THE INFLUENCE OF THE DISTRIBUTION FUNCTION ON THE ELECTRIC PROPERTIES OF WARM AND MAGNETIZED PLASMA

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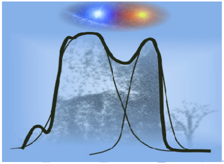

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Introduction

Undoubtedly the most powerful and effective theory to prospect the electric properties of plasma, is the transport theory started by Boltzmann, Ila Prigogine and Radu Balescu. It is a theory that describes the state of matter near the equilibrium. It serves to compute the transport coefficients, the electric properties, the correlation functions etc... In our work, we consider a very hot plasma composed with heavy atoms such that a fraction of atoms stay not ionized and play the role of diffusion centers. We compute the mobility of the free electrons of the plasma and the electric conductivity. We assume that the velocity of the free electrons is governed by the Juttner-Maxwell distribution, whereas by Maxwell distribution for the neutral atoms.

Kinetic Theory

Consider a kinetic plasma composed of heavy atoms such that for very high temperature, neutral atoms can exist. In these conditions the electron dynamics must be studied with relativistic description and the neutral with classical description. The charge current (of electrons or ions) is governed by the following transport equation

$$\frac{\partial f_e\left(\overrightarrow{p}, \overrightarrow{r}, t\right)}{\partial t} + \overrightarrow{v}\frac{\partial f_e\left(\overrightarrow{p}, \overrightarrow{r}, t\right)}{\partial \overrightarrow{r}}$$

$$+e\left[\overrightarrow{E} + \frac{1}{c}\left(\overrightarrow{V} \times \overrightarrow{B}\right)\right]\frac{\partial f_e\left(\overrightarrow{p}, \overrightarrow{r}, t\right)}{\partial \overrightarrow{p}} = \left(\frac{\partial f_e\left(\overrightarrow{p}, \overrightarrow{r}, t\right)}{\partial t}\right)_{\text{collision}} \quad (1)$$

To solve the last equation, we use the hypothesis that the plasma is very near the equlibrium state, that is to say we can write the distribution function as a small correction added to the steady distribution (the equlibrium distribution)

$$f_e\left(\overrightarrow{p},\overrightarrow{r},t\right)=f_{e0}\left(p\right)+f_{e1}\left(\overrightarrow{p},\overrightarrow{r},t\right) \tag{2}$$

$$f_{e1}\left(\overrightarrow{p},\overrightarrow{r},t\right)<<f_{e0}\left(p\right) \tag{3}$$

where $f_0\left(p\right)$ is the distribution function at equilibrium of the electron gas. It may be choosen, quantum distribution (Fermi-Dirac) if the gas is degenerated, or classical relativistic distribution (Juttner-Maxwell) if the gas is non-degenerated:

$$f_{e0}^{F-D}(p)\,dp = 4\frac{4\pi p^2 dp}{e^{pc/k_B T}+1} \quad (4)$$

degenerated case and $m << T$ that is the energy $\varepsilon \sim pc$

[Ref]: M.E. Carrington, T. Fugleberg, D. Pickering and M.H. Thoma, Canadian Journal of Physics, Vol. 82, p. 671-678 (2004).
If the gas is non-degenerated (the mean distance between electrons $r_e >> \lambda_{th} =$ thermal de De Broglie length), we must use a classical distribution in the case of the relativistic movement:

$$f_{e0}^{J-M}(\beta)\, d\beta = \lambda \frac{\gamma^5 \beta^2}{K_2(\lambda)} e^{-\lambda\gamma} d\beta \quad \text{non-degenerated case} \tag{5}$$

where

$$\lambda = mc^2/(k_B T) \tag{6}$$

$$\gamma = \frac{1}{\sqrt{1-\beta^2}};\ \beta = \frac{V}{c} \tag{7}$$

c is the light celerity in vaccum and m is the electron mass at the rest and k_B is Boltzmann constant, whereas $K_2(x)$ is a Bessel function of second kind. In our work, we consider the case of a classical relativistic case (non-degenerated).

Plasma Parameters

The most important parameter in transport phenomena in a plasma constituted by electrons and atoms is the electron-atom collision frequency ν_{e-a}. It is equal to $\nu_{e-a} = 2Z^{e-a}/N_e$ where Z^{e-a} is the number of collisions happening between electron and atom per seconde per unit volume at a point $\overrightarrow{r}$ of the plasma [Huang: Statistical mechanics]:

$$Z^{e-a} = \sigma_{tot}^{e-a} c^3 \int d\beta_1^e \int d\beta_2^a \left| \overrightarrow{\beta}_1^e - \overrightarrow{\beta}_2^a \right| f_{e0}^{J-M}\left(\beta_1^e\right) f_{a0}^{J-M}\left(\beta_2^a\right) \tag{8}$$

N_e is the electron density and σ_{tot}^{e-a} is the total cross section of the pair (electron-atom).

It is clear that the collision frequency is given now by

$$\nu_{e-a} = \frac{2\sigma_{tot}^{e-a} c^3}{N_e} \int d\beta_1^e \int d\beta_2^a \left| \overrightarrow{\beta}_1^e - \overrightarrow{\beta}_2^a \right| f_{e0}^{J-M}\left(\beta_1^e\right) f_{a0}^{J-M}\left(\beta_2^a\right) \tag{9}$$

In the case of Maxwellian distribution, the last integral can be perform analytically. In our case, this integral must be computed numerically. The same formula can be followed to compute the mobility of ions (change $e-a$ to $i-a$). Another simplifications we used too is to replace $f_{a0}^{J-M}\left(\beta_2^a\right)$ in the last formula by the Maxwell distribution because the atoms are not in relativistic movement. So the last formula is the key for the subsequent calculations.

Mobility and Conductivity

At first express the collision term seen in formula (1)

$$\left(\frac{\partial f_e\left(\overrightarrow{p},\overrightarrow{r},t\right)}{\partial t}\right)_{\text{collision}} = -\nu_{e-a}(f_e - f_{e0}) = -\nu_{e-a} f_{e1} \tag{10}$$

$$f_{e1} << f_{e0} \tag{11}$$

to compute the mobility μ (the conductivity $\sigma = -N_e e\mu$), for a weakly ionized plasma, assume that the equilibrium is reached, so $\frac{\partial f_e\left(\overrightarrow{p},\overrightarrow{r},t\right)}{\partial t} = 0$, then the equation (1) transforms to

$$\vec{V}\frac{\partial f_e\left(\vec{p},\vec{r},t\right)}{\partial \vec{r}}+e\left[\vec{E}+\frac{1}{c}\left(\vec{V}\times\vec{B}\right)\right]\frac{\partial f_e\left(\vec{p},\vec{r},t\right)}{\partial \vec{p}}=-\nu_{e-a}f_{e1}$$

$$\vec{V}\frac{\partial\left[f_{e0}+f_{e1}\right]}{\partial \vec{r}}+\frac{e}{m}\left[\vec{E}+\frac{1}{c}\left(\vec{V}\times\vec{B}\right)\right]\frac{\partial\left[f_{e0}+f_{e1}\right]}{\partial \vec{V}}+\nu_{e-a}f_{e1}=0(12)$$

neglecting the term containing $\vec{V}\frac{\partial f_{e1}}{\partial \vec{r}}$ and $\vec{E}\frac{\partial f_{e1}}{\partial \vec{V}}$, we find immediately

$$\frac{e}{m}\vec{E}\frac{\partial f_{e0}}{\partial \vec{V}}+\frac{e}{m}\left[\frac{1}{c}\left(\vec{V}\times\vec{B}\right)\right]\frac{\partial f_{e1}}{\partial \vec{V}}+\nu_{e-a}f_{e1}=0 \qquad (13)$$

Taking the external magnetic field towards z-axis, and multiply the last equation by $\overrightarrow{V}$ and integrating over $\overrightarrow{V}$, we find

$$\frac{e}{m_e}\overrightarrow{E} - \omega_c v_y \overrightarrow{i} + \omega_c v_x \overrightarrow{j} + \nu_{e-a}\overrightarrow{v} = 0 \qquad (14)$$

where

$$\overrightarrow{v} = \int \overrightarrow{V} f_{e1} d^3\overrightarrow{V} \qquad (15)$$

and $\omega_c = eB/m_e$ is the well-known cyclotron frequency. The equation (14), expanded on axis gives

$$\frac{e}{m_e}E_x - \omega_c v_y + \nu_{e-a} v_x = 0 \tag{16}$$

$$\frac{e}{m_e}E_y + \omega_c v_x + \nu_{e-a} v_y = 0 \tag{17}$$

$$\frac{e}{m_e}E_z + \nu_{e-a} v_z = 0 \tag{18}$$

The mobility towards the z axis is given by the flux in z-axis: $N_e v_z = -\frac{e}{m_e.\nu_{e-a}} N_e E_z = \mu_0 N_e E_z$, that is to say the mobility along z axis is unchanged by the presence of the magnetic field $\overrightarrow{B} = B\overrightarrow{k}$

$$\mu_0 = -\frac{e}{m_e.\nu_{e-a}} = \frac{\sigma_0}{-N_e e} \tag{19}$$

whereas (the electric field $\overrightarrow{E}$ is spaned in x-y plane) the perpendicular mobility changes as

$$\mu_{\perp} = \frac{\nu_{e-a}^2}{\nu_{e-a}^2 + \omega_c^2}\mu_0 = \frac{\sigma_{\perp}}{-N_e e} \quad (20)$$

and transverse mobility changes as

$$\mu_{\top} = \frac{\omega_c \nu_{e-a}}{\nu_{e-a}^2 + \omega_c^2}\mu_0 = \frac{\sigma_{\top}}{-N_e e} \quad (21)$$

as tensor, the mobility is given by 3x3 matrix as

$$[\mu] = \begin{bmatrix} \mu_{\perp} & \mu_{\top} & 0 \\ \mu_{\top} & \mu_{\perp} & 0 \\ 0 & 0 & \mu_0 \end{bmatrix} \quad (22)$$

Conclusion

In this work, we have introduced a new distribution function of the velocity. This function is more appropiate for ultra-relativitic particles (high speed compared to light celerity). We applied it to compute the electric properties of hot plasmas composed of heavy atomes with their ions and free electrons. In particular, the mobility and the conductivity tensor of the free electrons are computed. We expect, that the discrepancies must be more sensible when the temperature becomes large than $10^7 K$. Sightly, our investigation is of interest in the study of astrophysical plasmas.