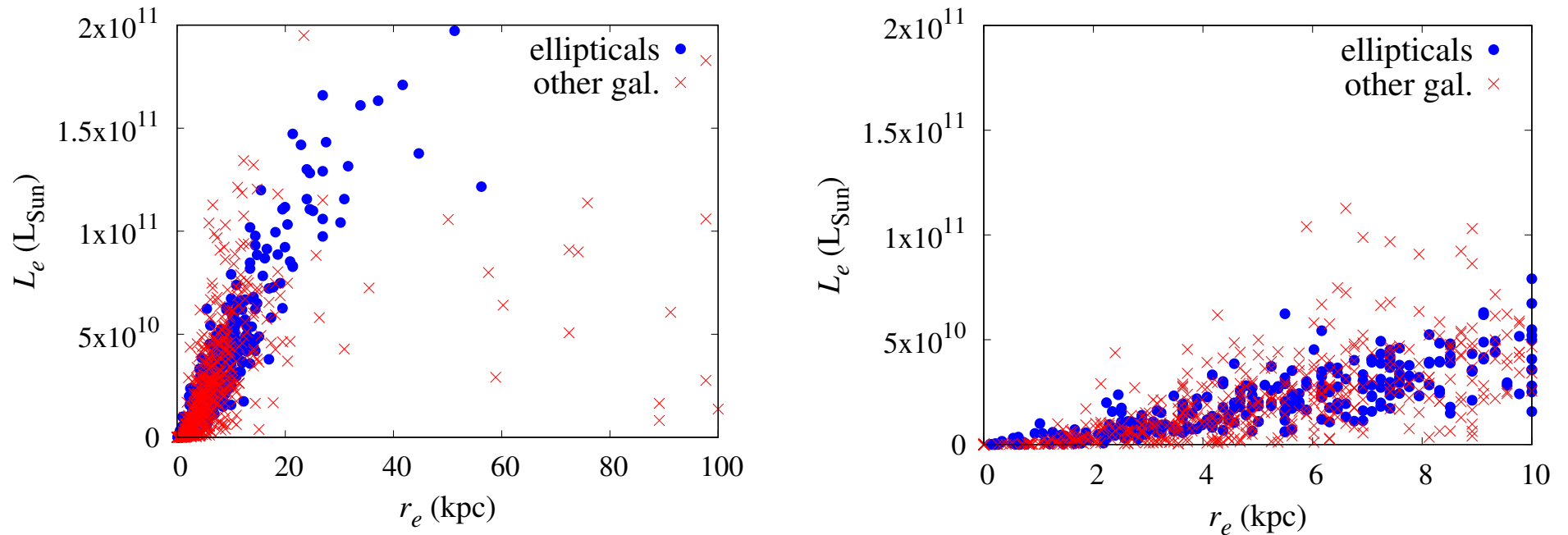


Fig. 2. (left): Galaxy luminosities  $L_e$  as a function of effective radius  $r_e$ , for elliptical and other types of galaxies. (right): Right panel shows a zoomed part of the figure, for  $r_e$  less than 10 kpc. Data are from Burstein et al. (1997).



# Results I- $R^n$ Gravity model

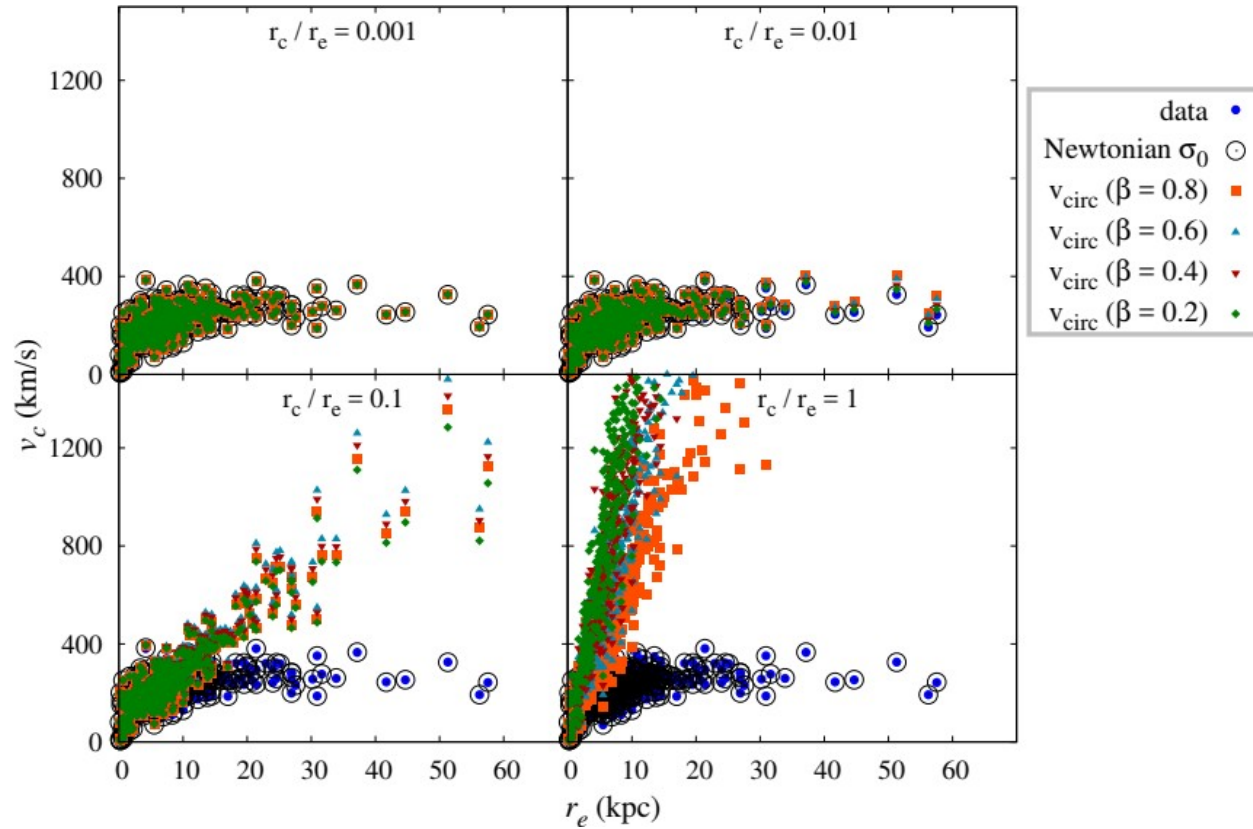


Fig. 3. Circular velocity  $v_c$  as a function of effective radius  $r_e$  for elliptical galaxies, for different values of ratio between  $R^n$  gravity scale-length and effective radius  $r_c/r_e$ : 0.001, 0.01, 0.1 and 1. For every of these ratios, the four values of  $\beta$  are presented: 0.2, 0.4, 0.6 and 0.8. Notice: all  $v_c$  values we calculated, except for blue full circles which are data given in Table 1 from Burstein et al. (1997).

# Results II-R<sup>n</sup> Gravity model

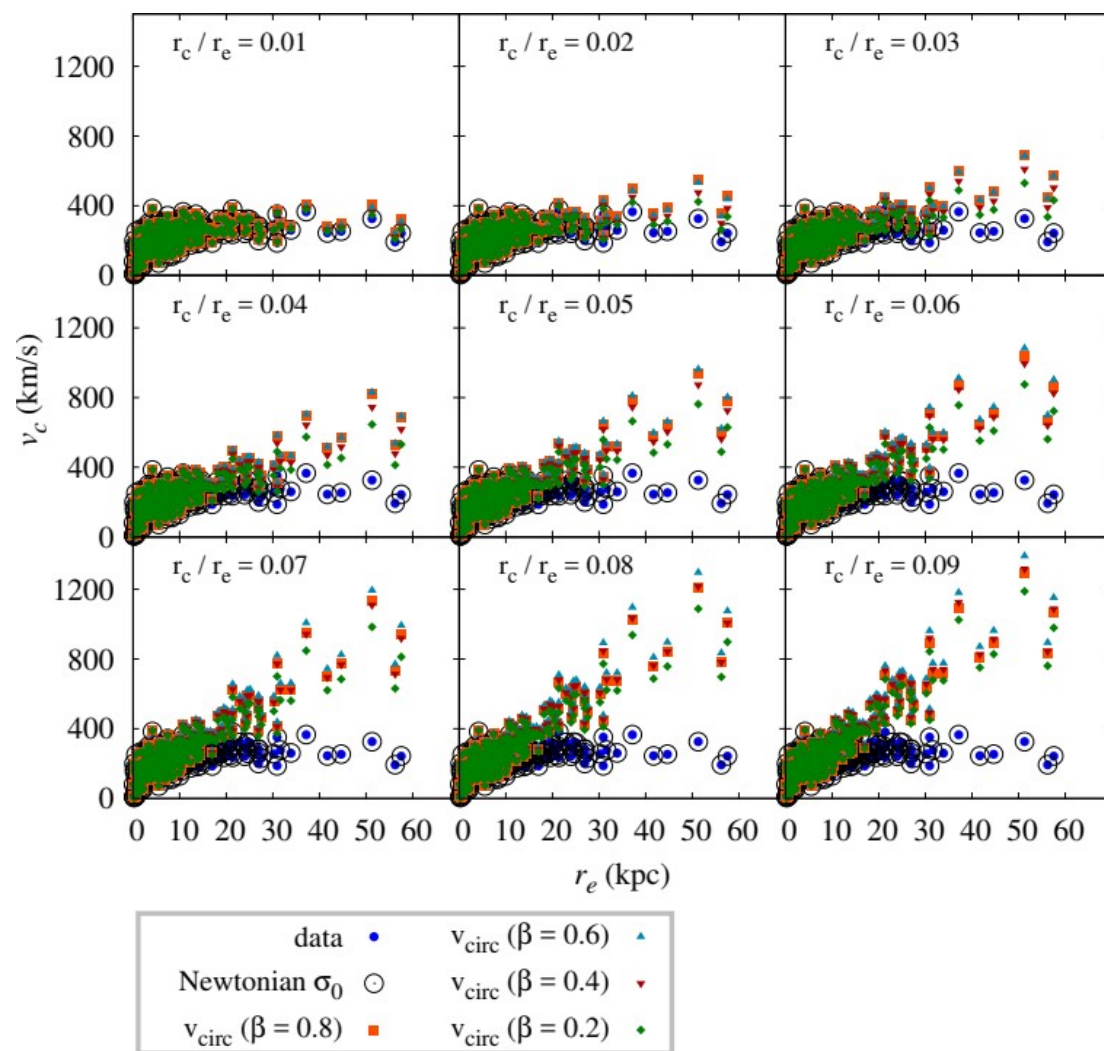


Fig. 4. The same as in Fig. 4, but for the following  $r_c/r_e$  ratios: 0.01, 0.02, 0.03, 0.04, 0.05, 0.06, 0.07, 0.08 and 0.09.



# Results III-Yukawa-like Gravity model

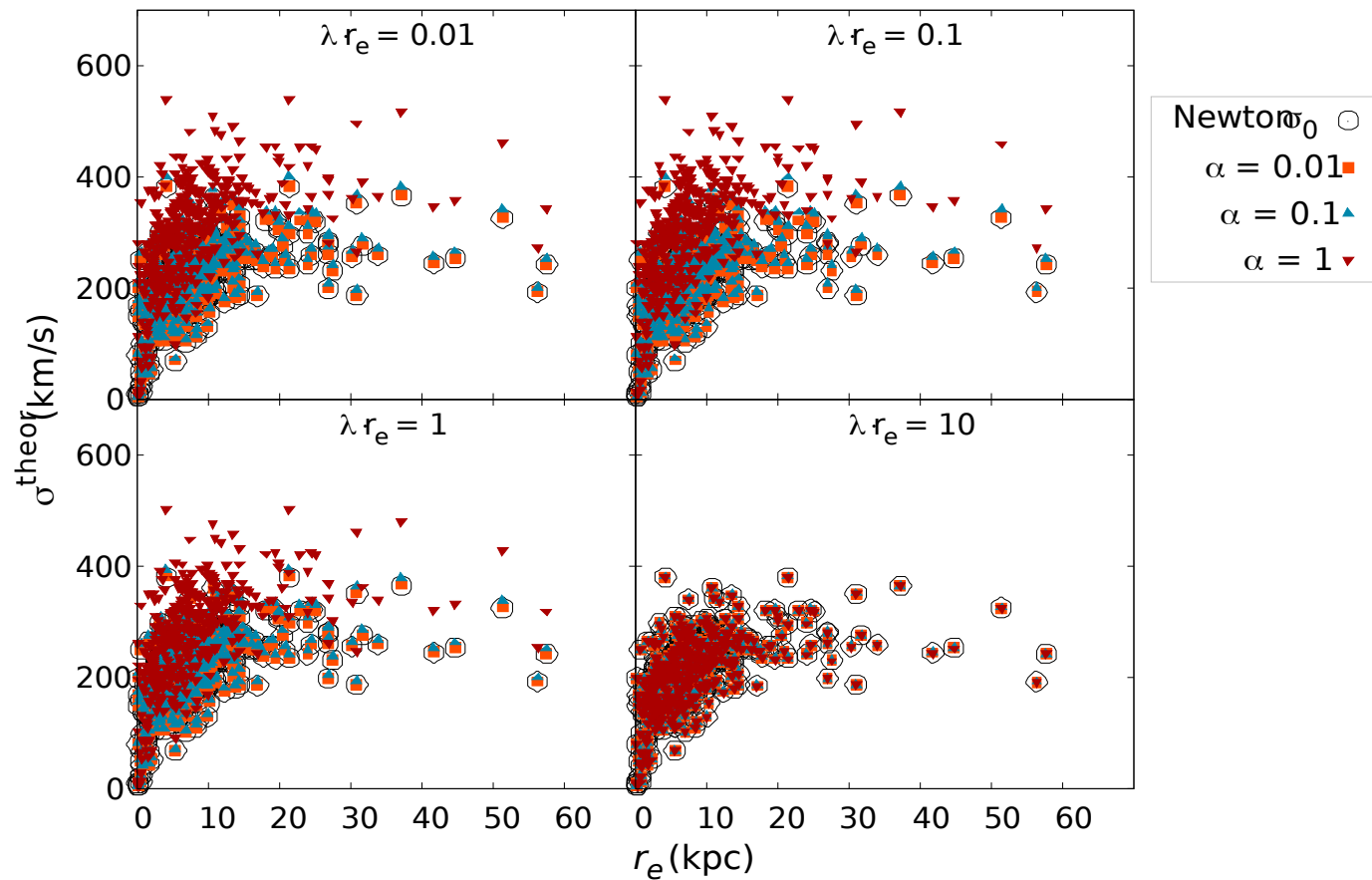


Fig. 5. Velocity dispersion  $\sigma^{\text{theor}}$  as a function of the effective radius  $r_e$  for elliptical Newtonian velocity dispersion inside the effective radius,  $\sigma_0$ , is taken from Burstein et al. (1997)., and parameter  $\alpha$ : 0.01, 0.1 and 1. galaxies, for four different values of the  $\lambda \cdot r_e$  product: 0.01, 0.1, 1 and 10.

# Results IV-Yukawa-like gravity model II

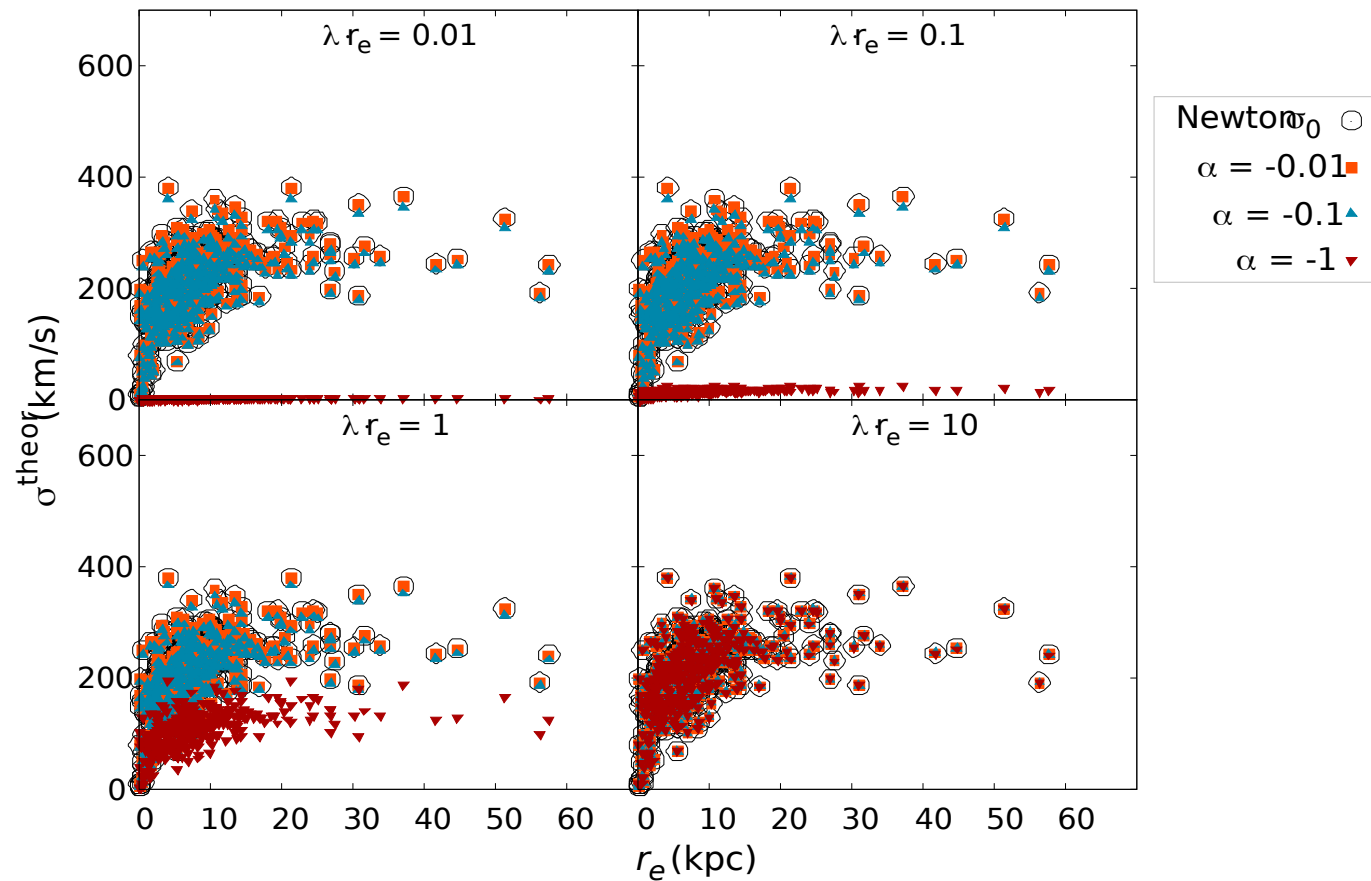


Fig. 6. The same as in Fig. 3, but for negative values of  $\alpha$  :  $-0.01$ ,  $-0.1$ ,  $-1$ .

# Results V-Hybrid Gravity model

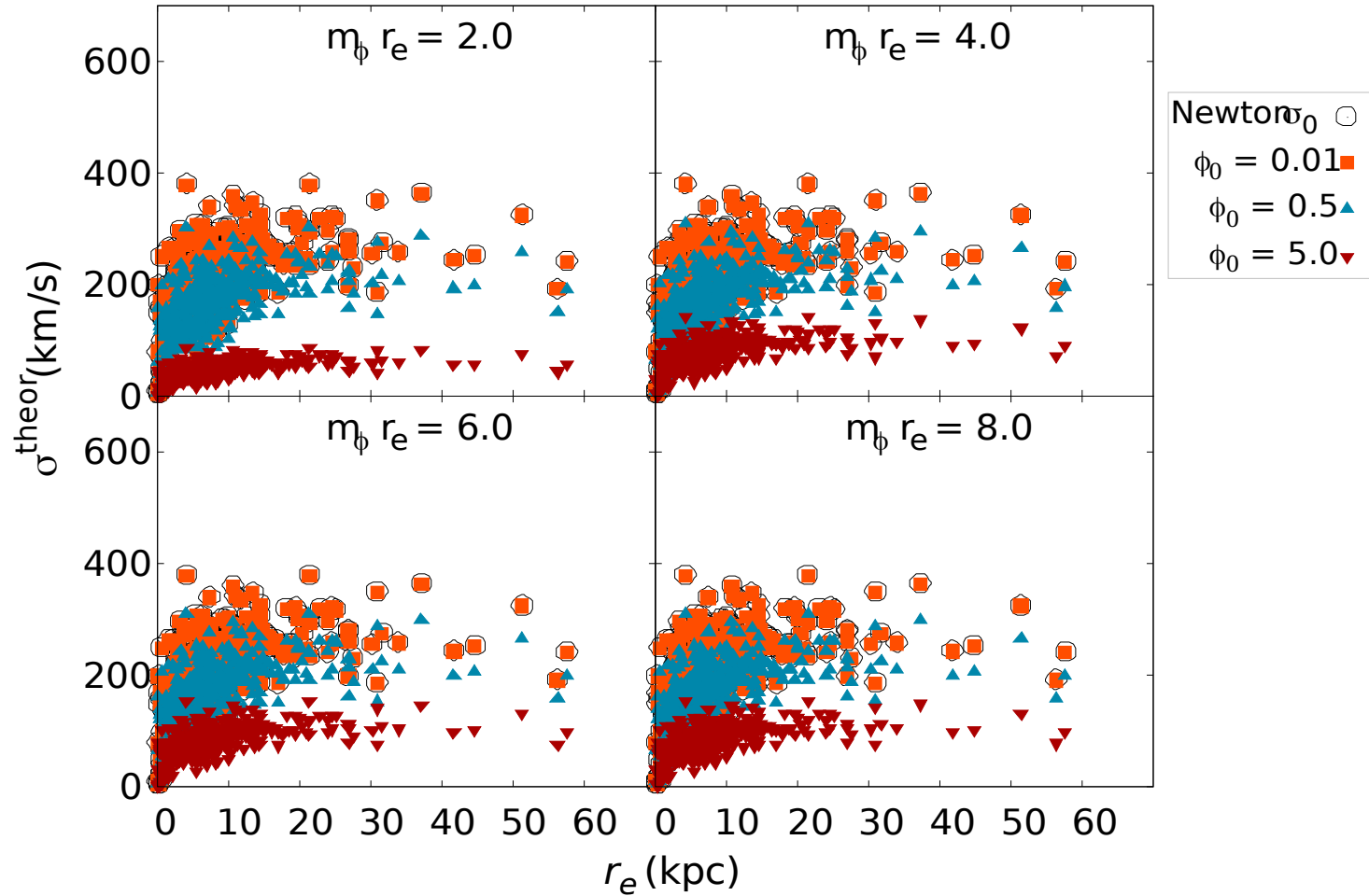


Fig. 7. Velocity dispersion  $\sigma^{\text{theor}}$  as a function of the effective radius  $r_e$  for elliptical galaxies, for four different values of the  $m_\phi \cdot r_e$  product: 2, 4, 6 and 8. The Newtonian velocity dispersion at the effective radius  $\sigma_0$  is taken from Burstein et al. (1997). Theoretical values of velocity dispersion  $\sigma^{\text{theor}}$  are calculated for the three values of Hybrid Gravity parameter  $\phi_0$ : 0.01, 0.5 and 5.0

# Results VI-Non-local Gravity model

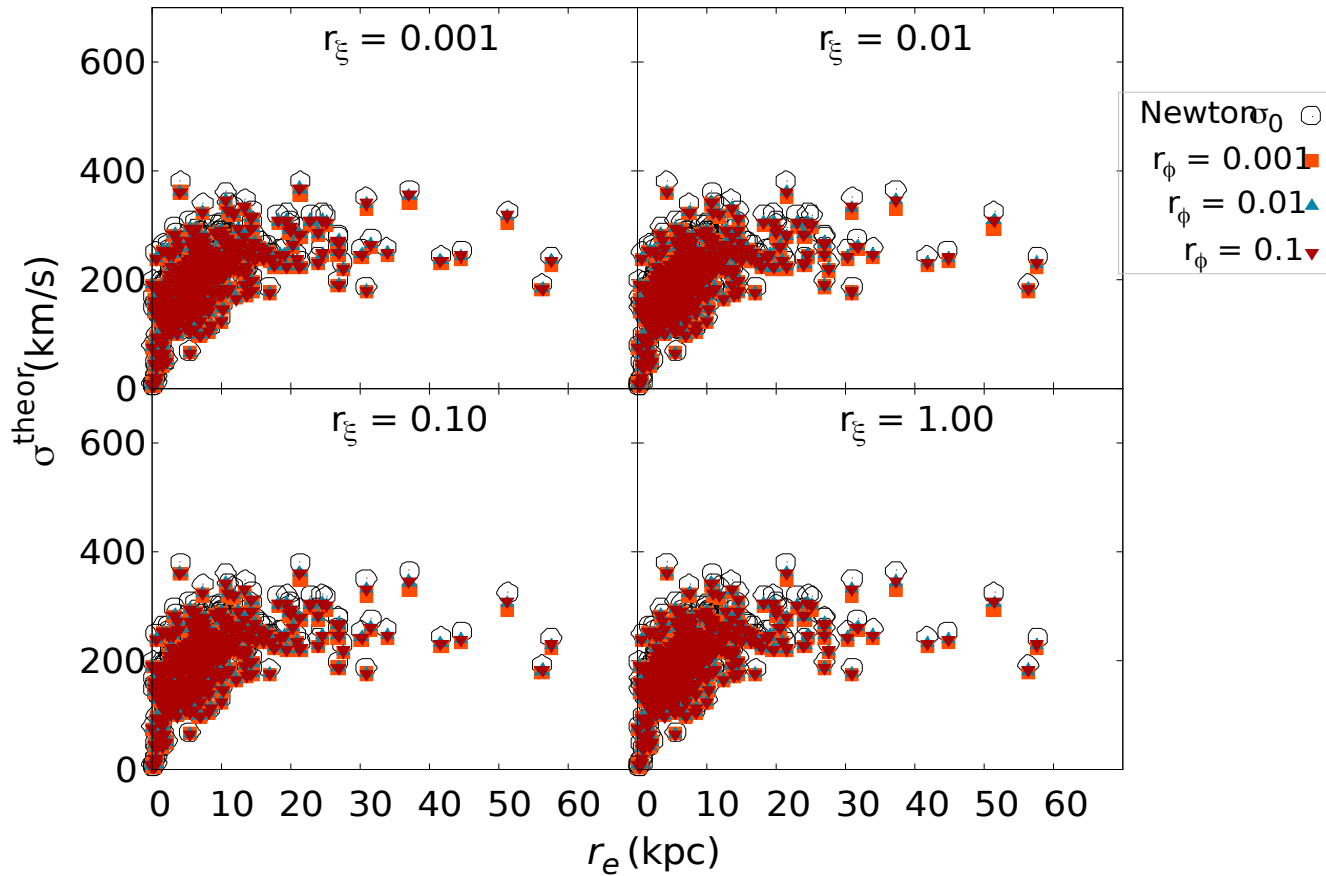


Fig. 8. Velocity dispersion  $\sigma^{\text{theor}}$  as a function of the effective radius  $r_e$  for elliptical galaxies, for four different values of the  $r_\xi$ : 0.001, 0.01, 0.10 and 1.00 kpc. The Newtonian velocity dispersion  $\sigma_0$  at the effective radius is taken from Burstein et al. (1997). Theoretical values of velocity dispersion  $\sigma^{\text{theor}}$  are calculated for the three values of non-local Gravity parameter  $r_\phi$ : 0.001, 0.01 and 0.1 kpc. Value of  $\phi_c$  is 0.9.

# Results VII-Non-local Gravity model

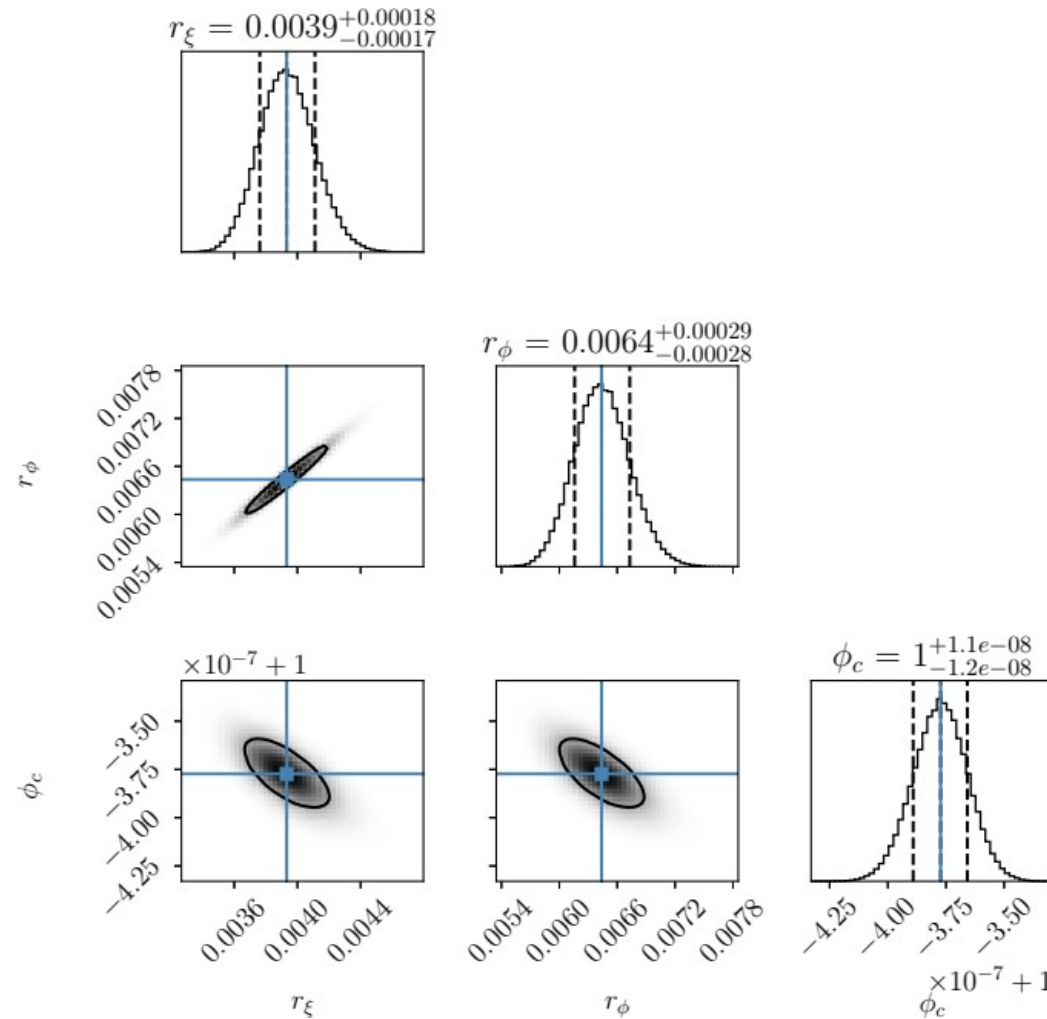


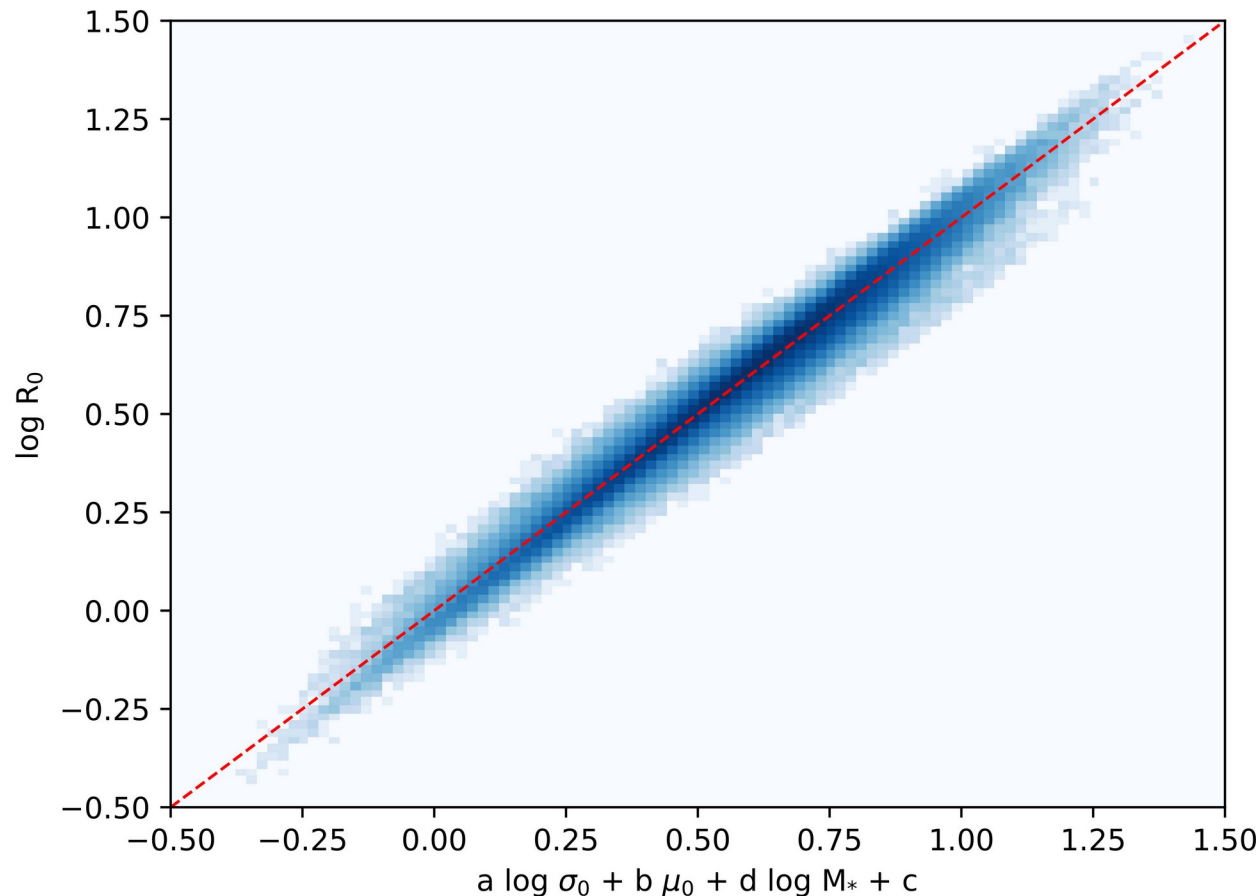
Fig. 9. The posterior probability distributions of the parameters of Non-local gravity model. The contours reported 68% confidence level for all studied parameters.

# Conclusions

- We compared theoretical predictions for circular velocity in above mentioned gravity models with the corresponding values from a large sample of observed elliptical galaxies. In our investigations, from this sample of data, we use surface brightnesses, effective radius and velocity dispersion.
- In this way, we are able to constrain the different gravity parameters.
- We showed that the gravity parameters are scale-length depending on the gravitational system properties.
- Also, we showed that analysed modified gravity models are able to reproduce the stellar dynamics in elliptical galaxies.
- These gravity models fit the observations very well, without the need for a dark matter. We believe that the approach we are proposing can be used to constrain the different modified gravity models.

# Future work: FP from SDSS - DR18

- SDSS (Sloan Digital Sky Survey) - Data Release 18
- DR18 is the first data release for the 5<sup>th</sup> phase of the SDSS (described in Almeida et al. 2023).



- **Saulder 2019**
  - SDSS DR15
  - arXiv:1905.12970v1
- (Fig 12: the expanded FP)
- FP distances to over 317000 early-type elliptical galaxies up to a redshift of  $z = 0.4$



## References

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THANK YOU FOR YOUR ATTENTION!

# MCMC analysis of data

## 5.1. Uncertainty analyses of the gravity parameters using Markov chain Monte Carlo analysis

Probabilistic data analysis has great impact on scientific research in the past decade. Its procedures involve computing and using the posterior probability density function for the parameters of the model or the likelihood function. Markov chain Monte Carlo (MCMC) methods provide sampling approximations to the posterior probability density function efficiently even in parameter spaces with large numbers of dimensions. For example, many problems in cosmology and astrophysics have benefited from MCMC because there are many free parameters, and the observations are usually low in signal-to-noise ratio (Foreman-Mackey et al., 2013).

In this subsection we estimated 68% confidence region for the Non-local Gravity parameters using MCMC (for more details see Foreman-Mackey et al., 2013; Audren et al., 2013; Sharma, 2017; Hogg and Foreman-Mackey, 2018, and references therein). For that purpose, we used an MIT licensed pure-Python implementation of Goodman & Weare's Affine Invariant Markov chain Monte Carlo Ensemble sampler (<https://emcee.readthedocs.io/en/stable/>). The explanation of the emcee algorithm and its implementation in detail are given in the following paper: Foreman-Mackey et al. (2013).

As a first step, we obtained the maximum likelihood values of Non-local Gravity parameters using the `optimize.minimize` module from SciPy for maximization of their likelihood function (or more precisely, for minimization of its negative logarithm), assuming Eq. (14) as our model for  $\sigma^{theor}$ . After that we used these maximum likelihood values of the parameters as a starting point for our MCMC simulations, which we performed in order to estimate the posterior probability distributions for the gravity parameters. These simulations were carried out using  $0 < r_\xi \wedge r_\phi < 100$  kpc and  $0.9 < \phi_c < 1.1$  as our priors.

Foreman-Mackey, D., Hogg, D.W., Lang, D., Goodman, J., 2013. emcee: The MCMC Hammer. *Publ. Astron. Soc. Pac.* 125 (925), 306–312. <https://doi.org/10.1086/670067>, arXiv:1202.3665.

Sharma, S., 2017. Markov Chain Monte Carlo Methods for Bayesian Data Analysis in Astronomy. *Ann. Rev. Astron. Astrophys.* 55 (1), 213–259. <https://doi.org/10.1146/annurev-astro-082214-122339>, arXiv:1706.01629.

Hogg, D.W., Foreman-Mackey, D., 2018. Data Analysis Recipes: Using Markov Chain Monte Carlo. *Astrophys. J. Suppl. Series* 236 (1), 11–18. <https://doi.org/10.3847/1538-4365/aab76e>, arXiv:1710.06068.

## Distance measurements to early-type galaxies by improving the fundamental plane

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### ABSTRACT

Using SDSS DR15 to its full extent, we derived fundamental plane distances to over 317 000 early-type galaxies up to a redshift of 0.4. In addition to providing the largest sample of fundamental plane distances ever calculated, as well as a well calibrated group catalogue covering the entire SDSS spectroscopic footprint as far a redshift of 0.5, we present several improvements reaching beyond the traditional definition of the fundamental plane. In one approach, we adjusted the distances by removing systematic biases and selection effects in redshift-magnitude space, thereby greatly improving the quality of measurements. Alternatively, by expanding the traditional fundamental plane by additional terms, we managed to remove systematic biases caused by the selection of our SDSS spectroscopic galaxy sample as well as notably reducing its scatter. We discuss the advantages and caveats of these various methods and calibrations in detail. We found that improving the fundamental plane distance estimates beyond the established methods requires a delicate balancing act between various systematic biases and gains, but managed to reduce the uncertainty of our distance measurements by about a factor of two compared to the traditional fundamental plane.



# Observational data VI

This paper has been withdrawn by Christoph Saulder [Submitted on 30 May 2019 ([v1](#)), last revised 25 Nov 2021 (this version, v2)]

## Distance measurements to early-type galaxies by improving the fundamental plane

Christoph Saulder, Ian Steer, Owain Snaith, Changbom Park

Using SDSS DR15 to its full extent, we derived fundamental plane distances to over 317 000 early-type galaxies up to a redshift of 0.4. In addition to providing the largest sample of fundamental plane distances ever calculated, as well as a well calibrated group catalogue covering the entire SDSS spectroscopic footprint as far a redshift of 0.5, we present several improvements reaching beyond the traditional definition of the fundamental plane. In one approach, we adjusted the distances by removing systematic biases and selection effects in redshift-magnitude space, thereby greatly improving the quality of measurements. Alternatively, by expanding the traditional fundamental plane by additional terms, we managed to remove systematic biases caused by the selection of our SDSS spectroscopic galaxy sample as well as notably reducing its scatter. We discuss the advantages and caveats of these various methods and calibrations in detail. We found that improving the fundamental plane distance estimates beyond the established methods requires a delicate balancing act between various systematic biases and gains, but managed to reduce the uncertainty of our distance measurements by about a factor of two compared to the traditional fundamental plane.

### Comments:

Paper got rejected and we have to admit there are some serious flaws in section 3.5 and 3.6 that propagate through the rest of the paper