# Simulations of positive and negative streamers in the AMReX environment

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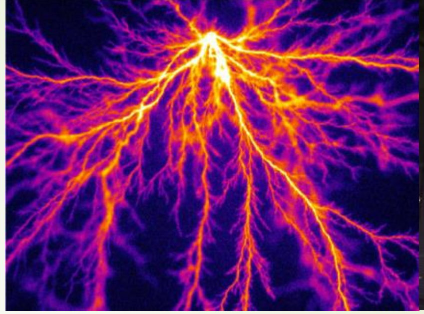
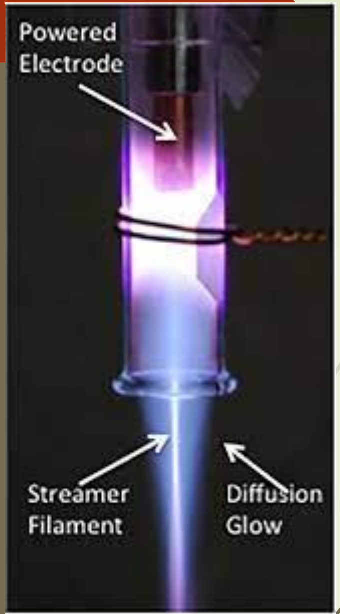
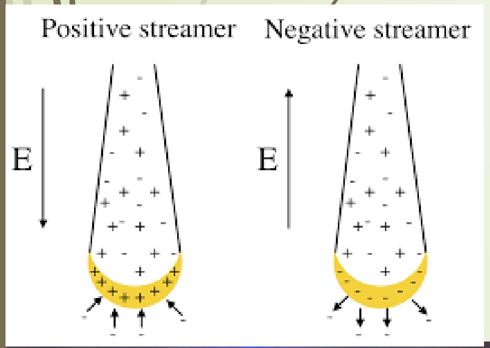
# What are streamers?

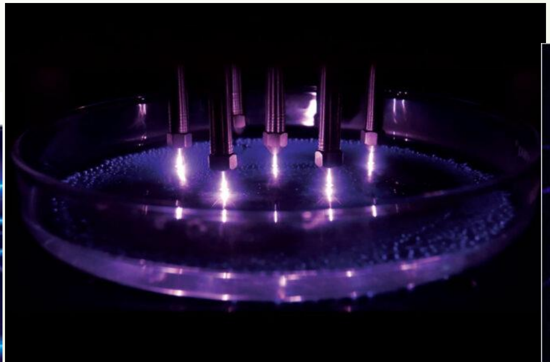


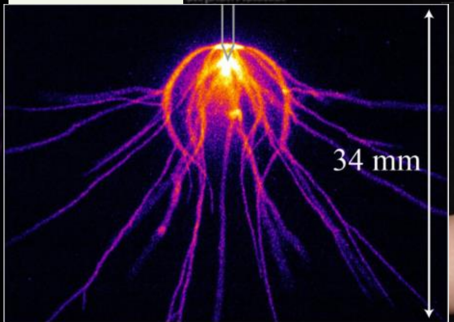
- Thin channels of weakly-ionized nonstationary plasma produced by an ionization front that moves through non-ionized matter
- Streamers in nature: lighting and sprite discharges in the upper planetary atmospheres
- Applications of streamers:
  - ignition of high-intensity discharge lamps
  - treatment of polluted gases and water
  - plasma medicine










# Streamer modelling

- Motivation:
  - better understanding of streamer physics
  - optimization of streamer applications
- Types of streamer models
  - Particle models
  - Fluid models
  - Hybrid models

# Fluid models of streamers

- Number densities of charged particles are represented by continuous functions
- Time evolution of these number densities are represented by the fluid equations
- Fluid models are generally more computationally efficient than the particle models and are much simpler than the hybrid models

# First-order fluid model

Advection diffusion reaction equation for the time evolution of the number density of electrons:

$$\frac{\partial n_e}{\partial t} + \nabla(n_e \boldsymbol{W} - \boldsymbol{D}\nabla n_e) = n_e(\alpha - \eta)|\boldsymbol{W}| + S_{ph}$$

Reaction equations for the time evolution of the number densities of ions:

$$\frac{\partial n_p}{\partial t} = n_e \alpha |\boldsymbol{W}| + S_{ph} \qquad \frac{\partial n_n}{\partial t} = n_e \eta |\boldsymbol{W}|$$

Local field approximation

Total electric field:

$$\mathbf{E} = \boldsymbol{E}_{applied} - \nabla\Phi_{space\_charge} \qquad \Delta\Phi_{space\_charge} = -q_e \frac{n_p - n_e - n_n}{\varepsilon_0}$$

# Photoionization

- In air photons emitted from excited nitrogen molecules can ionize oxygen molecules
- Zheleznyak model:

$$S_{ph}(\boldsymbol{r}) = \int d^3r' \frac{I(\boldsymbol{r}')f(|\boldsymbol{r}-\boldsymbol{r}'|)}{4\pi|\boldsymbol{r}-\boldsymbol{r}'|^2}$$

- The photon source term

$$I(\boldsymbol{r}) = \frac{p_q}{p+p_q}\xi^B \frac{\nu_u}{\nu_i} S_i(\boldsymbol{r})$$

- Ionization source term:

$$S_i(\boldsymbol{r}) = \alpha|\boldsymbol{W}|$$

- The absorption function:

$$\int_0^\infty f(r)dr = 1$$

# Photoionization

- The absorption function can be represented as:

$$f(|\boldsymbol{r}-\boldsymbol{r}'|) = p_{O_2}^2 |\boldsymbol{r}-\boldsymbol{r}'| \sum_{j=1}^{N} A_j^B e^{-\lambda_j^B p_{O_2} |\boldsymbol{r}-\boldsymbol{r}'|}$$

- Leading to the decomposition of the source term:

$$S_{ph} = \sum_{j=1}^{N} S_{ph,j}$$

$$S_{ph}(\boldsymbol{r}) = \frac{p_q}{p+p_q} \xi^B \frac{\nu_u}{\nu_i} \sum_{j=1}^{N} p_{O_2}^2 A_j^B \int d^3 r' \frac{S_i(\boldsymbol{r}') e^{-\lambda_j^B p_{O_2}^2 |\boldsymbol{r}-\boldsymbol{r}'|}}{4\pi |\boldsymbol{r}-\boldsymbol{r}'|}$$

- The components of the source term can be determined by solving a set of Helmholtz equations:

$$\left(\Delta - (p_{O_2}\lambda_j^B)^2\right) S_{ph,j} = -\left(A_j^B p_{O_2}^2 \frac{p_q}{p+p_q} \xi^B \frac{\nu_u}{\nu_i}\right) S_i$$

- Parametrizations: Luque, Bourdon 2 term, Bourdon 3 term

# Spatial discretization: Finite volume method

- Scalar variables are defined at cell centers
- Vector variables are defined at cell faces
- Electric field components at the cell centers are interpolated from the cell faces
- Interpolation of the number density of electrons from the cell centers to the cell faces, to calculate electron fluxes, is not trivial!
- First order upwind scheme creates too much numeric diffusion, while linear interpolation (central differencing) creates strong numeric oscillations.
- For this reason, flux limiting schemes are used.

# Koren flux limiter

- The flux of electrons is calculated as:

$$f_{i+\frac{1}{2}} = \begin{cases} W_{i+\frac{1}{2}}\left(n_i + \frac{1}{2}\phi\left(r^+_{i+\frac{1}{2}}\right)(n_i - n_{i-1})\right), & \text{for } W_{i+\frac{1}{2}} > 0 \\ W_{i+\frac{1}{2}}\left(n_{i+1} + \frac{1}{2}\phi\left(r^-_{i+\frac{1}{2}}\right)(n_{i+1} - n_{i+2})\right), & \text{for } W_{i+\frac{1}{2}} < 0 \end{cases}$$

- The Koren flux limiter is defined as:

$$\phi(r) = \max\left(0, \min\left(2r, \min\left(\frac{1}{3} + \frac{2r}{3}, 2\right)\right)\right)$$

- The upwind ratio of consecutive solution gradients:

$$r^+_{i+\frac{1}{2}} = \frac{n_{i+1} - n_i + \epsilon}{n_i - n_{i-1} + \epsilon}$$

$$r^-_{i+\frac{1}{2}} = \frac{n_i - n_{i+1} + \epsilon}{n_{i+1} - n_{i+2} + \epsilon}$$

# Time integration

- Time integration is performed by employing the 2nd order Runge-Kutta method.
- Electric potential of space charge, resulting electric field, and transport coefficients are calculated at each stage of the Runge-Kutta method
- Time step restrictions:
  - The CFL condition:

$$\Delta t\left(\sum_{i=1}^{N_{dim}}\frac{|W_i|}{\Delta x}+\frac{2N_{dim}D}{\Delta x^2}\right) < \mathrm{CFL},\quad \mathrm{CFL}=0.4$$

  - The dielectric relaxation time:

$$\Delta t < \frac{\varepsilon_0}{q_e\mu_e n_e},\quad \mu_e=\frac{|\boldsymbol{W}(|\boldsymbol{E}_{total}|)|}{|\boldsymbol{E}_{total}|}$$

# The AMReX library

- An open-source C++ library for massively parallel, block-structured adaptive mesh refinement (AMR) applications
- Has inbuild geometric multigrid solvers
- Has many inbuilt classes which enable a convenient implementation of both grid and particle data
- Allows both MPI and OpenMP parallelization, as well as parallelization on graphic processing units

# Adaptive mesh refinement

- Adaptive mesh refinement is used when high precision is required only in the subset of the calculation domain
- Using a uniform mesh with high resolution is impractical and inefficient under such circumstances
- The streamer dynamics is determined by the electron dynamics in the narrow region at the streamer front
- Thus, a high-resolution mesh is required at the streamer front, while a coarser mesh can be used in the streamer channel, and in the other parts of the domain, which are far from the streamer front

# Refinement criteria

- The refinement criterion due to ionization frequency:

$$\alpha_{eff}(c_1|\boldsymbol{E}_{total}|)\Delta x > c_0, \qquad \alpha_{eff} = \alpha - \eta \qquad c_1 = 1.2, \quad c_0 = 0.8$$

- The refinement criterion due to number density of space charges:

$$\frac{\Delta x^2|\rho|}{\epsilon_0} > c_2, \qquad c_2 = 0.1V$$

- The de-refinement criterion:

$$\alpha_{eff}(c_1|\boldsymbol{E}_{total}|)\Delta x < 0.1 \wedge \Delta x < 30\mu m$$

# Solving elliptic equations

- The Poisson equation and the Helmholtz equation are elliptic equations
- Their solution in each part of the domain depends on the values of the right-hand side in the entire domain
- Geometric multigrid method is very efficient in solving elliptic equations
- The method consists of applying a simple iterative procedure (like Gauss-Seidel or Jacobi) across a hierarchy of grids with reducing resolution
- AMReX includes inbuilt geometric multigrid solvers for the Poisson equation and the Helmholtz equations.
- These multigrid solvers can be easily applied across the entire hierarchy of adaptive mesh refinement levels

# Results: Simulation conditions

- Axisymmetric model
- Domain size: [0 mm, 16 mm] along both coordinates
- Number of points along each axis at the coarsest level: 64
- Boundary conditions:
  - For the number density of electrons: Zero Neumann conditions at all boundaries
  - For the electric potential of space charge and photoionization source terms: Zero Neuman conditions at boundaries perpendicular to the radial axis and zero Dirichlet conditions at boundaries perpendicular to the axial coordinate
- Results of our AMReX-streamer code are compared to the results of the open-source Afivo-streamer code

# Two-headed streamer in the Titan mixture

- The Titan mixture: 98.4% $N_2$, 1.6% $CH_4$
- The number of AMR levels 7 (from 0 to 6)
- No photoionization
- The applied reduced electric field: $E/N_0$ = 147 Td, 1Td = $10^{-21}$ V/m, $N_0 = 2.50475764 \cdot 10^{25}$ $m^{-3}$.

- The initial condition: Neutral Gaussian given by:

$$n_e = n_p = n_{background} + n_0 \cdot e^{-\frac{(z-z_0)^2+r^2}{\sigma^2}}, \quad n_n = 0m^{-3}$$

$$n_{background} = 10^{14}m^{-3}$$

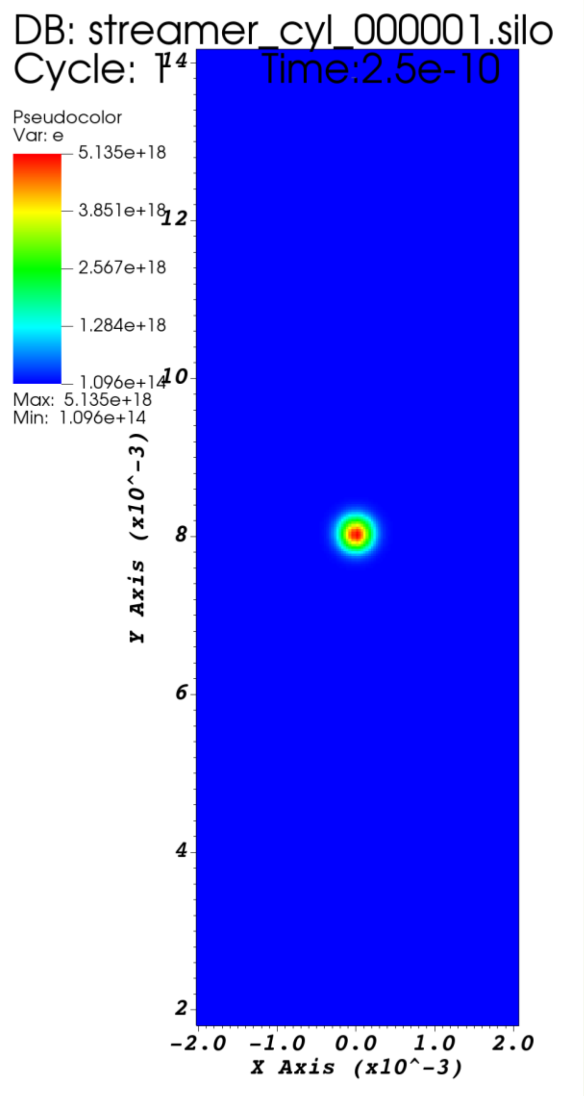
$$n_0 = 5 \cdot 10^{18}m^{-3}$$

$$z_0 = 8mm$$

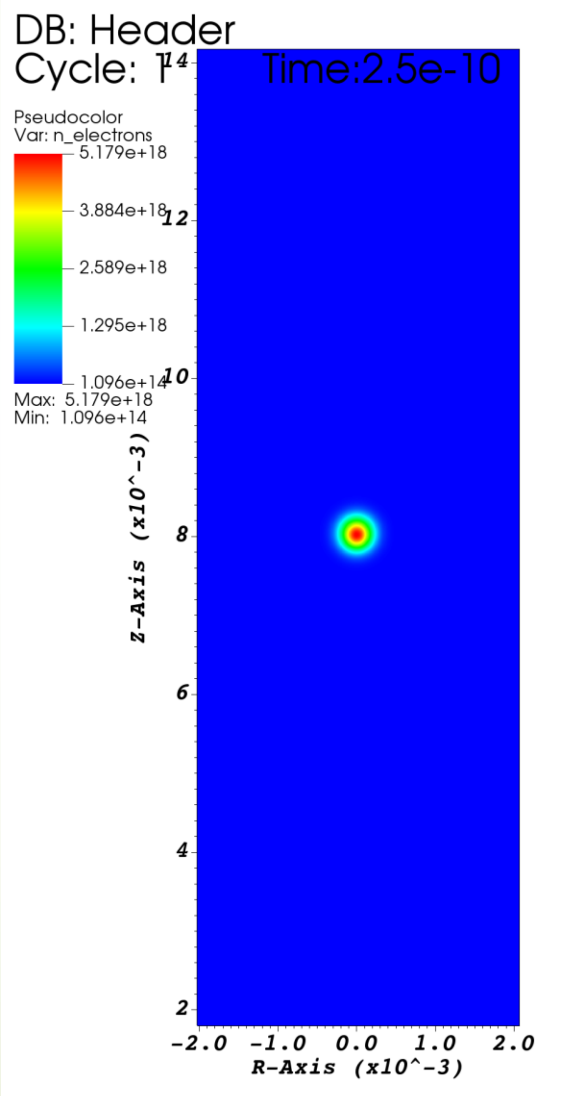
$$\sigma = 4 \cdot 10^{-4}m$$

# Number density of electrons at t = 0.25 ns

## Afivo-streamer

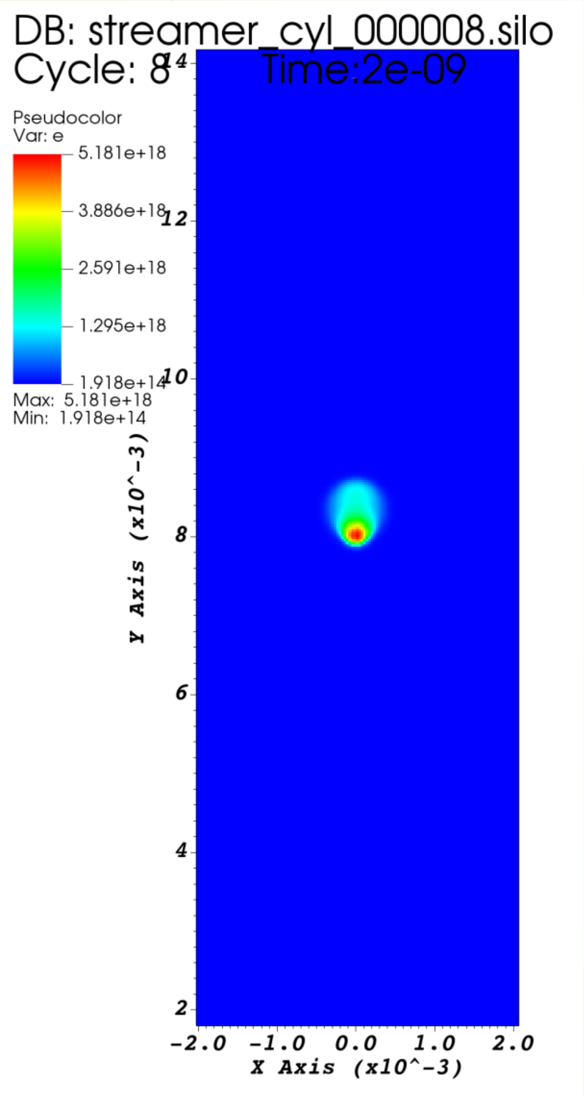



## AMReX-streamer

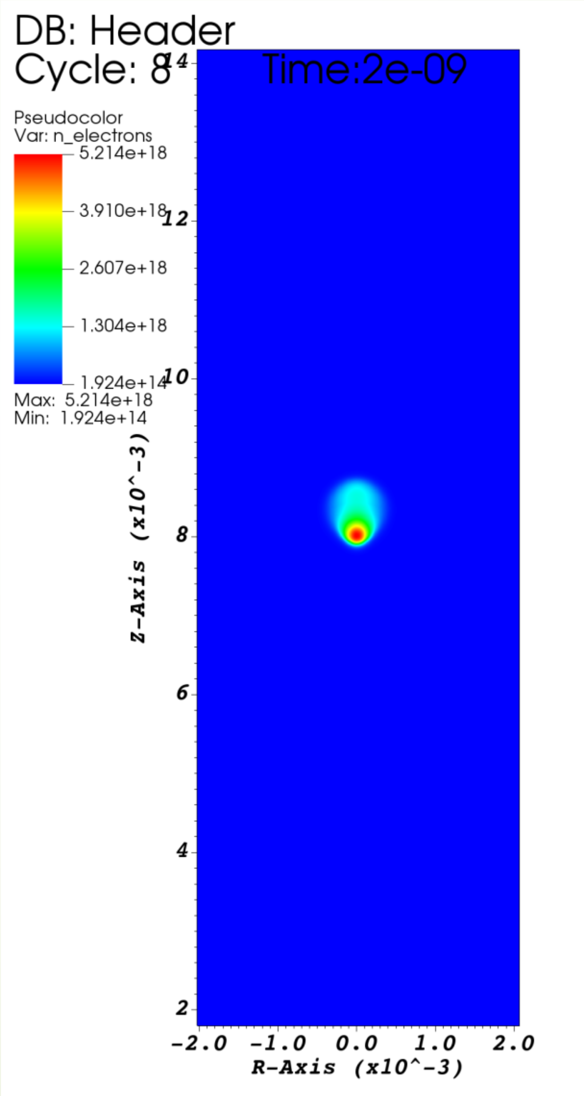


# Number density of electrons at t = 2 ns

Afivo-streamer

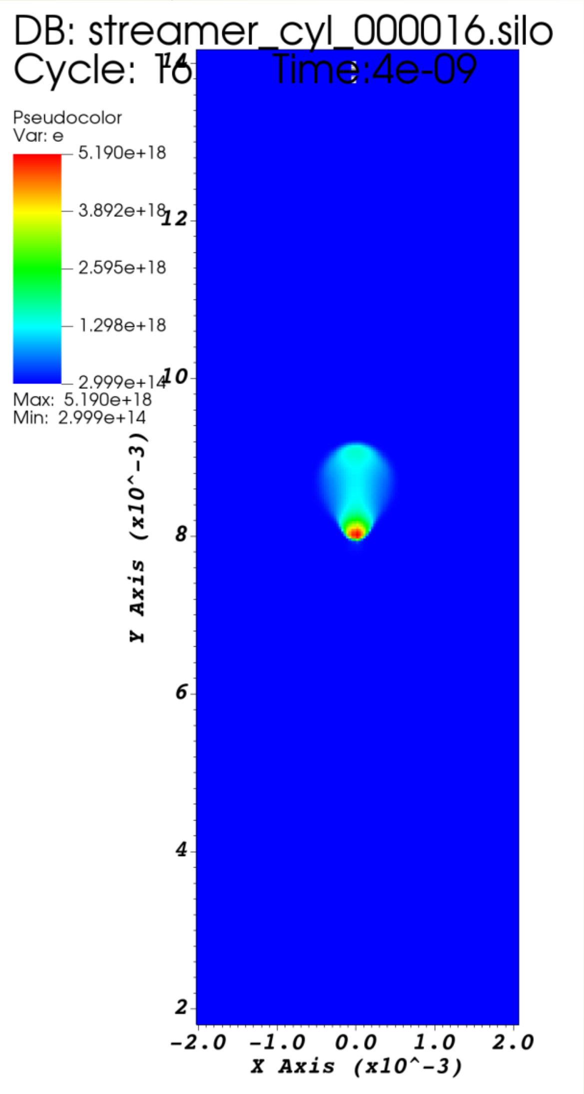



AMReX-streamer



# Number density of electrons at t = 4 ns

## Afivo-streamer

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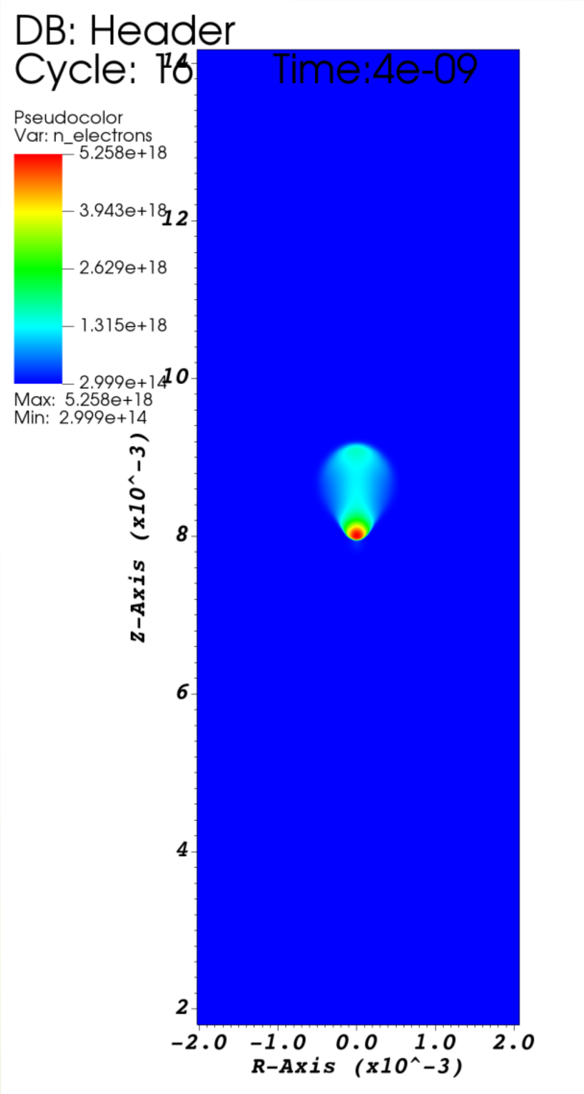
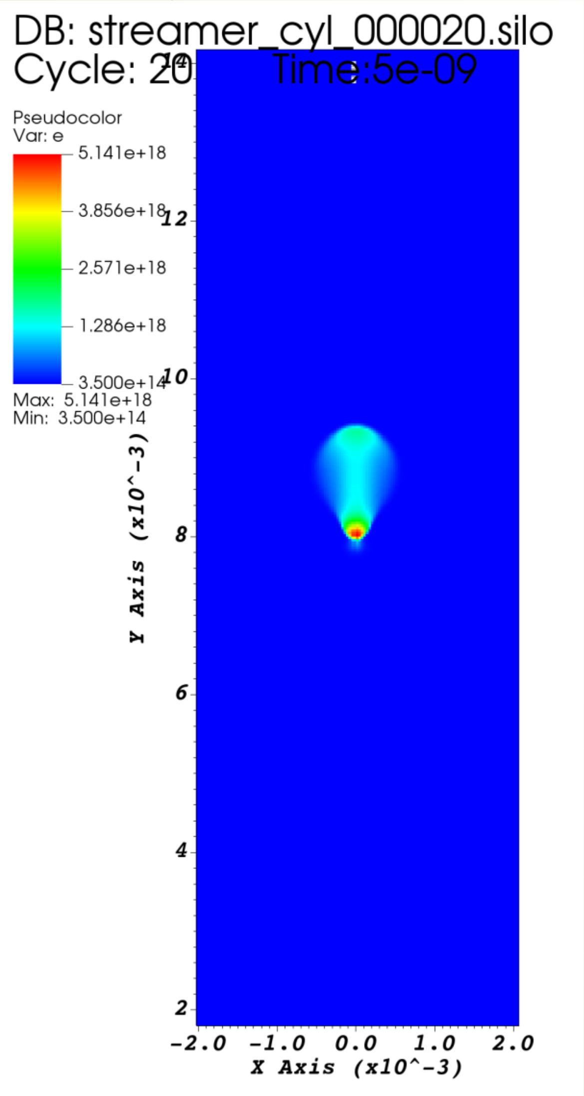


# Number density of electrons at t = 5 ns

## Afivo-streamer

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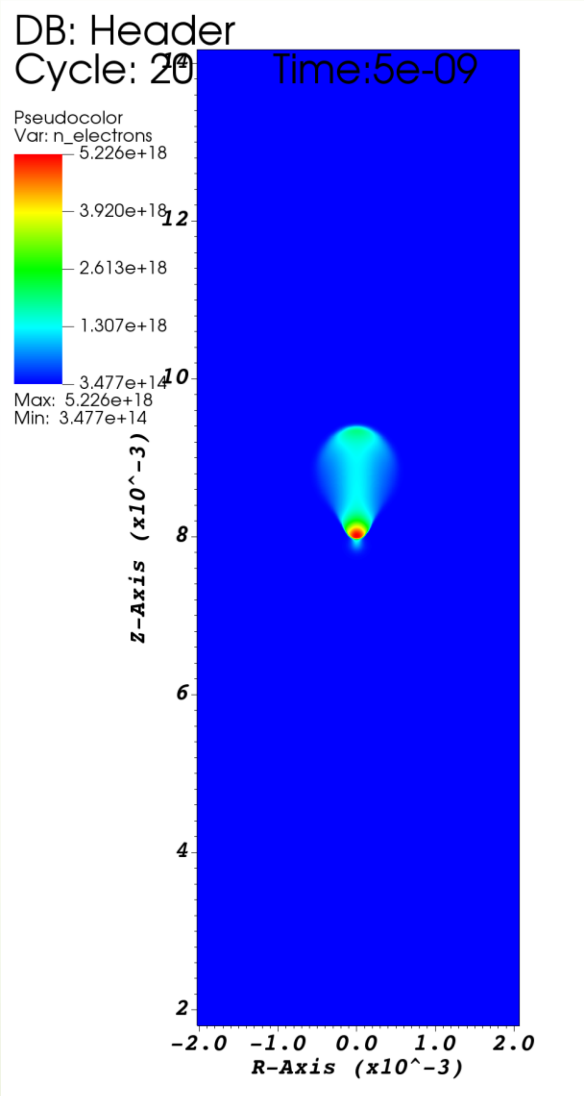
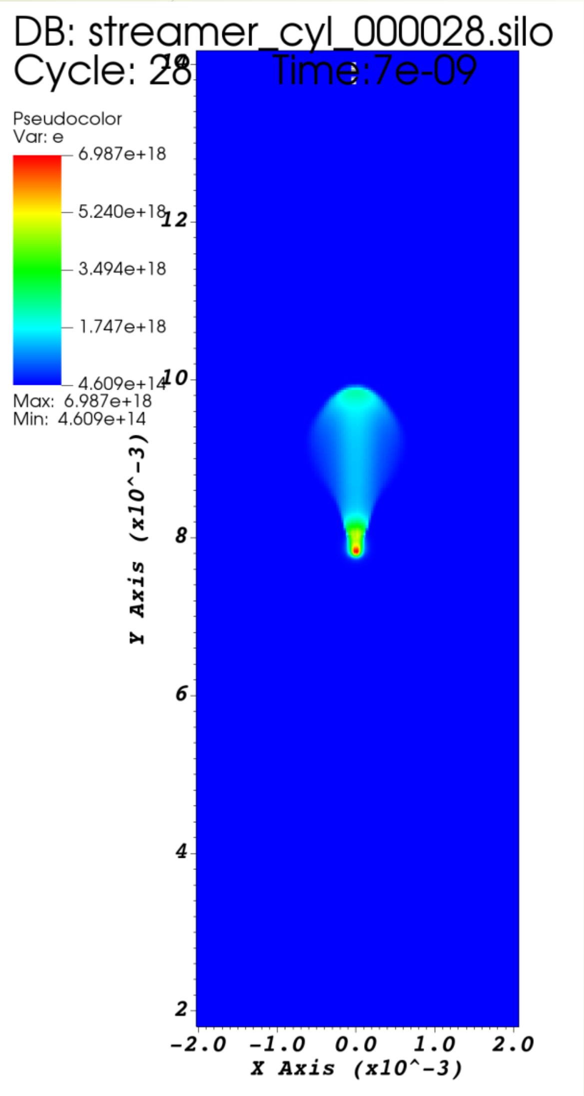


# Number density of electrons at t = 7 ns

## Afivo-streamer

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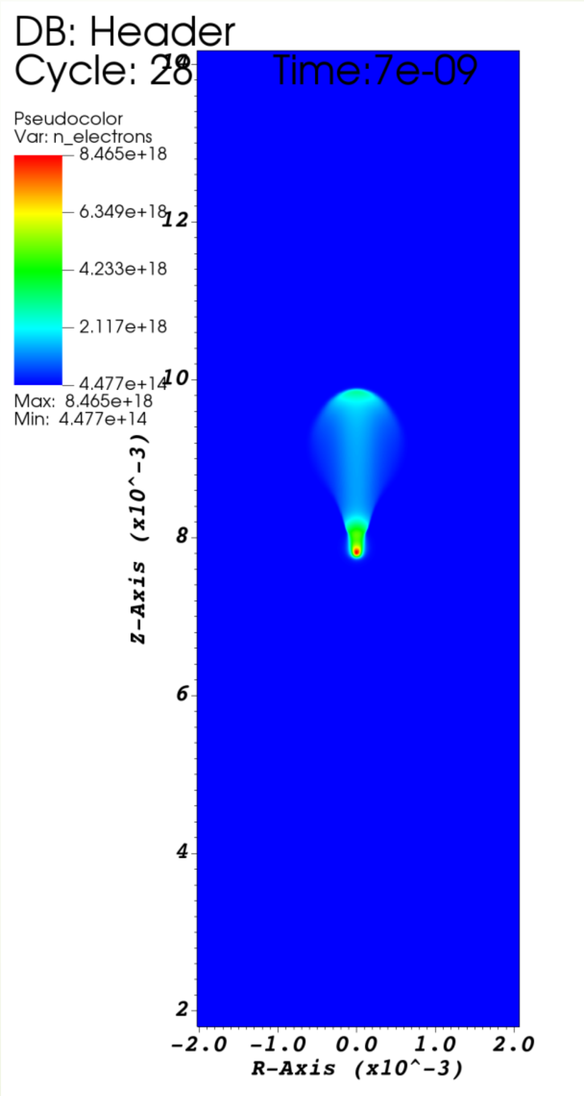
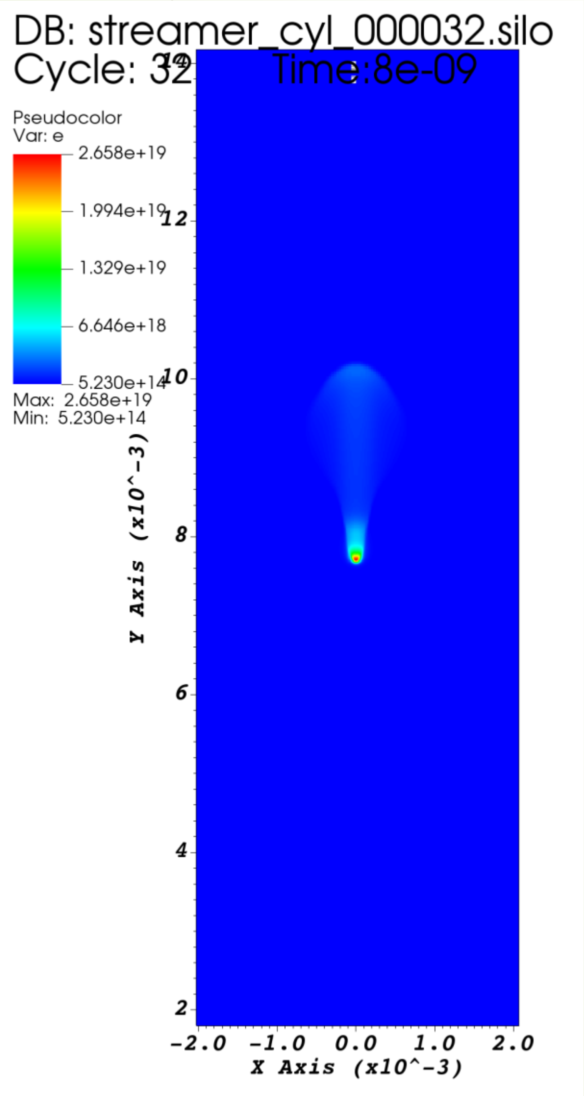


# Number density of electrons at t = 8 ns

## Afivo-streamer

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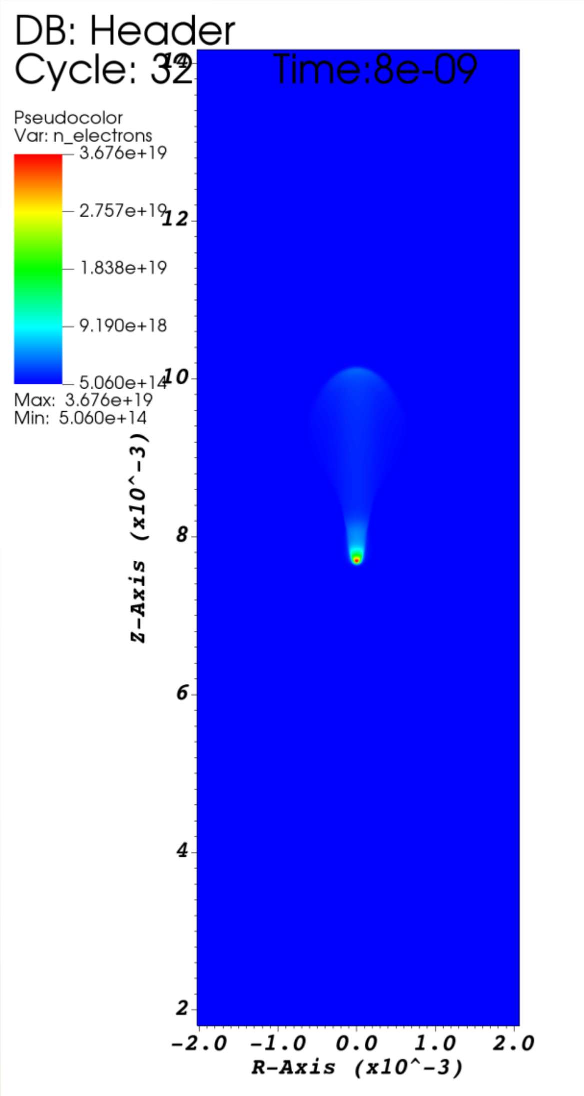
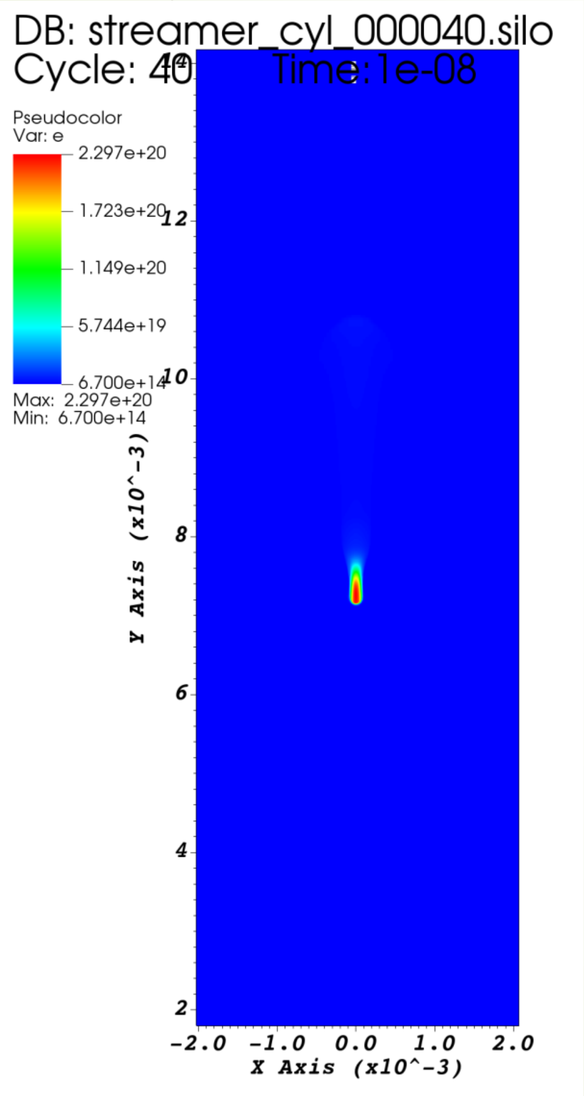
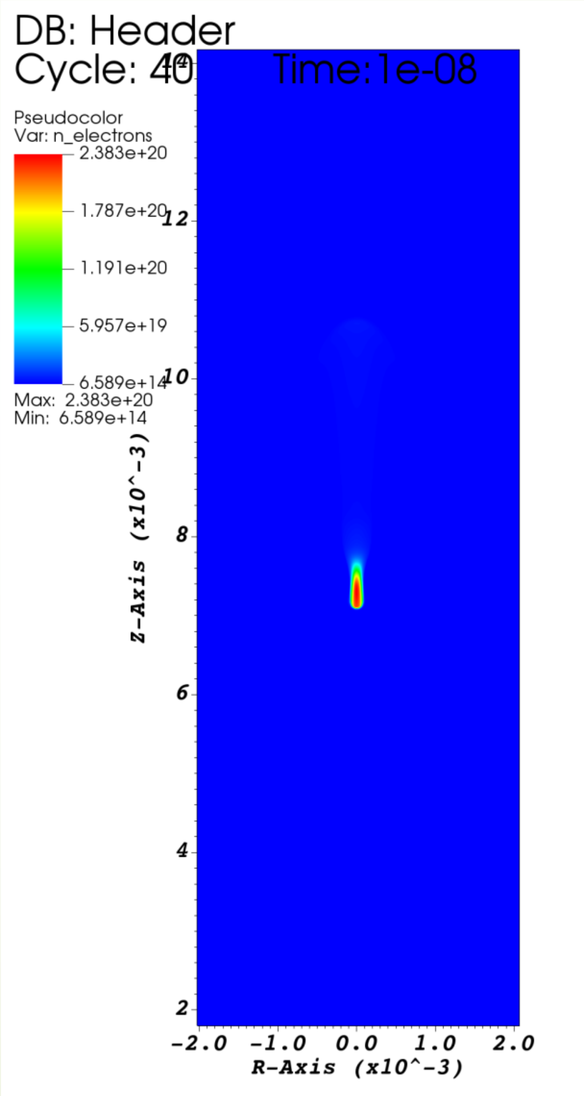


# Number density of electrons at t = 10 ns

## Afivo-streamer

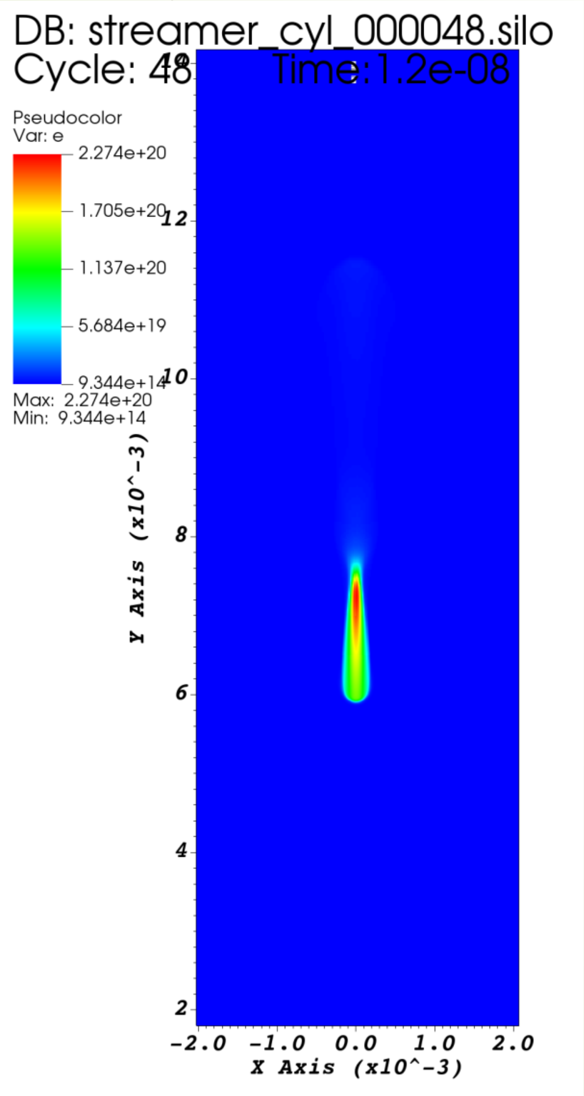



## AMReX-streamer

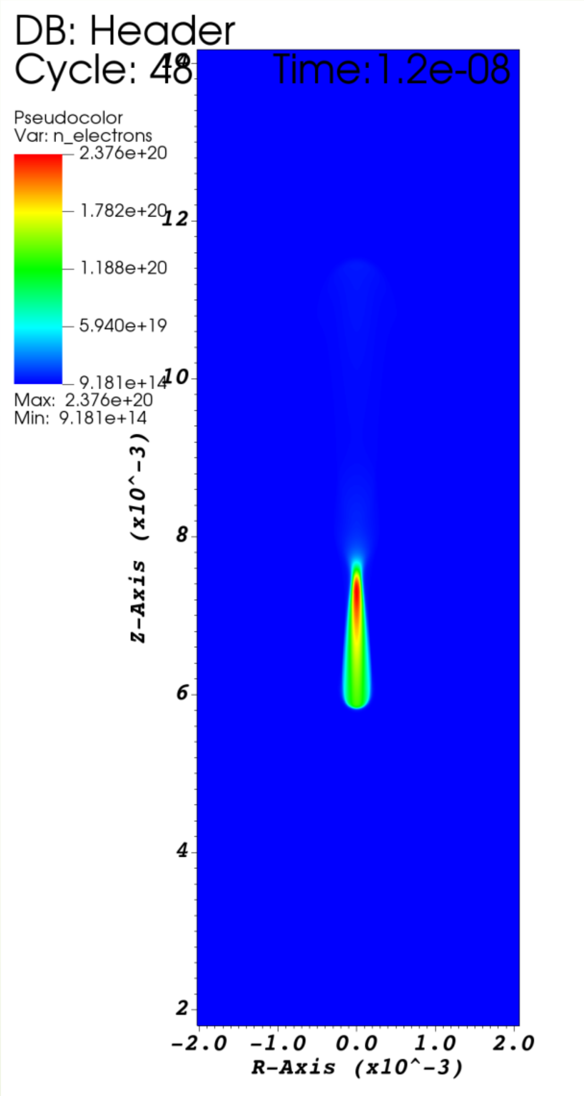


# Number density of electrons at t = 12 ns

## Afivo-streamer

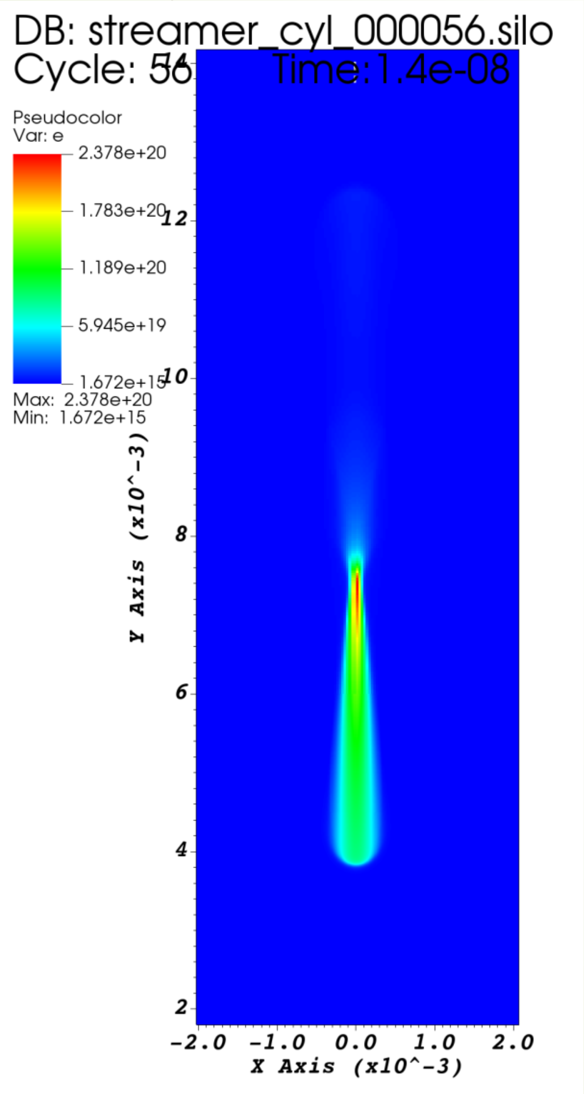



## AMReX-streamer

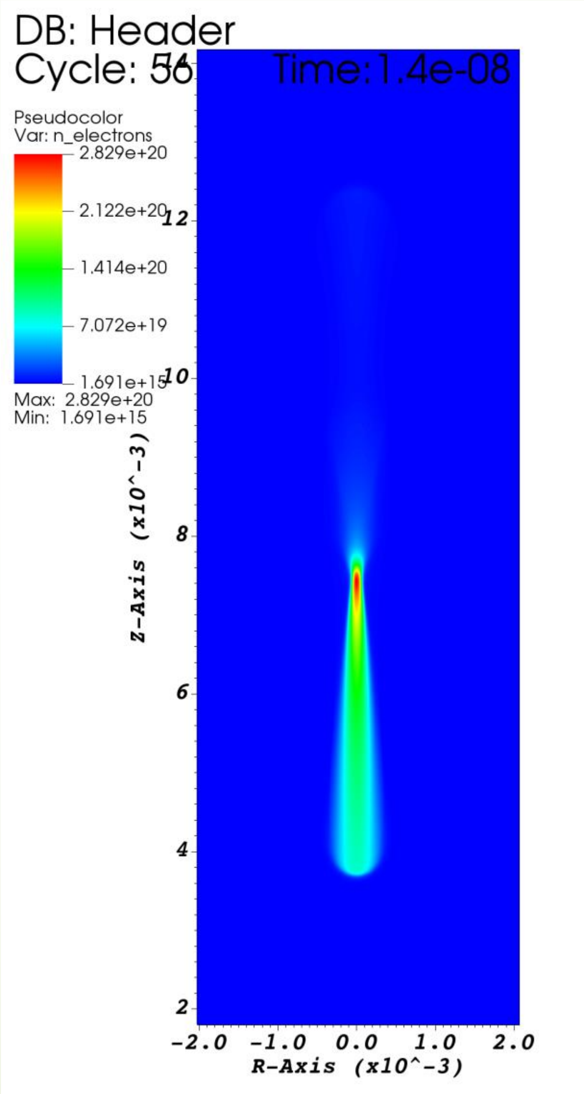


# Number density of electrons at t = 14 ns

## Afivo-streamer

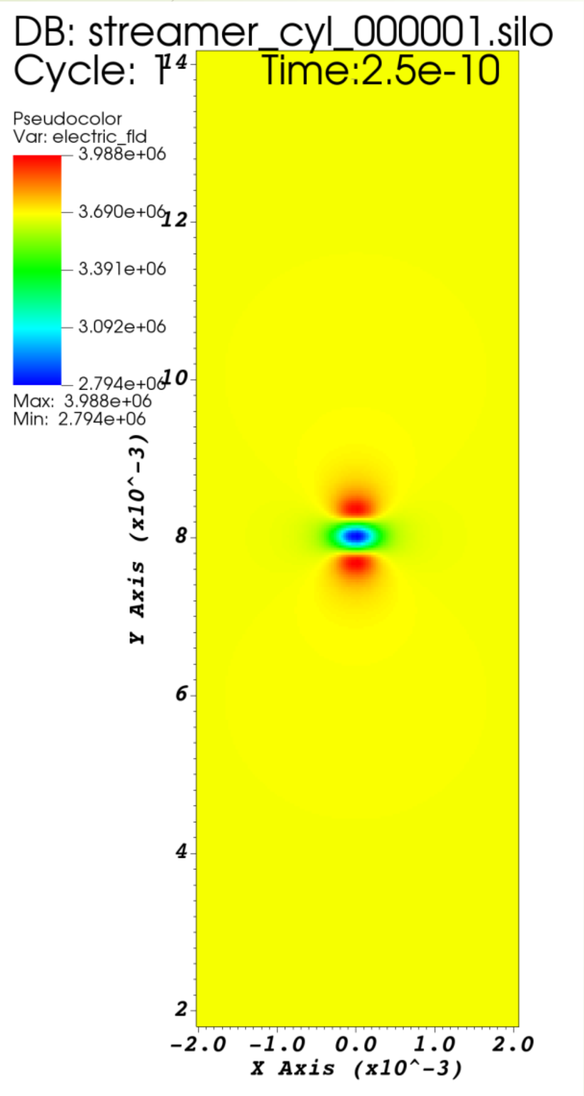



## AMReX-streamer

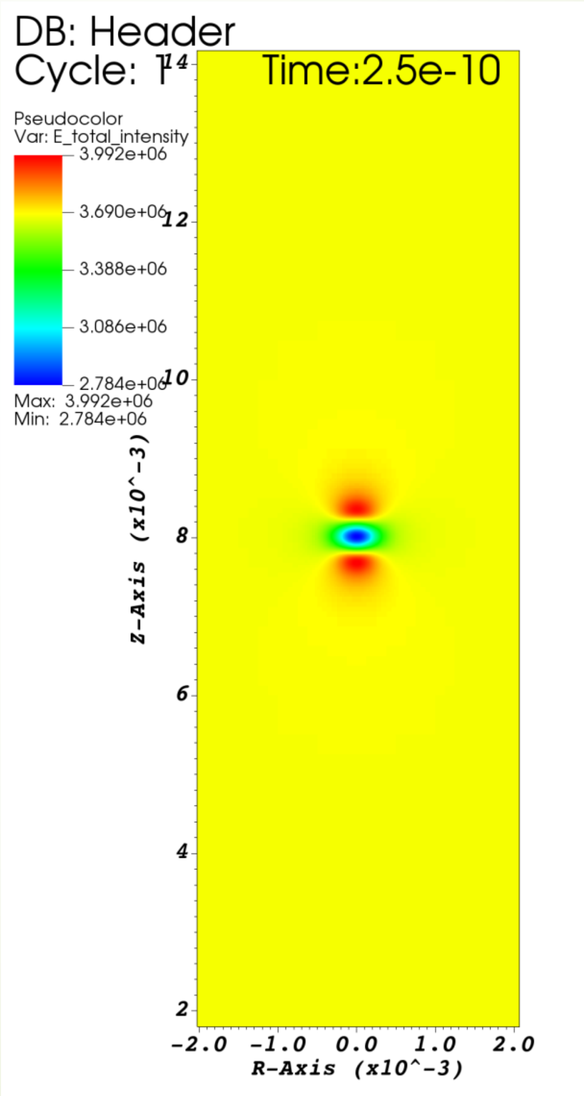


# Electric field intensity at t = 0.25 ns

## Afivo-streamer

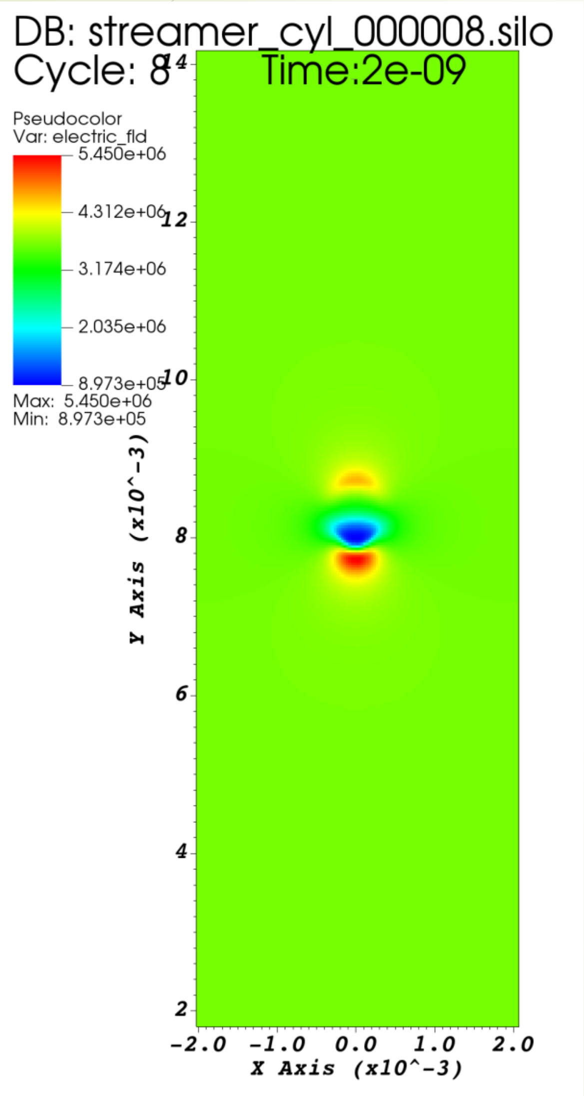



## AMReX-streamer

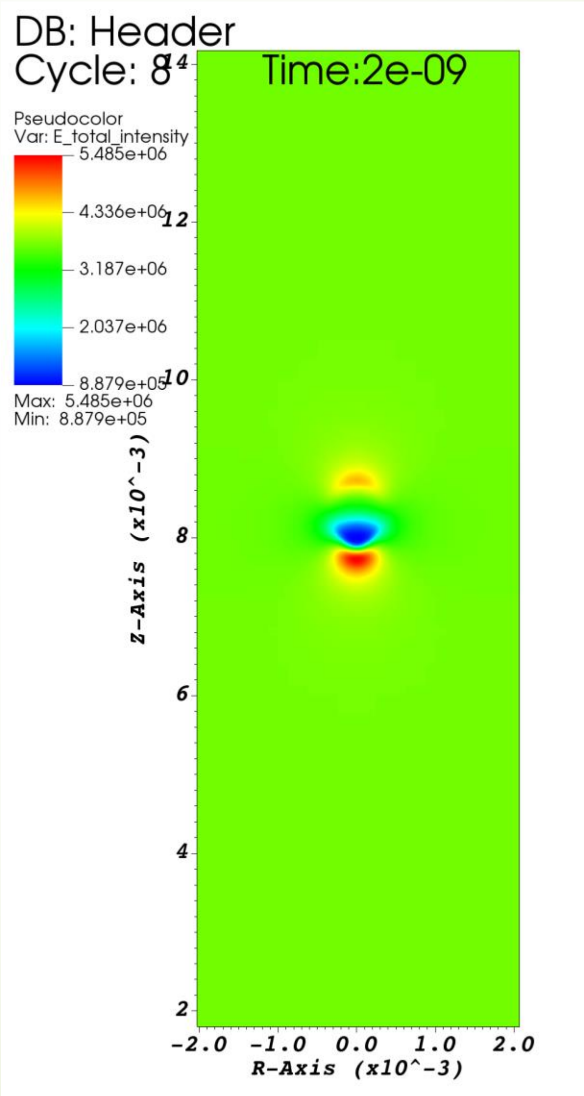


# Electric field intensity at t = 2 ns

## Afivo-streamer

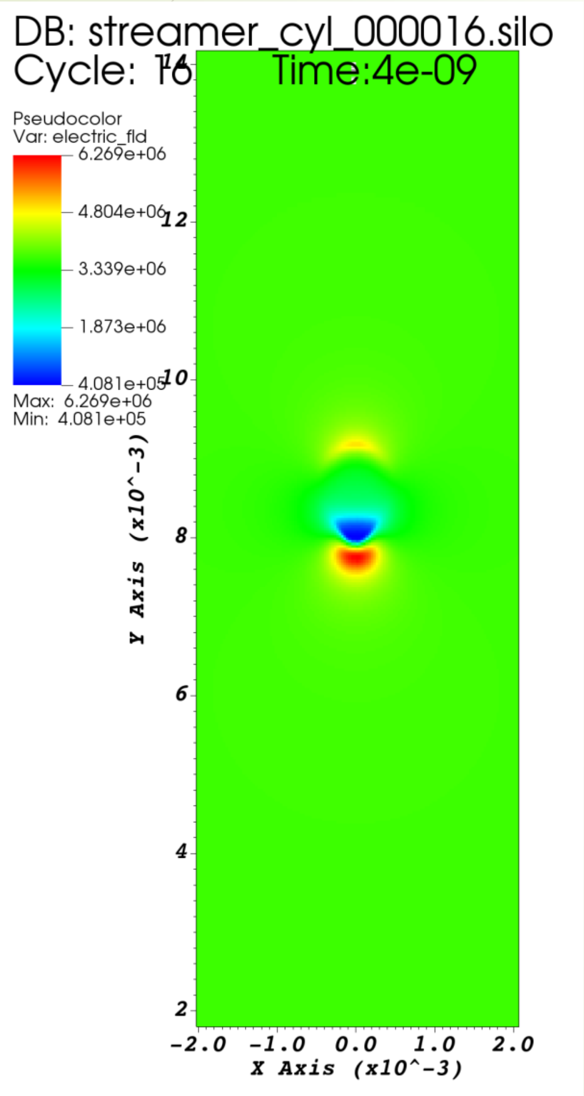



## AMReX-streamer

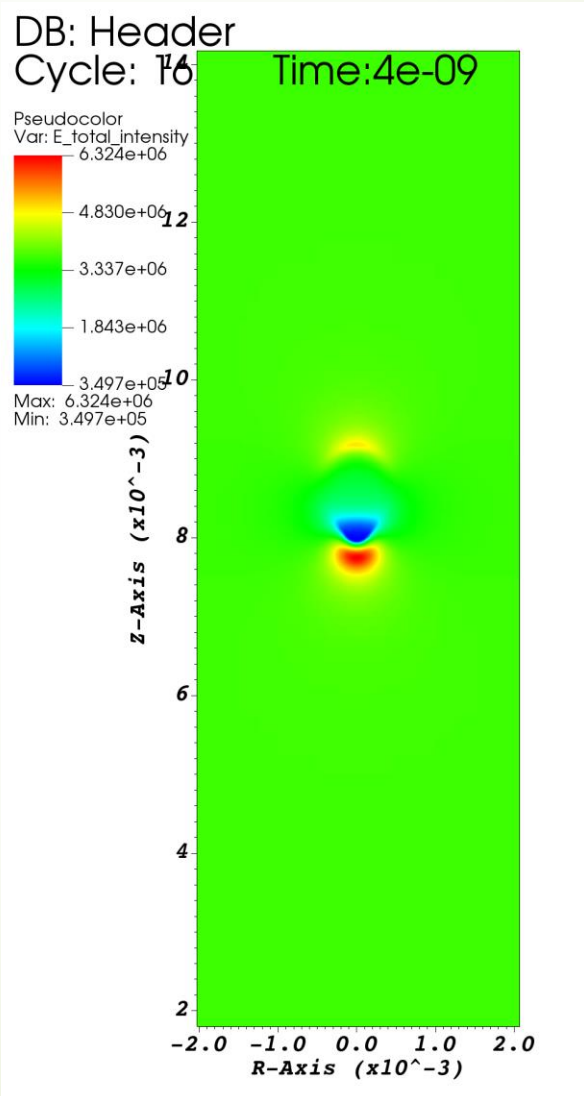


# Electric field intensity at t = 4 ns

Afivo-streamer

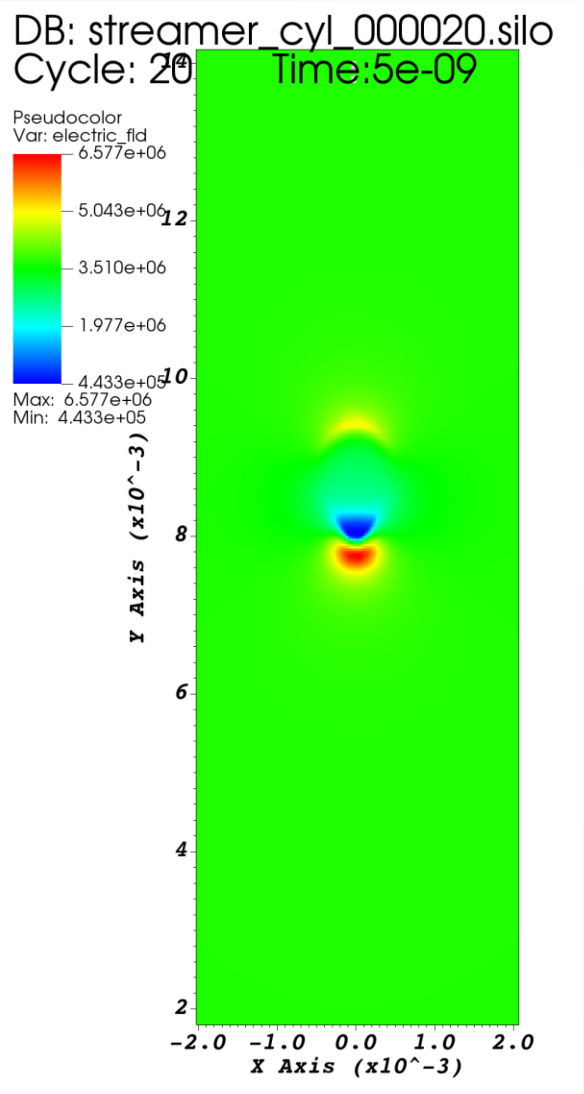



AMReX-streamer



# Electric field intensity at t = 5 ns

Afivo-streamer

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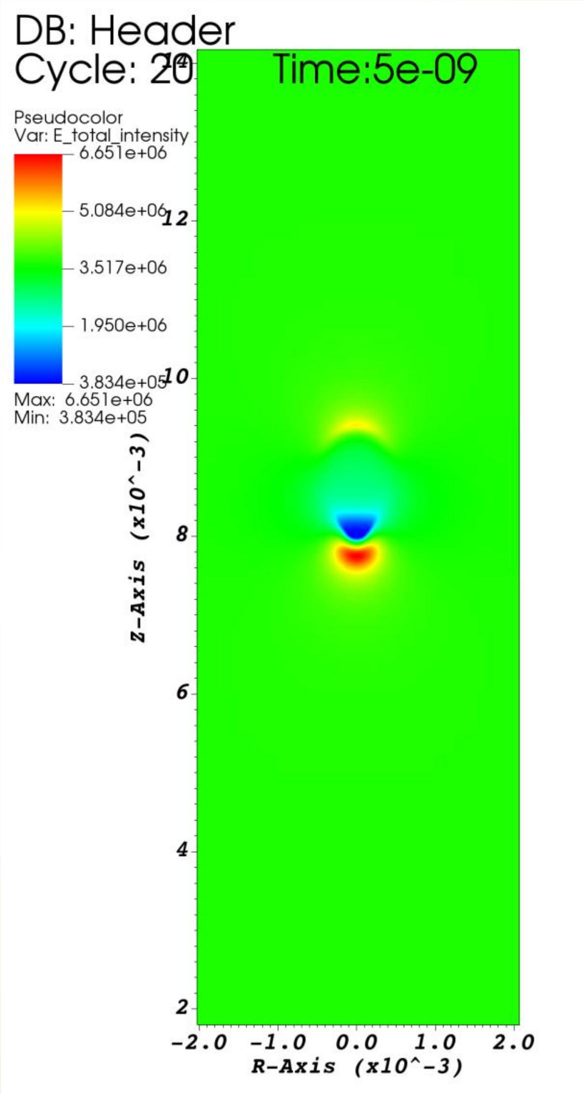
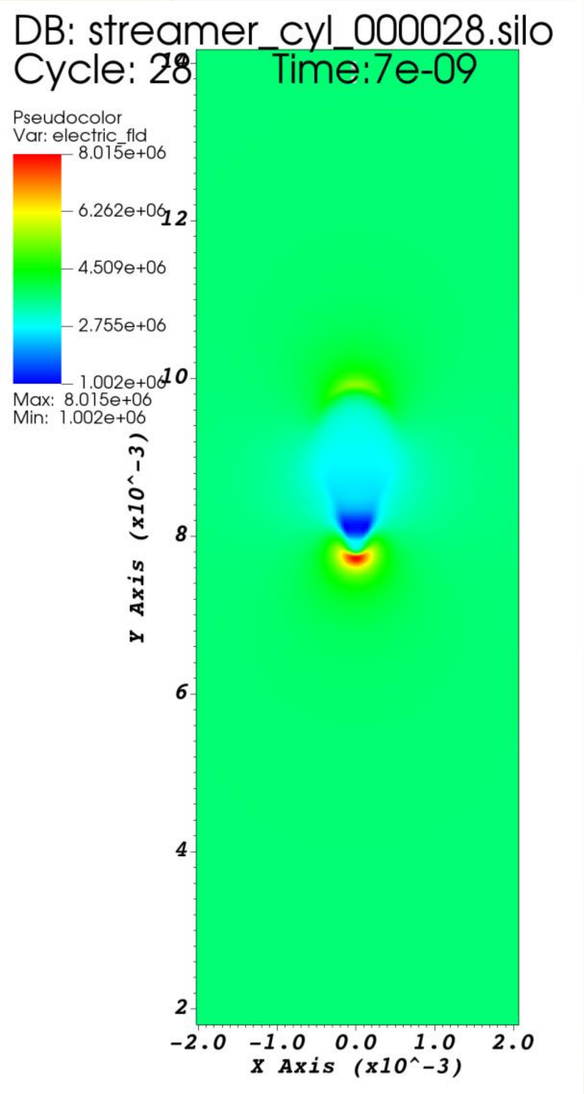
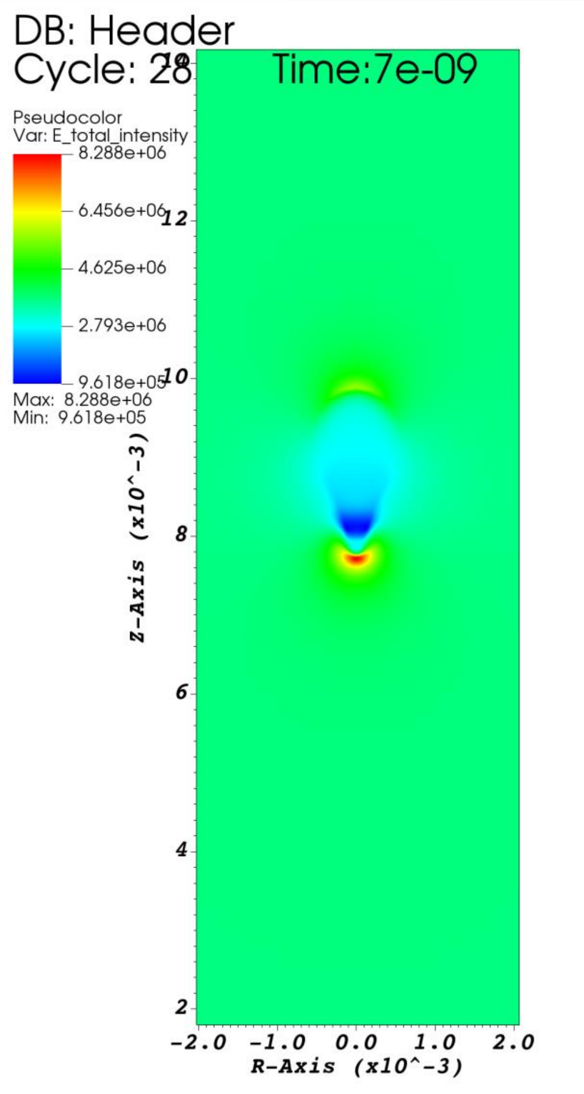


# Electric field intensity at t = 7 ns

## Afivo-streamer

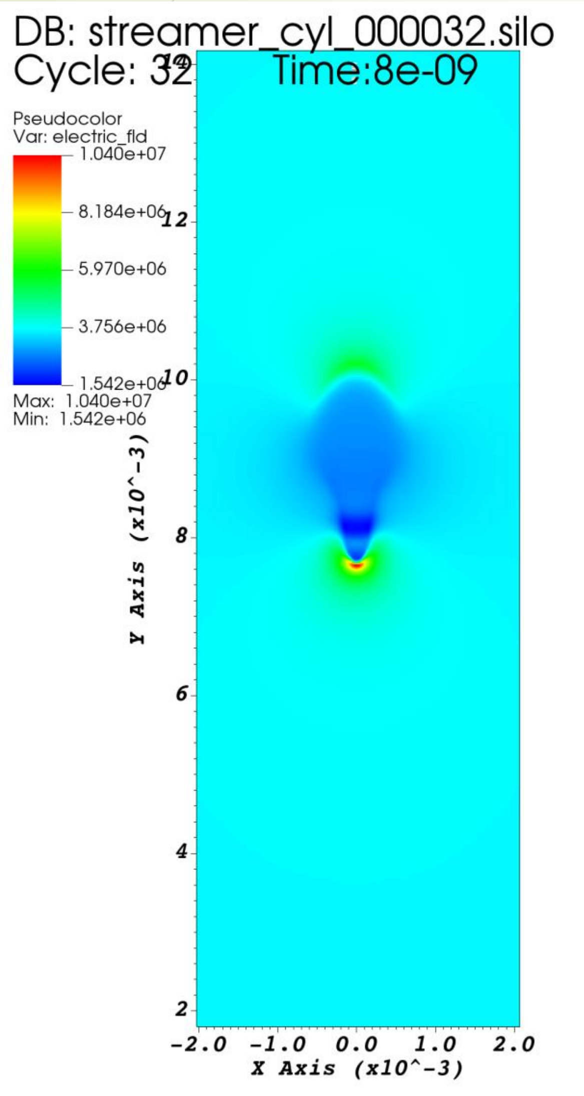



## AMReX-streamer



# Electric field intensity at t = 8 ns

## Afivo-streamer

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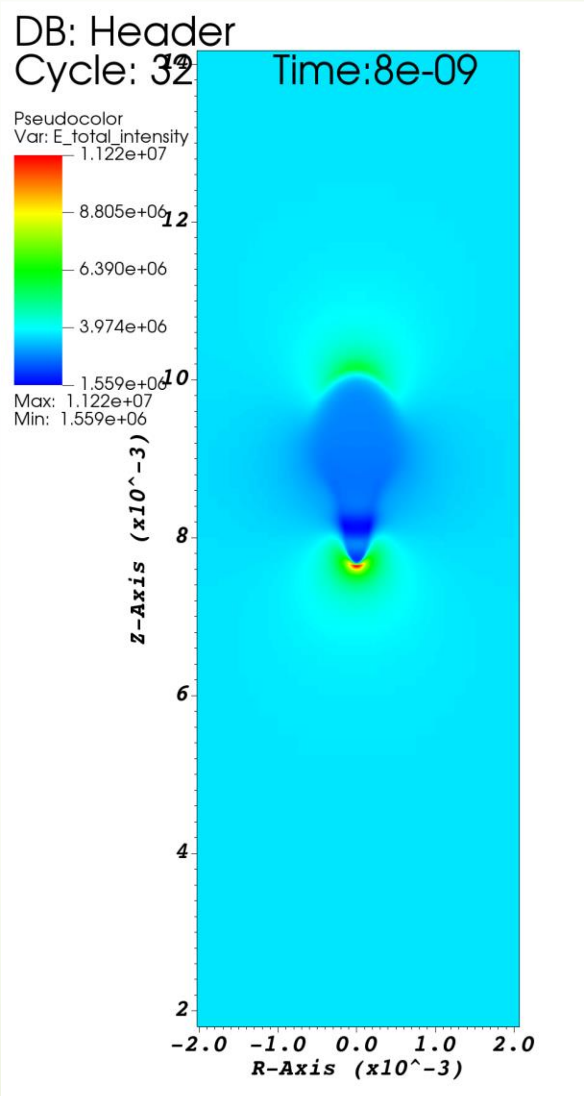
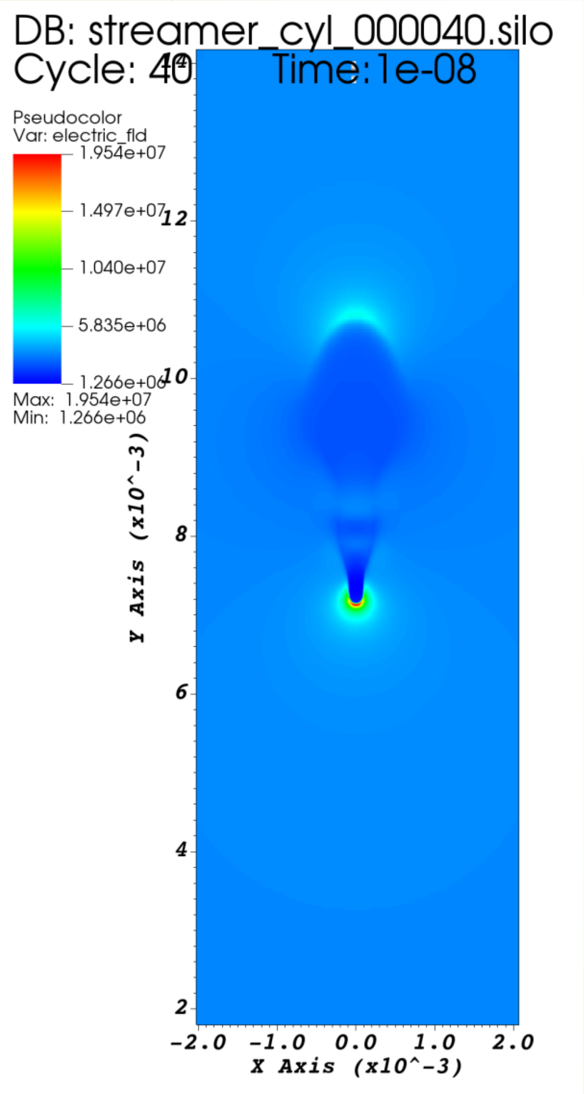


# Electric field intensity at t = 10 ns

## Afivo-streamer

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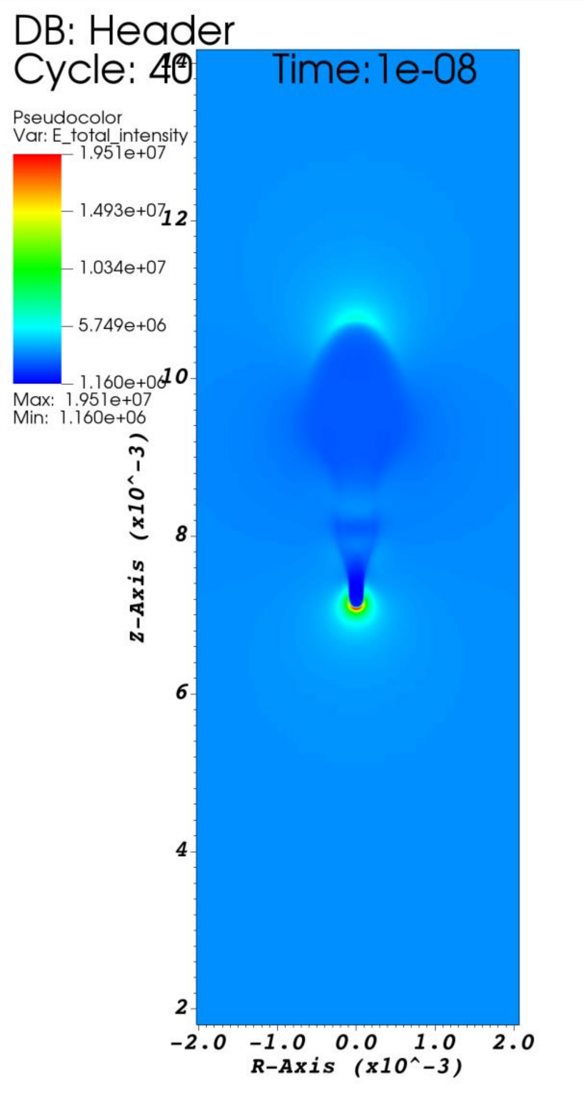
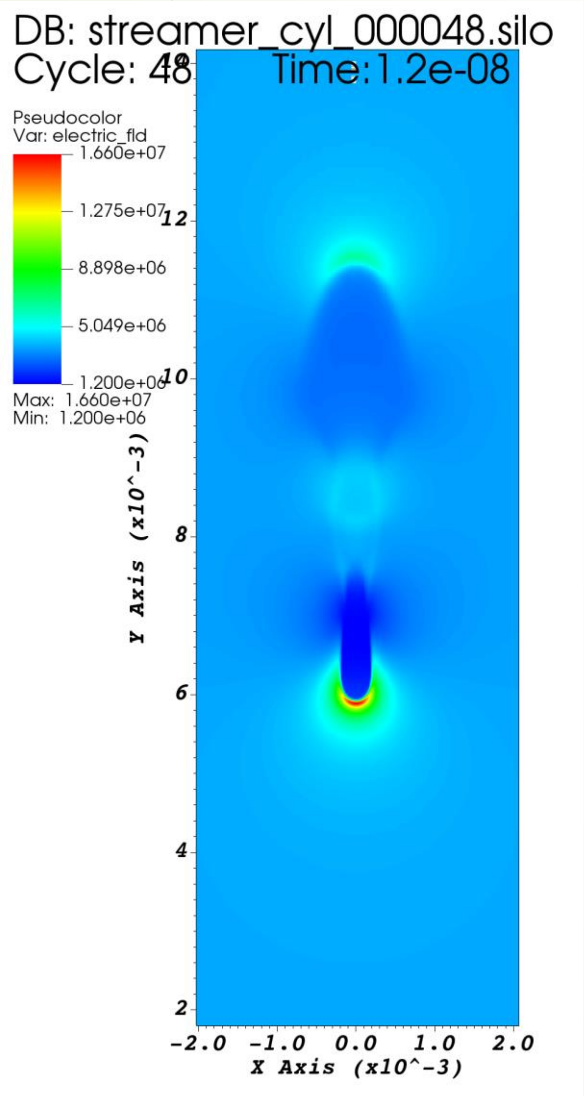
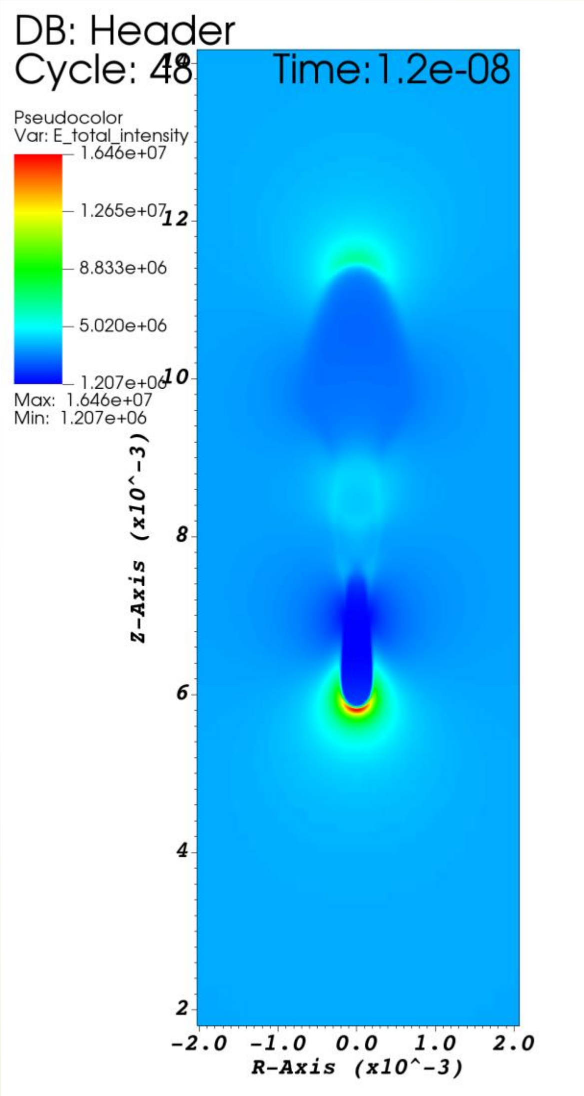


# Electric field intensity at t = 12 ns

Afivo-streamer

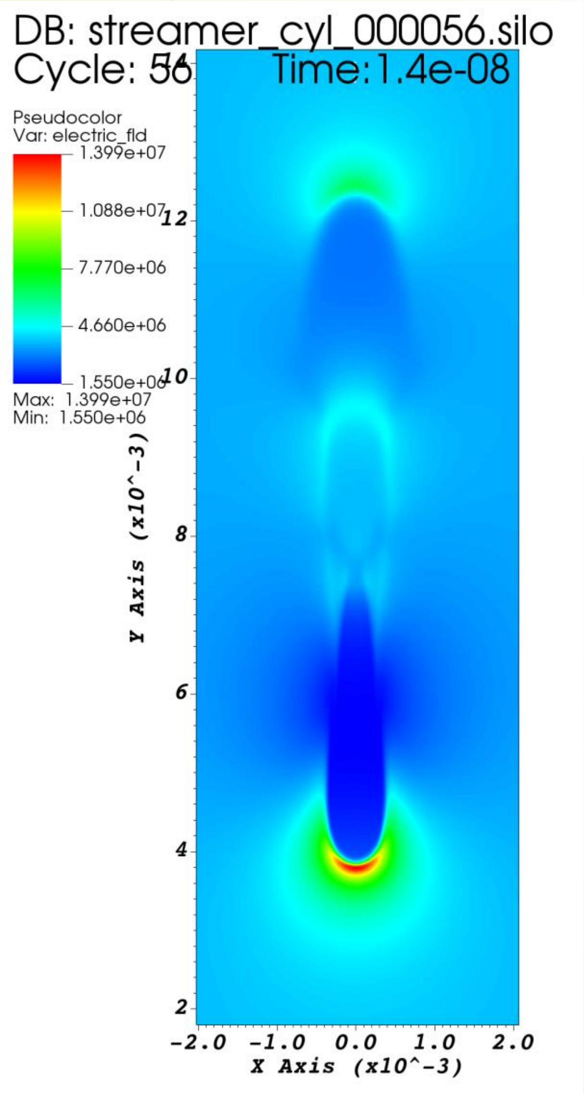



AMReX-streamer

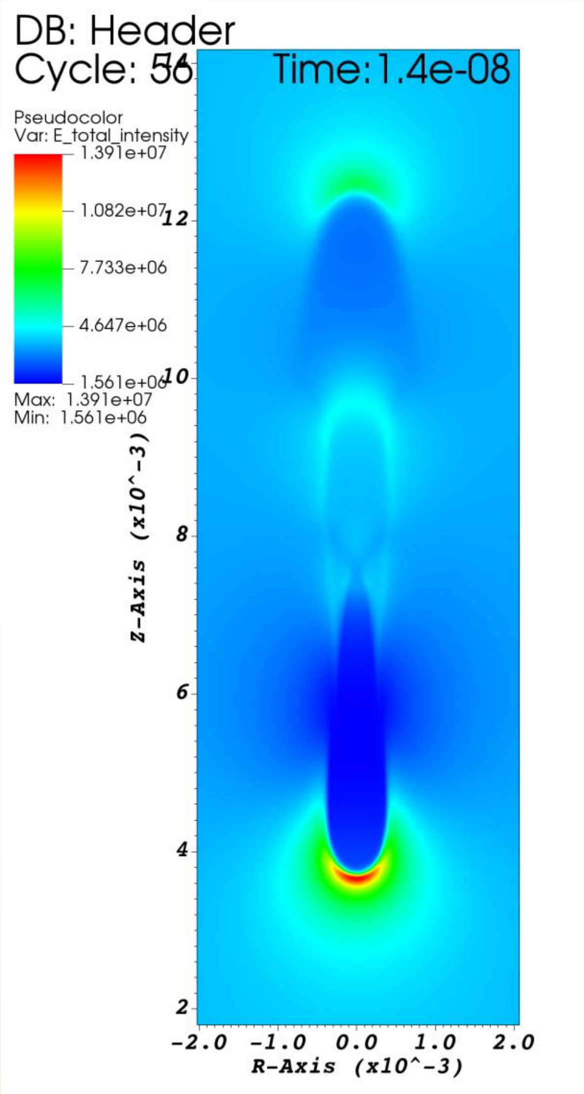


# Electric field intensity at t = 14 ns

Afivo-streamer




AMReX-streamer

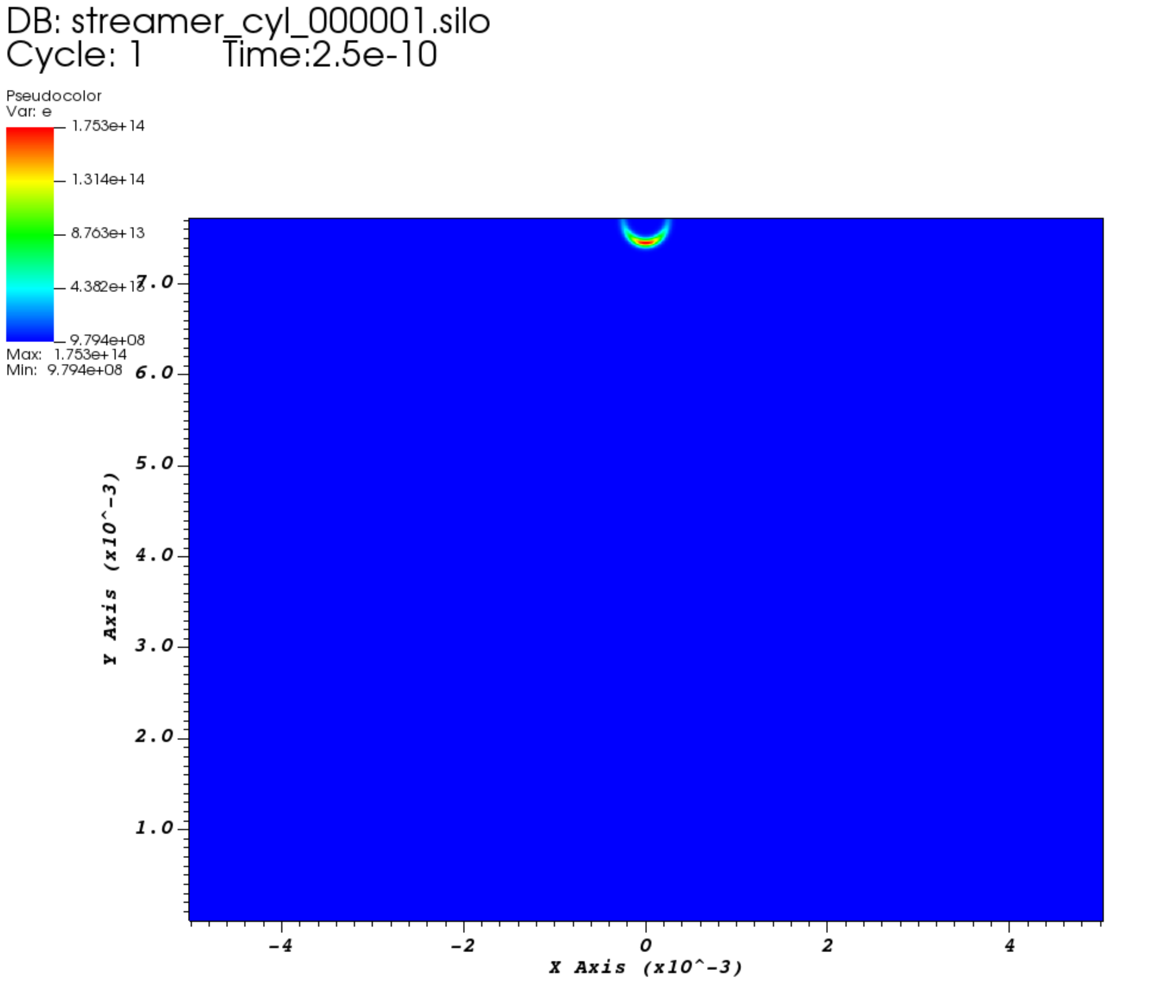


# Positive streamer in the air

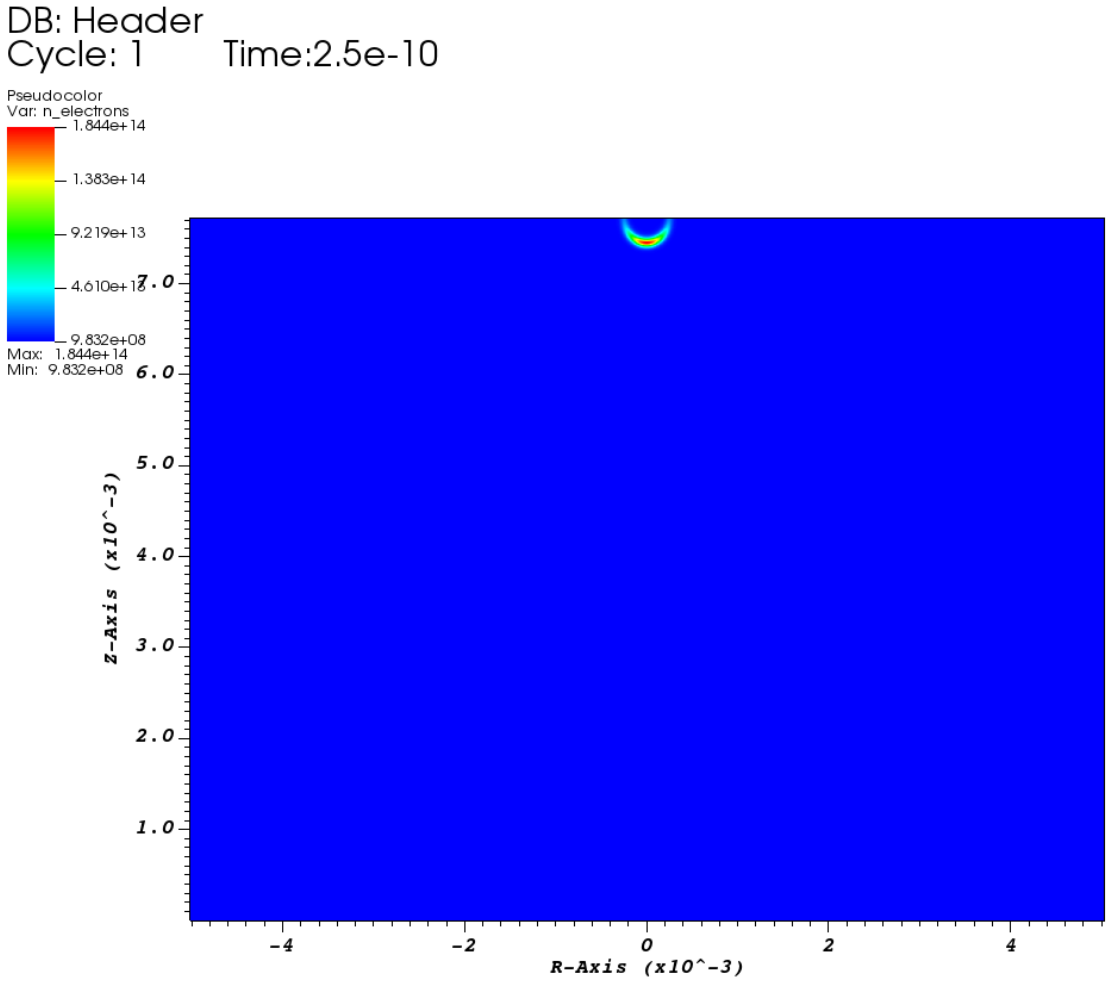
- The dry air: 80.0% $N_2$, 20.0% $O_2$ .
- The number of AMR levels 7 (from 0 to 6)
- Photoionization parametrization: Bourdon 3 term
- The applied reduced electric field: $E/N_0$ = 80 Td
- The initial condition: Positive line segment with a length of 0.64 mm and a width of 0.4 mm, centered at the center of the z coordinate range
- Background ionization: $10^9 m^{-3}$.
- Neutral gas density: $2.50475764 \cdot 10^{25}$ $m^{-3}$.

# Number density of electrons at t = 0.25 ns

Afivo-streamer

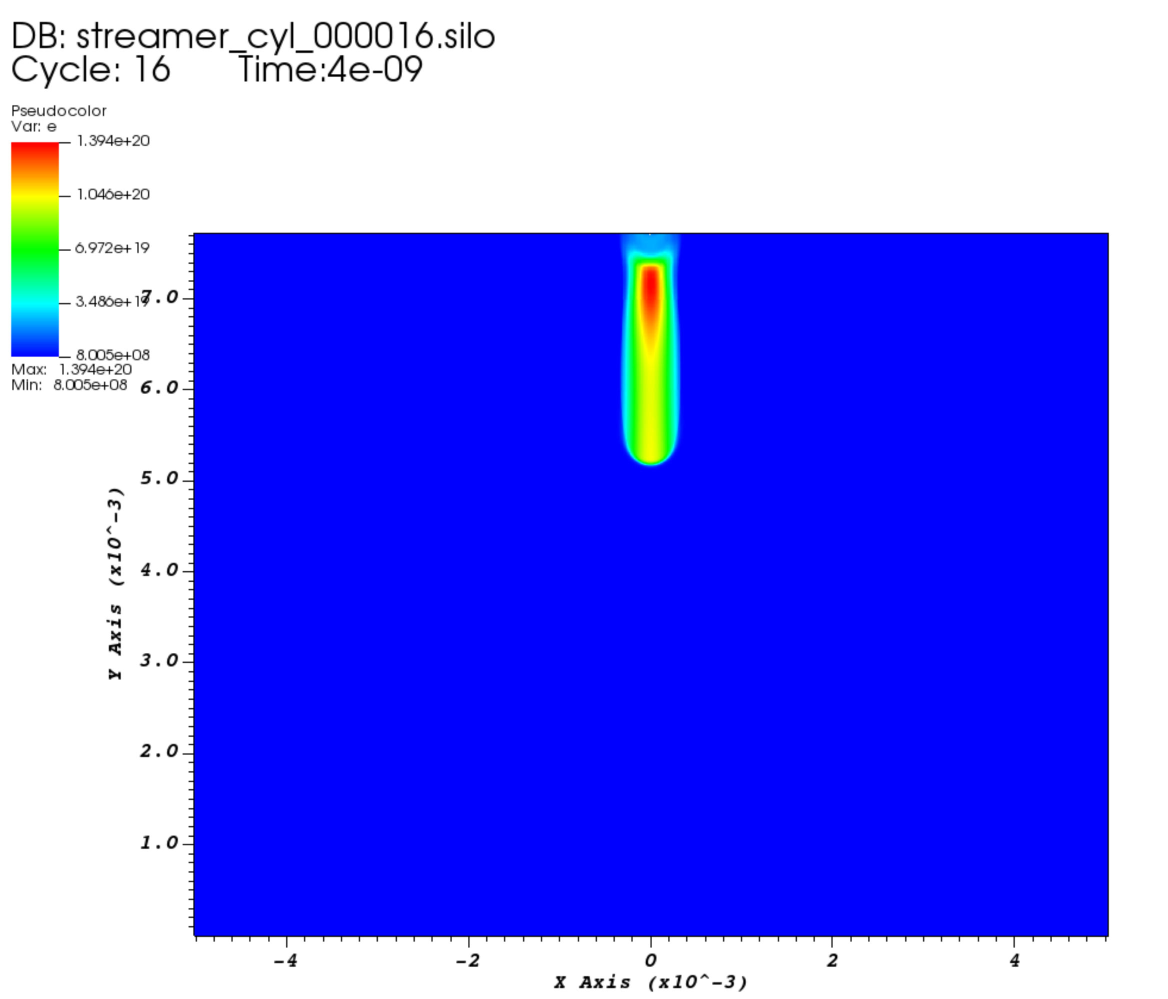



AMReX-streamer

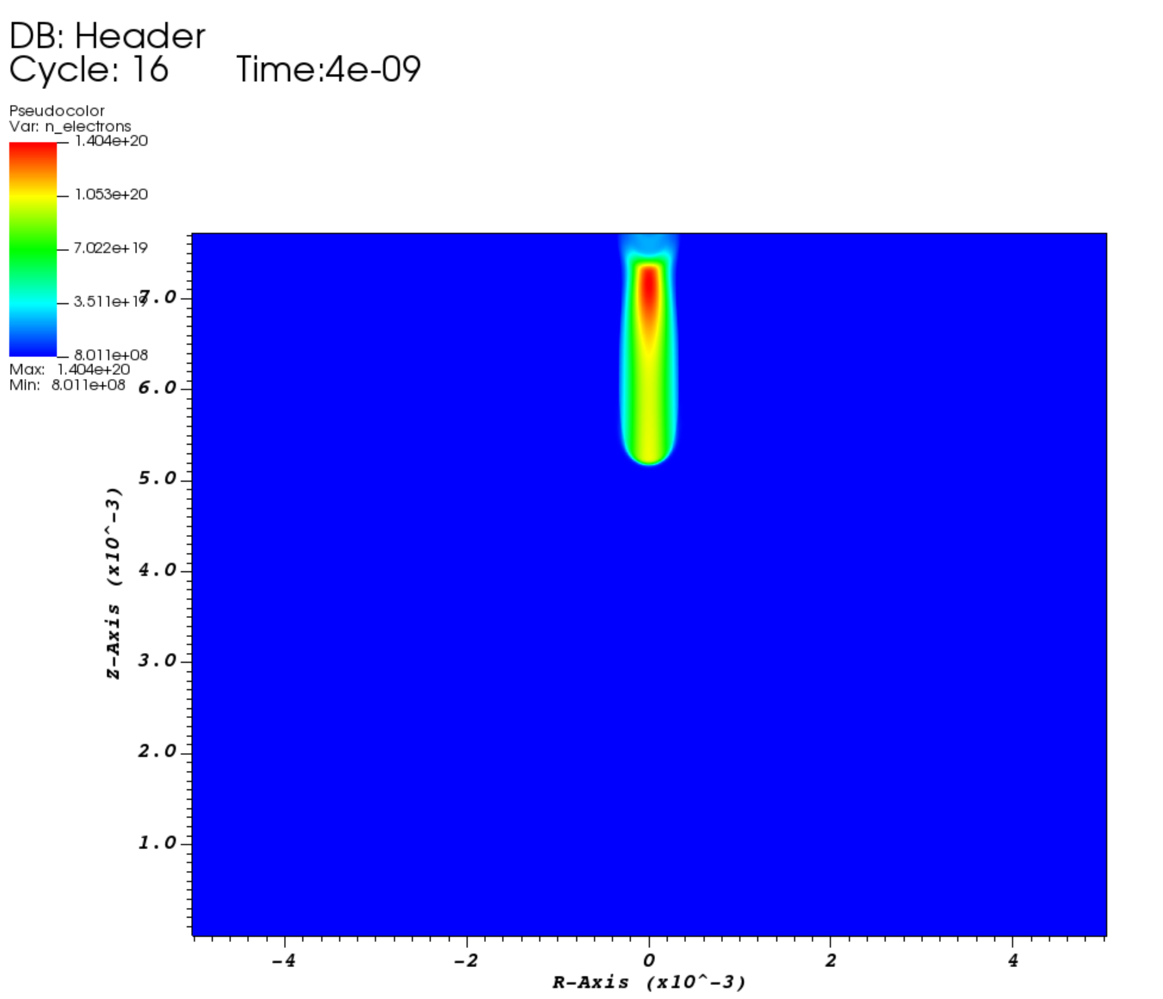


# Number density of electrons at t = 4 ns

Afivo-streamer

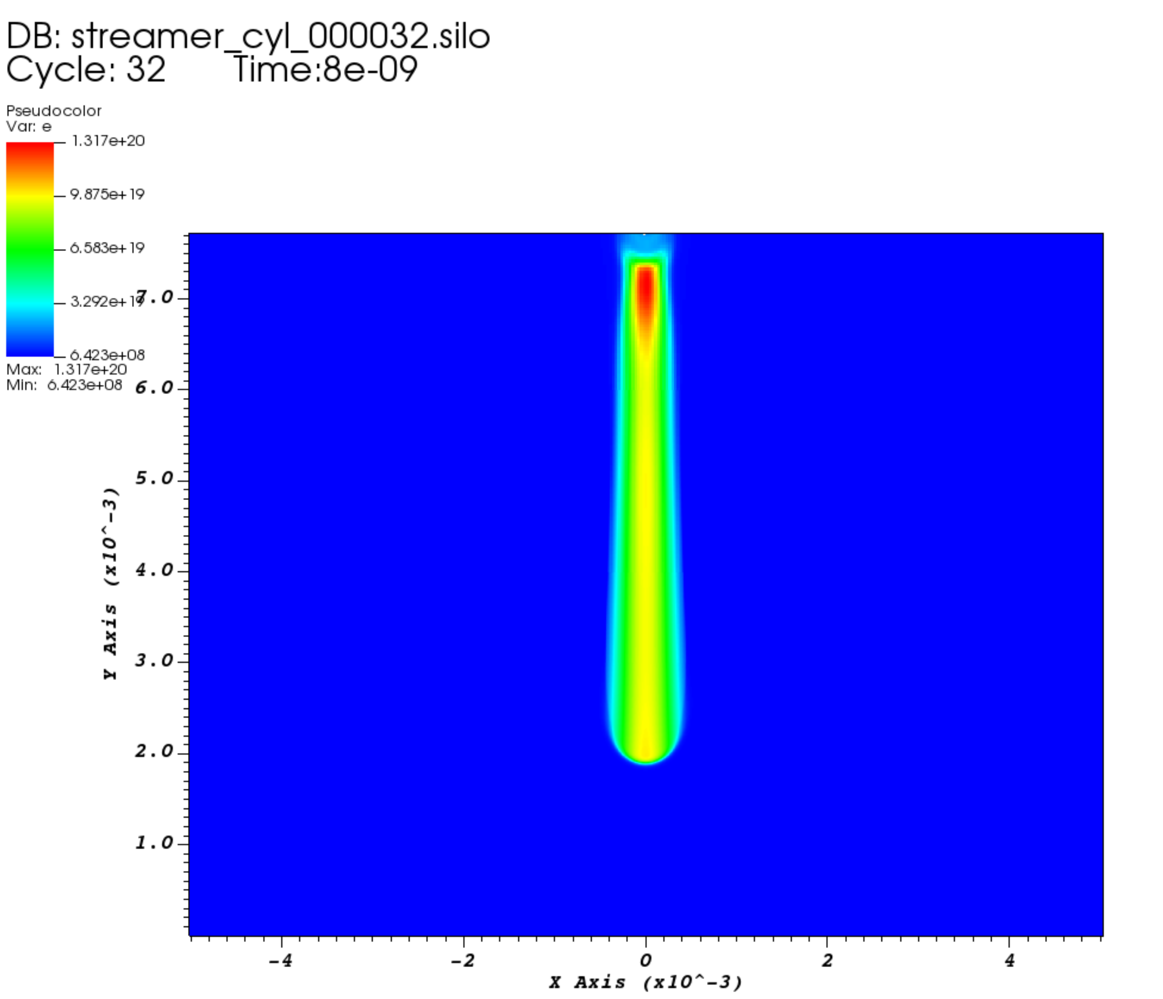



AMReX-streamer

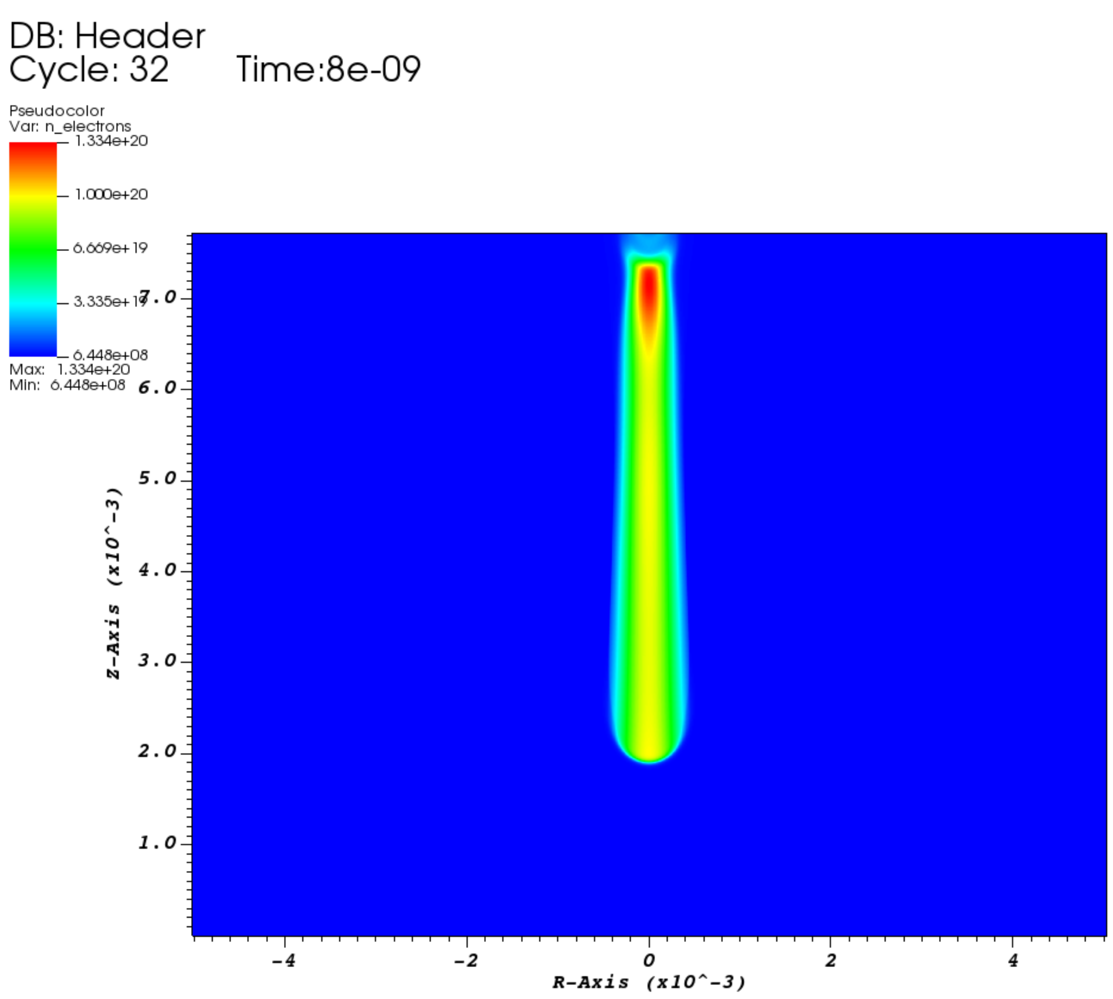


# Number density of electrons at t = 8 ns

## Afivo-streamer

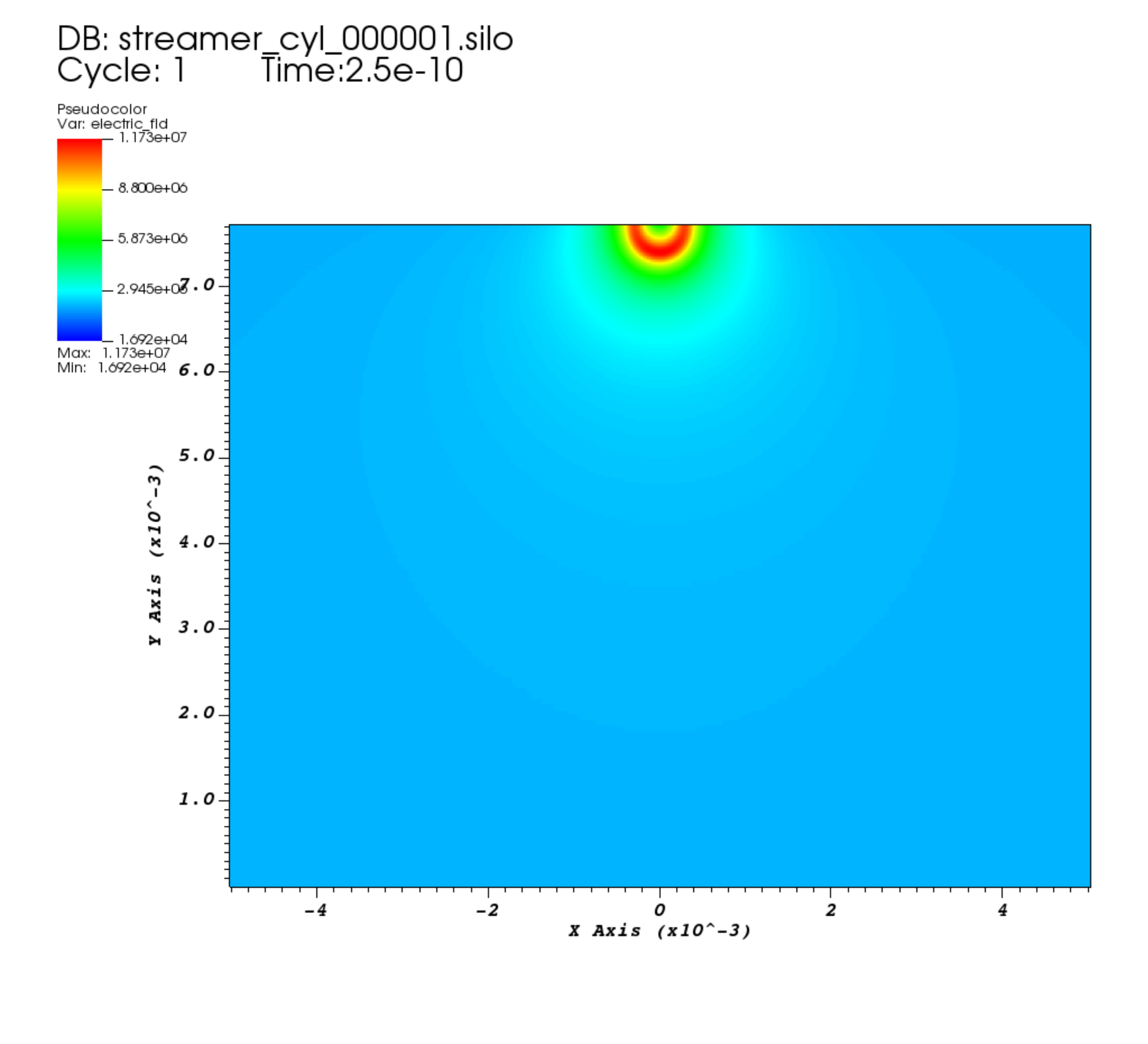



## AMReX-streamer

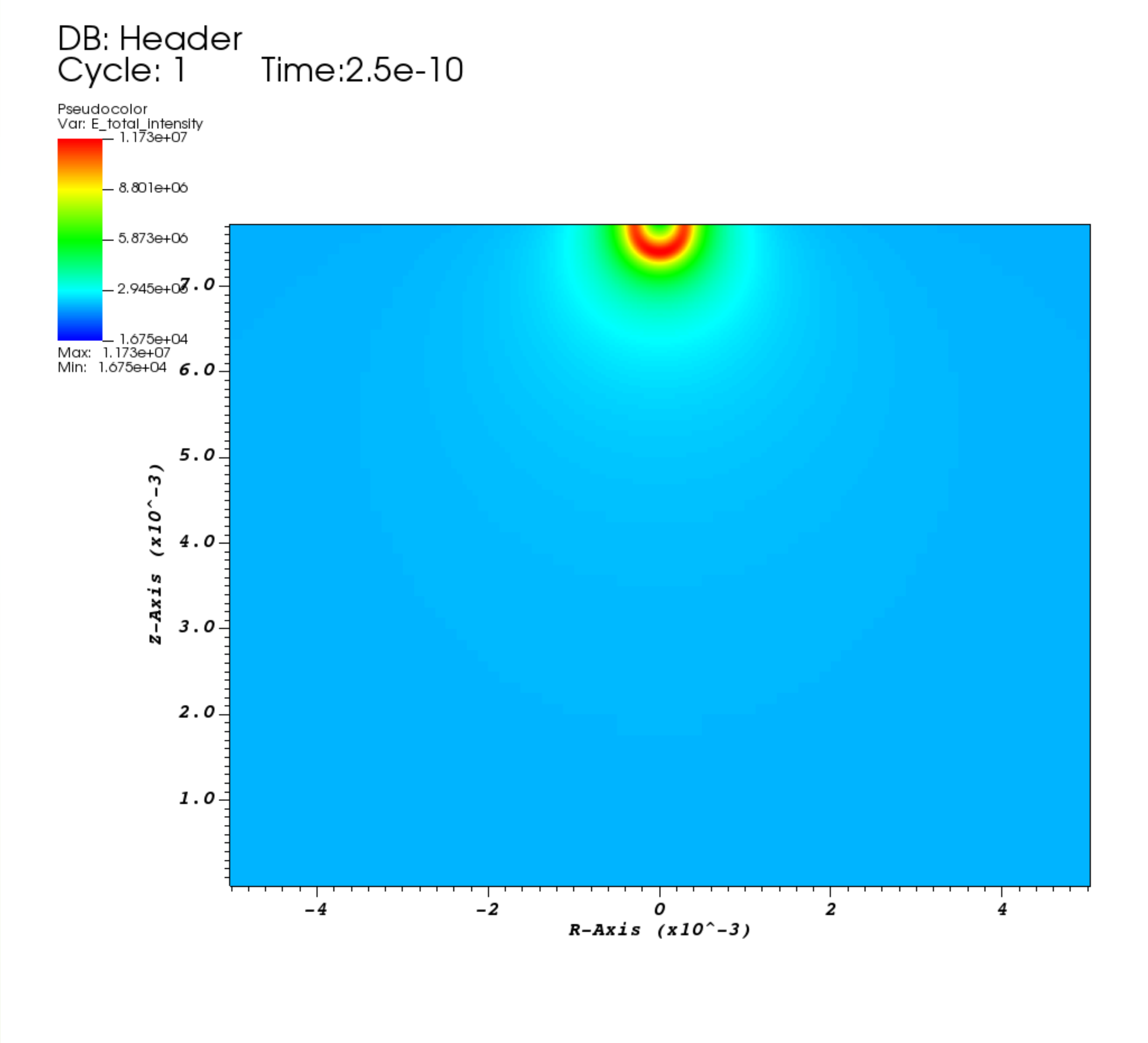


# Electric field intensity at t = 0.25 ns

Afivo-streamer

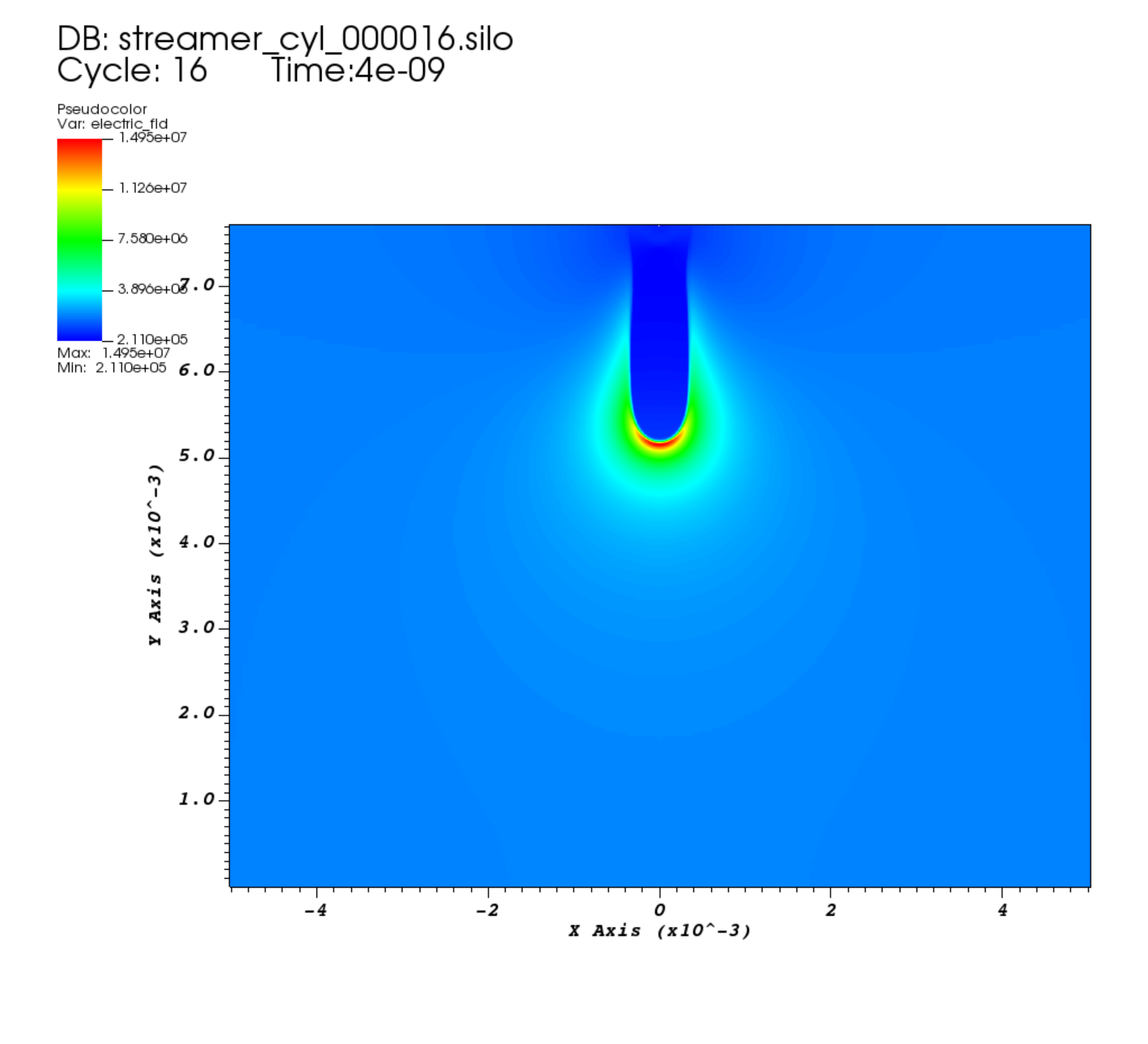



AMReX-streamer

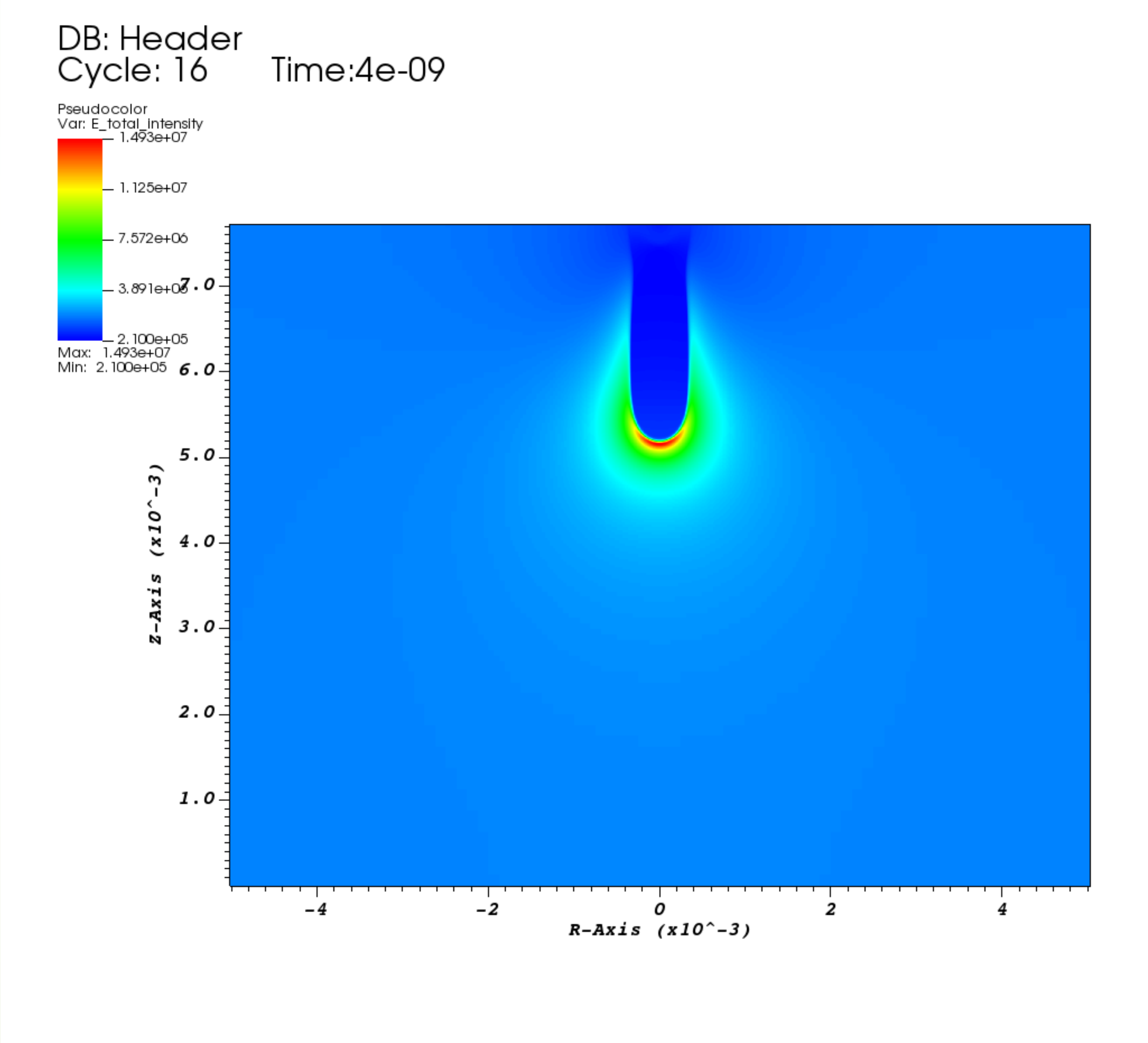


# Number density of electrons at t = 4 ns

Afivo-streamer

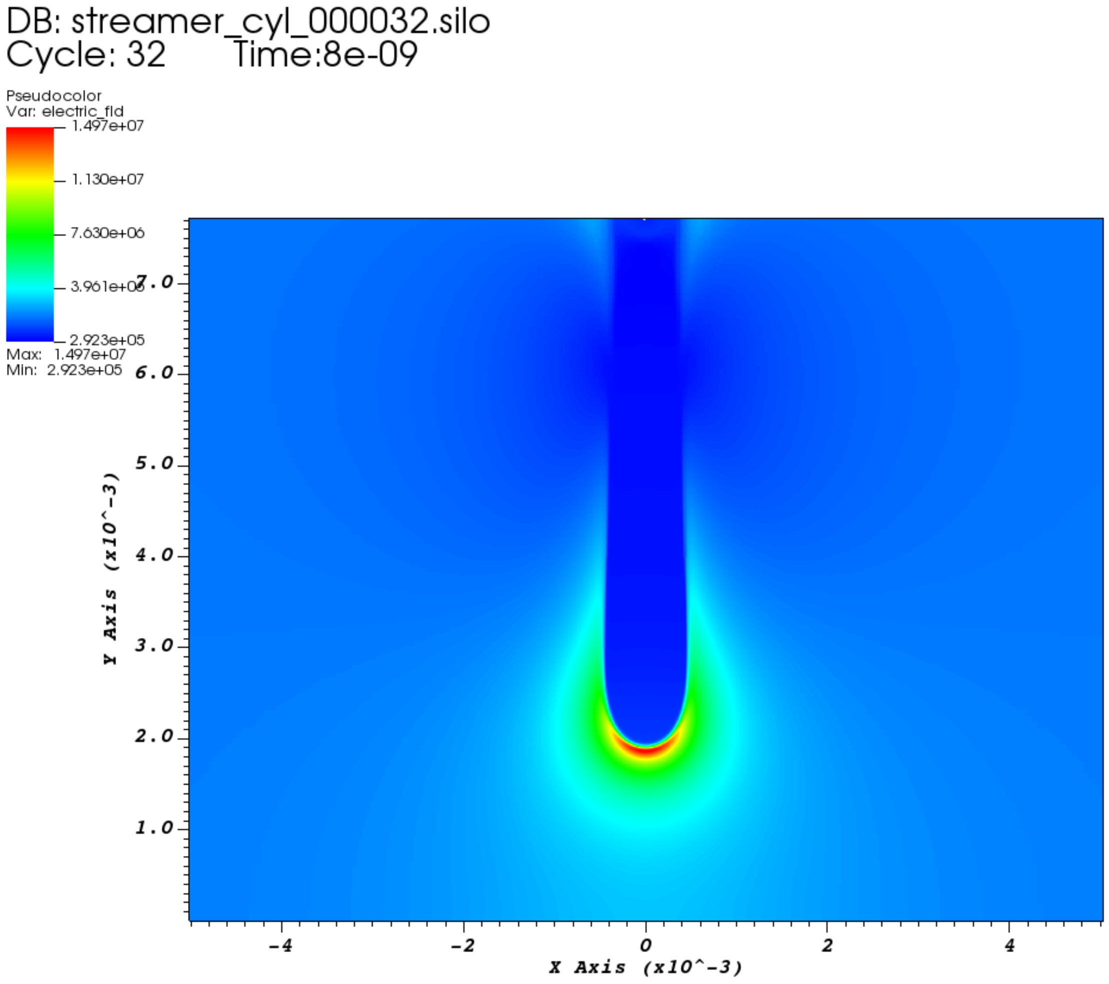



AMReX-streamer

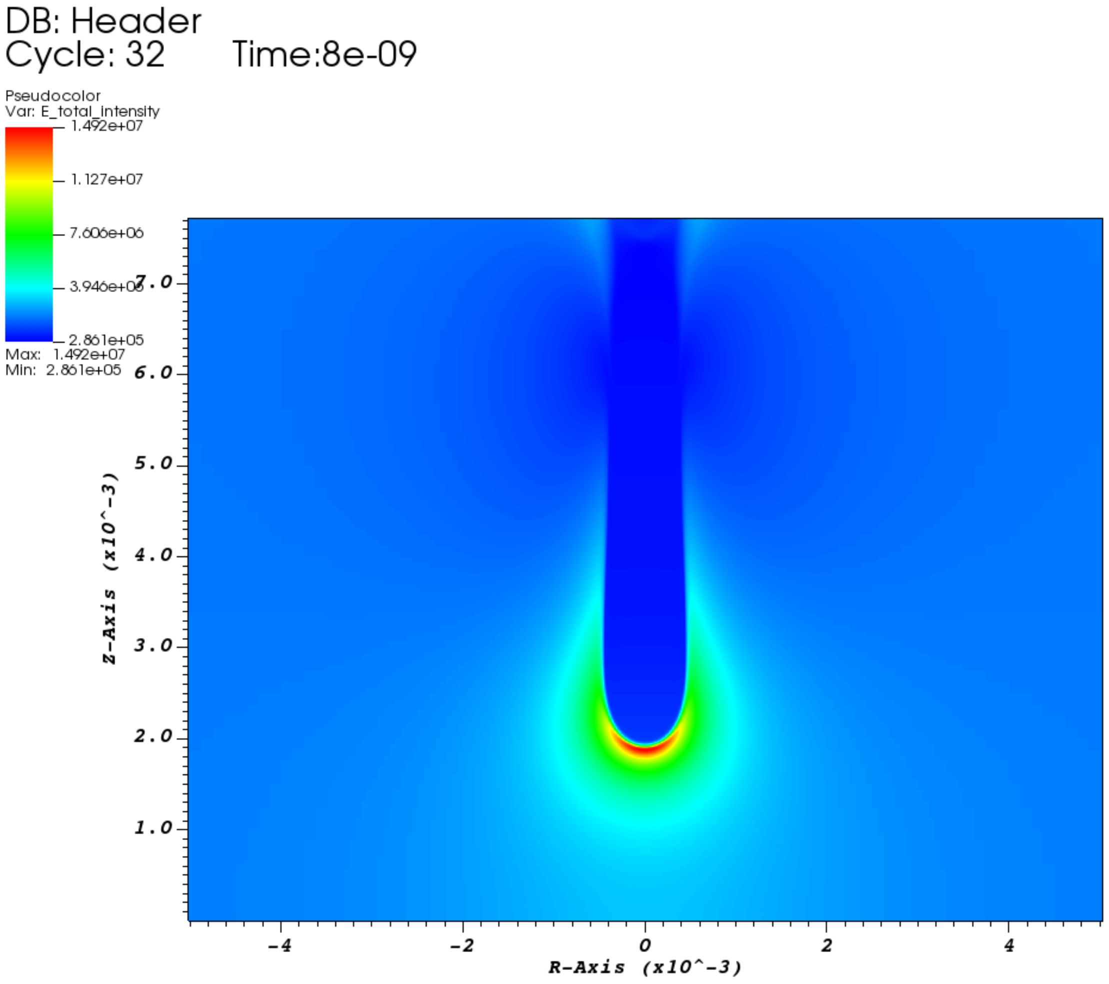


# Number density of electrons at t = 8 ns

Afivo-streamer




AMReX-streamer



# Conclusions

- We have implemented an axisymmetric fluid model in the AMReX library
- Our model is based on the first-order fluid model with local field approximation
- Photoionization is implemented by solving a system of Helmholtz equations
- The good agreement between the results of our program and the Afivo-streamer open-source program confirms the validity of our code.

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**THANK YOU**