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Vrdnik, Serbia

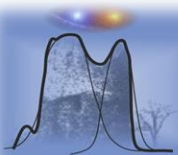
**12<sup>th</sup> Serbian Conference on  
Spectral Line Shapes  
in Astrophysics**

***Collisional contribution to the Spectral line shape in  
magnetized plasmas***

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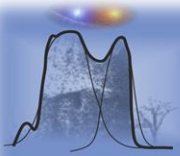
# INTRODUCTION

*\* The collisional contribution to the spectral line shape is modified by the presence of the magnetic field, in the case of electronics perturbers, the resolution of the stochastic equation is based on the theory of impact of the interaction [M. Baranger].*

*\*\* In the standard model, the emitter is subjected to a succession of independent collisions carried out by the electrons,*

*\*\*\* The effect of electrons perturbers is presented by a phenomenological operator of electronic collisions [H. Huddlestone et al], which can be calculated by the relaxation method [Griem et al],*

*\*\*\*\* This collision operator  $\Phi(\vec{v}, \vec{B})$  must take into account the influence of the magnetic field on the collision : the trajectory of the perturbers is modified in the presence of the magnetic field, as well as the velocity distribution function.*



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# *Collisional contribution to the Spectral line shape in magnetized plasmas*

The collisional contribution to the profile is also modified by the magnetic field

When the perturbers are electrons, the solution of the stochastic equation is usually achieved by using the impact theory of the interaction [M. Baranger].

In the standard model, the emitter supplied to successive independent collisions with electrons

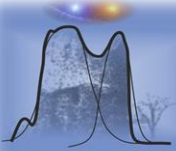
The collective picture of the interactions with the electrons is generally described by a screened potential with Debye length

The solution of the stochastic equation is given in the impact regime : corresponding to a long time of interest greater than the mean collision time,

## In the presence of magnetic field :

The collision operator must take in consideration the influence of  $\vec{B}$  on the collision :  
in fact the perturbers trajectory is modified, and the velocities distribution too.

The trajectory of electron are helicoidal parallel to  $\vec{B}$ , and Larmor radius



# *Collisional contribution to the Spectral line shape in magnetized plasmas*

Define **Larmor mean radius**  $\rho_L$  corresponding to the most probable movement of the perturbers with thermal velocity  $v$

$$\rho_L = \frac{m_e}{e |\vec{B}|} v$$



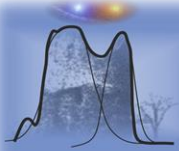
$$v = \sqrt{\frac{2kT}{m_e}}$$

 In the impact theory, the **Debye length**  $\rho_D$  constitutes the upper limit on the impact parameters of the cross sections of the collisions

$$\rho_D = \sqrt{\frac{kT}{4\pi N_e q_e^2}}$$

If  $\rho_D$  is greater than  $\rho_L$ :

**The influence of the trajectory curvature is not negligible in the presence of the magnetic field**



# Collisional contribution to the Spectral line shape in magnetized plasmas

Debye Length :

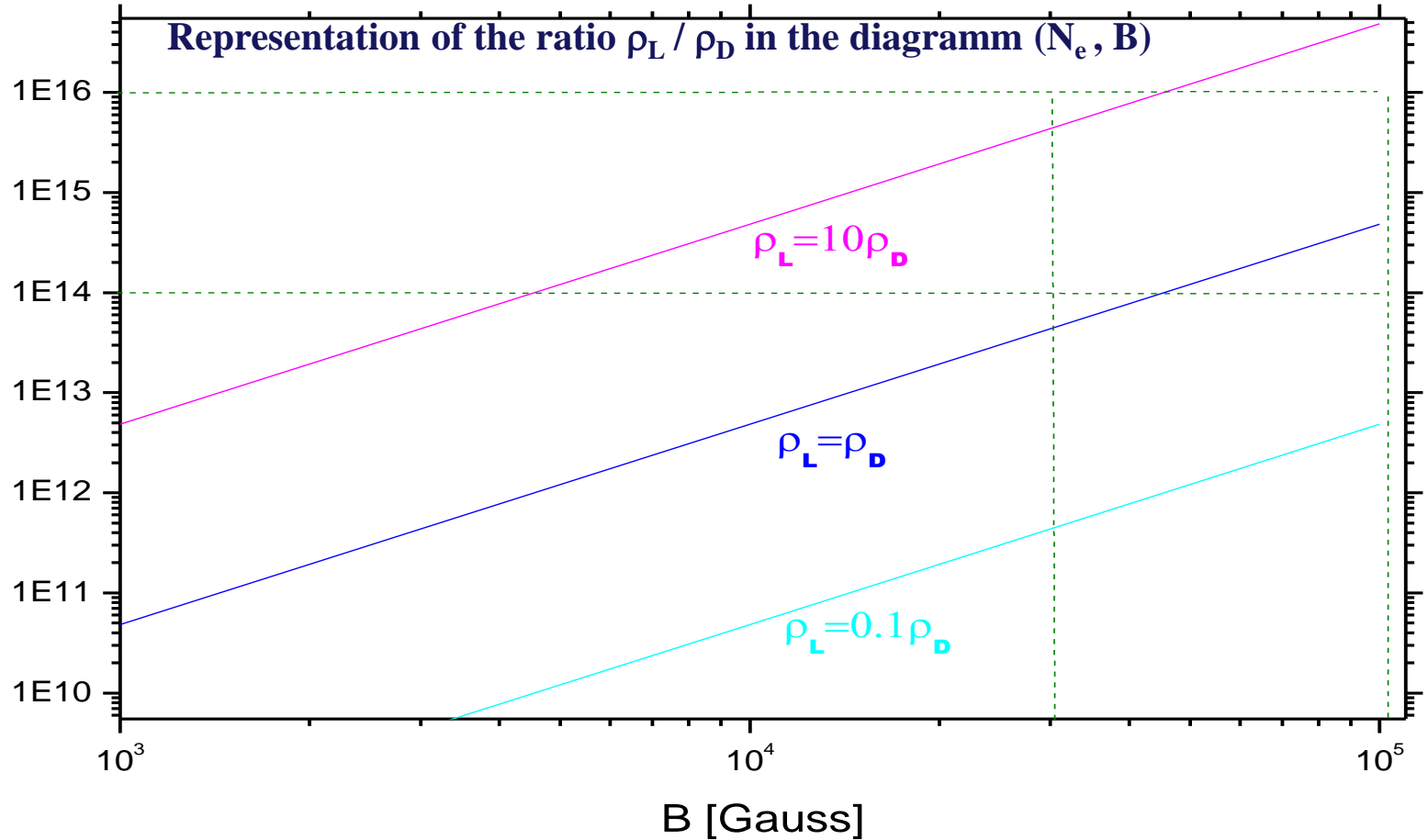
$$\rho_D = \sqrt{\frac{kT}{4\pi N_e e^2}}$$

Larmor radius:

$$\rho_L = \frac{m_e c}{e |\vec{B}|} v$$

The ratio  $\rho_D / \rho_L$   
is independant of the  
temperature

$N_e [\text{cm}^{-3}]$



*In this figure, the lines show this ratio in the diagramm ( $N_e, B$ ). This diagramm shows the non consideration of the trajectory effect is valid only when the radius of Larmor  $\rho_L$  is greater than the length of Debye  $\rho_D$  (laboratory plasmas), for the rest of the cases, especially at low densities  $N_e < 5 \cdot 10^{14} \text{ cm}^{-3}$  and for very strong fields  $B > 10^5 \text{ G}$  (astrophysical plasmas) the influence of the curvature of the trajectories is not negligible and then we have to take it into account.*

# The collision operator in the presence of the magnetic field

The effects of electron collisions on the profil are treated in the impact approximation :

→ a perturbative expansion (Dyson series) [H. Huddleston et al ] of the system emitter- perturbors

→ a multipolar developpment of the interaction allows to write

$$\Phi = -\pi N_e \int \mathbf{v} f(\mathbf{v}) d\mathbf{v} \left[ \rho_{\min}^2 + \frac{4}{3} \left( \frac{\hbar}{m} \right)^2 \frac{\vec{r} \cdot \vec{r}}{v^2} \ln \frac{\rho_{\max}}{\rho_{\min}} \right]$$

$\vec{r}$  Is the position operator of the bounded electron

$\mathbf{v}$  Is the velocity of the perturbing electrons

$f(\mathbf{v})$  Is the distribution function of velocities

$\rho_{\max} \rho_{\min}$  are the limits of the integral over impact parameter in the classical path approximation [Griem et al]

Weisskopf radius [Griem et al] :

$$\rho_{\min} = \frac{n^2 \hbar}{Z m_e v}$$

$n$  is the principal quantum number  
 $Z$  is the nuclei charge

En présence du champ magnétique  $\rho_{\max}$  vaut :

$$\rho_{\max} = v \frac{1}{\sqrt{(\Delta\omega^2 + \Delta\omega_p^2 + \Delta\omega_s^2 + \Delta\omega_Z^2)}}$$

$$\rho_{\max} = v \frac{1}{\sqrt{(\Delta\omega^2 + \Delta\omega_p^2 + \Delta\omega_s^2)}} + \frac{v}{\omega_L}$$

# The collision operator in the presence of the magnetic field

$\Delta\omega$  Is the frequency separation in absence of any perturbation,

$\Delta\omega_p$  Is the electron plasma frequency

$\Delta\omega_s$  Is an estimation of the frequency shift due to linear Stark effect

$$\Delta\omega_s \approx 13 \frac{n^2 \hbar}{Zm} N_e^{\frac{2}{3}}$$

$\Delta\omega_Z$  Is the frequency separation of two extreme Zeeman components

$$\Delta\omega_Z = \frac{q_e}{cm_e \omega_0^4} |\vec{B}|$$

$\omega_0$  Is the central frequency of the line

By using expressions of  $\rho_{\max}$  and  $\rho_{\min}$  the collision operator takes the following form :

$$\Phi \approx - \left( \frac{4\pi}{3} \right) \left( \frac{2m}{\pi k_B T} \right)^{\frac{1}{2}} N_e \left( \frac{\hbar}{m} \right)^2 \vec{r} \cdot \vec{r} \left( \frac{1}{2} \int_y^\infty \exp(-x) \frac{dx}{x} \right)$$

where

$$y \approx \left( \frac{\hbar n^2}{2Z} \right)^2 \frac{\Delta\omega^2 + \Delta\omega_p^2 + \Delta\omega_s^2 + \Delta\omega_Z^2}{E_H k_B T}$$

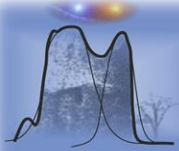
Such that  $E_H = \frac{q_e^2}{2a_0}$  is ionisation energy of hydrogen atom

The matrix elements of  $\Phi$  are proportionnal :

$$\langle n\ell m | \vec{r} \cdot \vec{r} | n\ell' m' \rangle = \frac{9}{4} n^2 [n^2 - (\ell^2 - \ell + 1)] \delta_{\ell\ell'} \delta_{mm'}$$

Such that  $m = m_\ell$  because the operator is spin independent

The electron collision operator is then diagonal in the basis  $(|n\ell s m_\ell m_s\rangle)$



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# Conclusion

## In presence of the magnetic field:

- the non consideration of the trajectory effect is valid only when one is in the domain where the radius of Larmor  $\rho_L$  is greater than the length of Debye  $\rho_D$  (*laboratory plasmas*), for the rest of the cases, especially :low densities  $N_e < 5 \cdot 10^{14} \text{ cm}^{-3}$  and for very strong fields  $B > 10^5 \text{ G}$  (*astrophysical plasmas*) the influence of the curvature of the trajectories is not negligible and then we have to take it into account.
- The electron collision operator is diagonal in the basis  $(|n\ell s m_\ell m_s\rangle)$
- The diagram  $(N_e, |\vec{B}|)$  presents a description that allows us to precise the order of magnitude. Also it allows to precise the ranges of the validity of the various hypothesis. For example the non consideration of the trajectory effect is appropriate for certains laboratory plasmas.

## Références

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