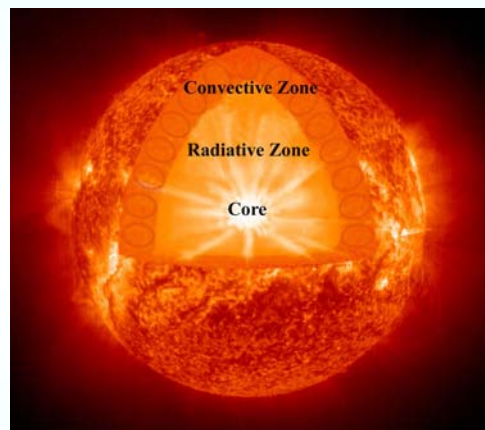




*8th Serbian Conference*  
"Spectral Line Shapes in Astrophysics"  
*Divčibare, Serbia, June, 2011*



# Plasma Polarization in Massive Astrophysical Objects



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[astro-ph:0901.2547](#)

[astro-ph:0902.2386](#)

*High Temperature, 48, 766 (2010)*

# Basic Idea

Gravitation attracts (heavy) ions and does not attract electrons.  
It leads to a small violation of electroneutrality and polarizes plasma in MAO  
( *Sutherland, 1903* )

Polarization field compensates (totally or partially) gravitational (and any other mass-acting) force in thermodynamically equilibrium state  
( *macroscopic screening* )

Comment: Ions in thermodynamic equilibrium are suspended, figuratively speaking, in electrostatic field of strongly degenerated and weakly compressed electrons

## Expected consequences

Polarization **always** accompanies gravitation

Polarization field must be of the **same order** as gravitation field (*per one proton*)

Polarization field must be **congruent** to gravitation field

**Any mass-acting force** must be accompanied by polarization

**Rotation** – centrifugal force  $F_c \Leftrightarrow ( F_E \sim -\alpha F_c )$

**Expansion or compression** – inertial force  $F_a \Leftrightarrow ( F_E \sim -\alpha F_a )$

**Vibration**  $\Leftrightarrow$  no pure acoustic oscillations  $\Leftrightarrow$  (*+ electromagnetic oscillations*)

# Basic Idea

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## Basic statement

( *J. Phys. A: Math. & Theor. 2009* )

New “**Coulomb non-ideality force**” is third “**participant**” in competition between gravitation and polarization forces in equilibrium MAO.

In most cases this new force **increases** final **electrostatic field** in comparison with that of ideal-gas solution.

[astro-ph:0901.2547](#)

[arXiv:0902.2386v1](#)

Iosilevskiy I. / Int. Conf. “*Physics of Neutron Stars*”, St.-Pb. Russia, 2008

Iosilevskiy I. / Int. Conf. “*Physics of Non-Ideal Plasmas*”, Moscow, Russia, 2009

# Plasma Screening

*(historical comments)*

Gouy G. *J. Phys. Radium* **9** 457 (**1910**)

Chapman D. *Phil. Mag.* **25** 475 (**1913**)

# Micro- & Macro- Screening

## Microscopic screening (*ideal plasma*)

Debye - Hückel screening ( $n\lambda^3 \ll 1$ )

Thomas - Fermi screening ( $n\lambda^3 \gg 1$ )

$$F_{av}(\mathbf{r}) = F_{ext}(\mathbf{r}) + F_{scr}(\mathbf{r}) \approx F_{ext}(\mathbf{r}) \exp\{-r/r_{scr}\} \rightarrow 0$$

$$r \rightarrow \infty$$

## Macroscopic screening (*ideal plasma*)

Pannekoek - Rosseland screening ( $n\lambda^3 \ll 1$ )

Bildsten *et al* screening ( $n\lambda^3 \gg 1$ )

$$F_{av}^{(Z)}(\mathbf{r}) = F_{grav}^{(Z)}(\mathbf{r}) - F_{scr}^{(Z)}(\mathbf{r}) \approx 0$$



Peter Debye



Erich Hückel

## What is the problem ?

Micro-scopic screening: - Correct screening for **non-ideal** plasma at **micro-** level

Macro-scopic screening: - Correct screening for **non-ideal** plasma at **macro-** level

$$e\nabla\varphi_E(\mathbf{r}) = -\nabla\varphi_G(\mathbf{r}) \frac{\langle Z | \mathbf{D}_\mu^n | \mathbf{M} \rangle}{\langle Z | \mathbf{D}_\mu^n | Z \rangle}$$



$$\mathbf{D}_\mu^n = (\mathbf{D}_\mu^n)^{id} + \Delta\mathbf{D}_\mu^n$$

$$\mathbf{D}_\mu^n - \text{Jacobi matrix } \left[ \left[ \frac{\delta n_j}{\delta \mu_k} \right]_{T, \mu_i (i \neq k)} \right] \quad (j, k = 1, 2, 3, \dots)$$

# Historical comments

- Plasma polarization at **micro**-level – Debye and Hückel, *Phys. Zeitschr.*, **24**, 8, 1923.
- Plasma polarization at **macro**-level – Pannekoek A. *Bull. Astron. Inst. Neth.*, 1 (**1922**)  
 == «» == – Rosseland S. *Mon. Roy. Astron. Soc.*, **84**, (**1924**)

## Pannekoek - Rosseland electrostatic field

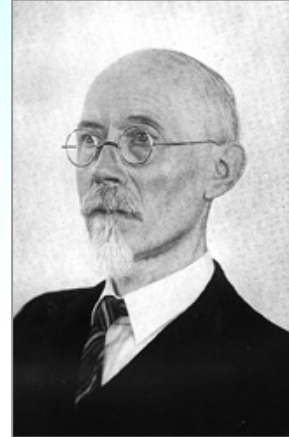
Application to plasma:

- 1) - **ideal**
- 2) - **non-degenerate**
- 3) - equilibrium
- 4) - **isothermal** ( $T = \text{const}$ )
- 5) - electroneutral

$$\{ n_+(r) = n_-(r) \}$$

$$\begin{aligned} dP_e/dr &= -GMm_e n_e/r^2 - n_e eE \\ dP_i/dr &= -GMm_i n_i/r^2 + n_i qE \end{aligned}$$

$M$  – mass of the Sun,  
 $G$  – gravitational constant,  
 $m_e, m_p$  – electron & proton masses



A. Pannekoek

$$F_E^{(p)} = -(1/2)F_G^{(p)}$$

$$F_E^{(e)} = +(1/2)F_G^{(p)}$$

## Generalization to ideal plasma of ions ( $A, Z$ ) and electrons

$$F_E^{(p)} = -\frac{A}{(Z+1)}F_G^{(p)}$$

$$F_E^{(Z)} = -\frac{Z}{(Z+1)}F_G^{(Z)}$$



N. Bohr & S. Rosseland

(\*)  $F_E^{(p)}, F_G^{(p)}, F_E^{(Z)}, F_G^{(Z)}$ , - electrostatic and gravitational forces acting on one proton (p) and ion ( $A, Z$ )

# Extension for strongly degenerated plasma

The model of **L. Bildsten *et al.*** (2001 – 2007)

L. Bildsten & D. Hall // *Ap.J.*, 549: (2001) *Gravitational settling of  $^{22}\text{Ne}$  in liquid white dwarf interior*  
 P. Chang & L. Bildsten // *Ap.J.*, 585 (2003) *Diffusive nuclear burning in neutron star envelopes*

$$\frac{dP_e}{dr} = -n_e(r) \{m_e g(r) + eE\}$$

$$\frac{dP_i}{dr} = -n_i(r) \{A_i m_p g(r) - Z_i eE\}$$

- 1) - **ideal**
- 2) - **strongly degenerated electrons**
- 3) - isothermal ( $T = \text{const}$ )
- 4) - electroneutral  
 $\{ n_+(r) = n_-(r) \}$   
 -----
- 5) - equilibrium

The SUN

$(p^+ + e^-)$

$$F_E^{(p)} \approx -(1/2)F_G^{(p)}$$

White Dwarfs

$(_{16}\text{O}^{8+}, _{12}\text{C}^{6+}, _4\text{He}^{2+})$

$$F_E^{(p)} \approx -2F_G^{(p)}$$

$$F_E^{(Z)} \approx -F_G^{(Z)}$$

Accuracy ~ small parameter  $x_c$

$$x_c \equiv \left( \frac{\partial n_e}{\partial p_e} \right)_T \bigg/ \left( \frac{\partial n_i}{\partial p_i} \right)_T$$

**NB!**

- Average electrostatic field must be of the same order as gravitational one\*

(\* - counting per one proton)

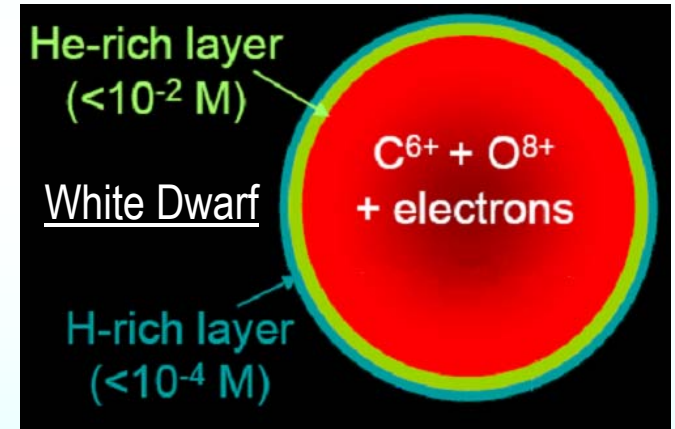
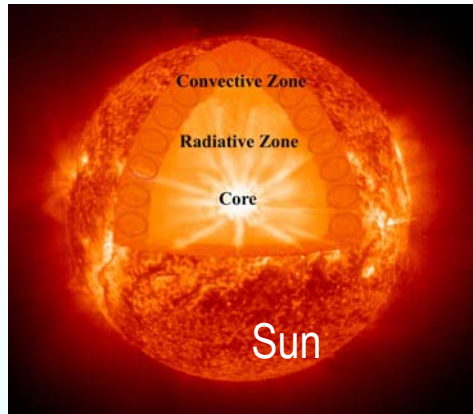
**Question:** (Bally & Harrison, 1978)

? - Do both limiting cases (ideal non-degenerate and degenerate electrons) restrict interval of possible ratio of gravitational and electrostatic forces - ?

$$F_E^{(p)} = -(1/2)F_G^{(p)}$$



$$F_E^{(p)} = -2F_G^{(p)}$$



**Answer:**

**! Yes :** - if one takes into account the electron degeneracy only !

**! No :** - if one takes into account non-ideality effects additionally !

(see below)

It may be

$$|F_E^{(p)} / F_G^{(p)}| \geq 2$$

i.e.

$$|F_E^{(Z)} / F_G^{(Z)}| \geq 1$$

(“Overcompensation”)

I.L.I. “Physics of NS”, S-Pb. Russia, 2008

J. Phys. A, 42, 2009 // [astro-ph:0901.2547](https://arxiv.org/abs/astro-ph/0901.2547)



# Milestones

1903 // W. Sutherland – Discussed basic idea of gravitational polarization in MAO

1922 // A. Pannekoek

1924 // S. Rosseland

Obtained key relation for proportionality of average gravitational and electrostatic fields (counting per proton) for the case of ideal non-degenerated plasma of the Sun atmosphere  $\{ F_E = \frac{1}{2} F_G \}$

1924 // E. Milne – Net charge on the star // Discussed basic idea of non-electroneutrality of stars

1926 // A. Eddington – Respected these ideas in his book

1968 // L. Rosen – Discussed gravitational polarization in the stars as a standard

1976 // T.Montmerle & A.Mishaud

1979 // A.Mishaud & G.Fontain

Idea: - protons are “repelled out” by electrostatic field from helium star envelope due to gravitational polarization

1978 // J.Bally & E.Harrison – *The Electrically Polarized Universe* // Idea of non-electroneutrality for all self-gravitating objects in the Universe (stars, galaxies and their clusters, black holes etc...)

1980 // C.Alcock – *Electric field of a chemically inhomogeneous star* / Electrostatic pollution of hydrogen from helium envelope of white dwarfs

1986 // C.Alcock, Fachri, Olinto – *Electric field on the Strange Star Surface* / Idea of huge local charge densities and average electrostatic field at the surface of the “strange” star

1992 // N.Glendenning / Introduced concept of «Structured Mixed Phase» for quark-hadron phase transition / *Compact Stars: Springer, 2000.*

1996 // D. Kirzhnits – Gravitational polarization gives no noticeable observable effects!

2001-2005 // L. Bildsten *et al* – Extended the idea of influence of gravitational polarization on diffusion of heavy ions in interiors of white dwarfs. Influence on star cooling and evolution

2003-2005 // S.Ray *et al.*

2005 // A.Mattei

2007 // A.Di Prisco *et al.*

Exotics: Ideas of ultra high charges and fields, charged black holes, charged gravitational collapse . . . *etc.*

*And many other papers probably missed in this list . . .*

# Macroscopic screening *in MAO*

J. Bally & E. Harrison, *Astrophys. Journal*, 220, 1978

## The Electrically Polarized Universe

### THE ELECTRICALLY POLARIZED UNIVERSE

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Received 1977 September 8; accepted 1977 September 22

#### ABSTRACT

It is shown that all gravitationally bound systems—stars, galaxies, and clusters of galaxies—are positively charged and have a charge-to-mass ratio of  $\sim 100$  coulombs per solar mass. The freely expanding intergalactic medium has a compensating negative charge. The immediate physical consequences of an electrically polarized universe are found to be extremely small.

*Subject headings:* cosmology — galaxies: intergalactic medium — hydromagnetics

Eddington (1926; see also Rossland 1924) showed in *The Internal Constitution of the Stars* that a star has an internal electric field

$$-\nabla\phi = \alpha(m_p/e)\nabla\psi, \quad (1)$$

where  $\phi$  is the electrical potential,  $\psi$  is the gravitational potential,  $m_p$  is the mass, and  $e$  is the charge of a proton. For a nondegenerate electron gas

$$\alpha = \sum n_i A_i / \sum n_i (1 + Z_i)$$

where the summations are over ion species,  $n_i$ , atomic weight  $A_i$ , and effective charge  $Z_i$  of a fully ionized gas of arbitrary composition,  $\frac{1}{2} \leq \alpha \leq 2$ . When radiation pressure degeneracy are included,  $\alpha$  has similar general  $\alpha \sim 1$ .

From the divergence of equation (1)

$$\sigma/\rho = Gam_p/e,$$

where  $\sigma$  is the positive gravitationally induced charge density and  $\rho$  is the mass density. For a star of total charge  $Q$  and mass  $M$  the charge-to-mass ratio is

$$Q/M = Gam_p/e, \quad (4)$$

and with  $\alpha \sim 1$ , is of order 100 coulombs per solar mass. This positive charge exists because electrons, despite their low mass, contribute substantially to the pressure, and an electric field is therefore needed to hold in the electron gas. In effect, some electrons escape (most electrons have velocities exceeding the escape velocity), and the remaining electrons are retained by the positively charged star.

It has previously seemed reasonable to suppose that the positive charge within a star is screened by a negatively charged atmosphere containing the expelled electrons. It can be shown, however, that screening occurs in the atmosphere only when the scale height is less than a Debye length.

By allowing for the difference in charge densities in the hydrostatic equations, we find

$$\nabla^2\sigma = -\lambda_D^{-2}(\sigma - Gam_p/e), \quad (5)$$

in place of equation (3), where

$$\lambda_D = (kT/4\pi n_e e^2)^{1/2} \sim 10(T/n_e)^{1/2} \text{ cm}, \quad (6)$$

is the Debye length and  $n_e$  is the electron density in a gas of temperature  $T$ . Thus, if  $L$  is a scale height, and  $\nabla^2 \sim L^{-2}$ , then equation (3) is recovered whenever  $\lambda_D \ll L$ . The charge density  $\sigma$  can only become negative in tenuous outer regions of a stellar atmosphere

**All gravitationally bound systems—stars, galaxies, and clusters of galaxies—are positively charged, and the freely expanding intergalactic medium between clusters of galaxies contains the expelled electrons and is therefore negatively charged.**

pared with the Debye length of their interstellar media. Our equations neglect—among other things—rotational inertial forces and are therefore not correct for rotationally supported gaseous systems. The charge-to-mass ratio of equation (4) does apply, however, to spiral galaxies in which the interstellar gas accounts for only a small fraction of their total mass.

Possibly most galaxies are rotationally bound clusters. Since the charge-to-mass ratio of equation (4) does apply, however, to spiral galaxies in which the interstellar gas accounts for only a small fraction of their total mass.

All gravitationally bound systems—stars, galaxies, and clusters of galaxies—are positively charged. The freely expanding intergalactic medium between clusters of galaxies contains the expelled electrons and is therefore negatively charged. The Sun has center-to-surface potential differences of  $\sim 10^3$  V, giant galaxies have potential differences of  $\sim 10^3$  V, and rich clusters such as

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BALLY AND HARRISON

two examples illustrate how small are the physical consequences of an electrically polarized universe.

Blackett (1947) advanced the hypothesis that all massive rotating bodies have magnetic moments of

$$P = \beta G^{1/2} J/c, \quad (7)$$

where  $J$  denotes angular momentum,  $c$  is the speed of light, and  $\beta$  is a dimensionless constant of order unity. In Blackett's words: "It is suggested tentatively that the balance of evidence is that the above equation represents some new and fundamental property of rotating matter." It is now known that numerous astronomical objects (planets, magnetic variable stars, pulsars, etc.) do not obey equation (7) with  $\beta \sim 1$ . All gravitationally bound systems, however, having the

generating seed magnetic fields (Harrison 1970, 1973).

Two charged stars in orbit about each other emit electromagnetic radiation; and if they have different charge-to-mass ratios denoted by  $\alpha_1$  and  $\alpha_2$ , then

$$L_{EM}/L_G \sim (\alpha_1 + \alpha_2)^2 \beta^2 \sim 10^{-36}, \quad (9)$$

where  $L_{EM}$  is the magnetic dipole radiation luminosity and  $L_G$  is the gravitational radiation luminosity. In the case of electric dipole radiation

$$L_{EM}/L_G \sim (\alpha_1 - \alpha_2)^2 \beta^2 (cP/a)^2, \quad (10)$$

where  $P$  is the orbital period and  $a$  is the separating distance of the two stars. It is again apparent that the results derived are of no astrophysical importance.

The picture presented consists of positively charged astronomical systems embedded in an intergalactic sea of negative charge. It provides a theoretical basis for Blackett's hypothesis, although the magnetic fields are much weaker than Blackett anticipated. We find the picture of an electrically polarized universe intriguing, and yet, rather surprisingly, we have so far failed to discover any physically significant effects of immediate consequence.

REFERENCES

- Harrison, E. R. 1970, *M.N.R.A.S.*, **147**, 279.
- . 1973, *M.N.R.A.S.*, **165**, 185.
- Rossland, S. 1924, *M.N.R.A.S.*, **84**, 308.

JOHN BALLY and E. R. HARRISON: University of Massachusetts, Department of Physics and Astronomy, GR Tower B, Amherst, MA 01002

**... We find the picture of an electrically polarized universe intriguing, and yet, rather surprisingly, we have so far failed to discover any physically significant effects of immediate consequence.**

## Electrostatics of a star

Proportionality (congruence) of average electrostatic and gravitational potentials

Excess charge profile in a star is similar (*proportional*) to their density profile

$$Q(\mathbf{r}) \propto \rho(\mathbf{r})$$

**NB!**

$$Q(\mathbf{r}) \ll \rho(\mathbf{r})$$

Primitive estimation:

- Maximal value of electrostatic field (*at the surface*) –  $E_{max}(r=R)$
- Maximal value of electrostatic potential (*in the centre*) –  $U_{max}(r=0)$

$$E_{max} \cong gm_p/e = (GMm_p/R^2e) \approx 2.85 \cdot 10^{-8} \cdot [M^*/(R^*)^2] \text{ V/cm}$$

$$U_{max} \cong gR/2 = (GMm_p/2R) \approx 1 \cdot 10^3 (M^*/R^*) \text{ eV}$$

$$M^* \equiv M/M_{\odot}; R^* \equiv R/R_{\odot}$$

$M_{\odot} \cong 1.99 \cdot 10^{33} \text{ g}$ .  $R_{\odot} \cong 6.96 \cdot 10^{10} \text{ cm}$   
 -- mass and radius of the Sun

### Electrostatic potential parameters:

	SUN $M \equiv M_{\odot}$ $R \equiv R_{\odot}$	White Dwarf $M_{WD} = M_{\odot}$ $R_{WD} = R_{Earth}$	Neutron Star $M_{NS} = M_{\odot}$ $R_{NS} = 10 \text{ km}$	Black Hole
$U_{max}$ [eV]	1 keV	1 MeV	70 MeV	$\infty$ (?)
$E_{max}$ [V/cm]	$3 \cdot 10^{-8}$	0.03	150	$\infty$ (?)

# Widely used approach (*standard*)

From unique equation of hydrostatic (i.e. mechanical) equilibrium of electro-neutral matter in gravitational field . . .

$$\frac{dP_{\Sigma}}{dr} = -\{n_e(r)m_e + n_i(r)m_i\}g(r) = -\rho(r)g(r)$$



. . . to the set of separate equations of hydrostatic equilibrium for each charged specie (*in terms of partial pressures*)

$$\frac{dP_e}{dr} = -n_e\{m_e g(r) + eE\}$$



$$\frac{dP_i}{dr} = -n_i\{A_i m_p g(r) - Z_i eE\}$$

## What is non-correct ?

**NB!**

- partial pressures and separate equations of "hydrostatic" equilibrium are not well-defined quantities in non-ideal plasmas of compact stars

## What should be done instead ?

# Quasi-stationary state in non-ideal self-gravitating body

(the problem in general)

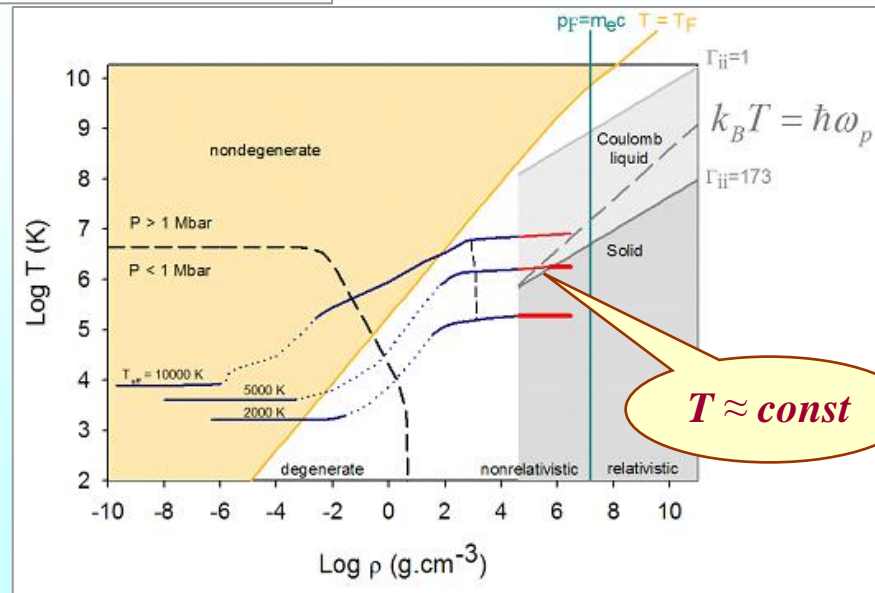
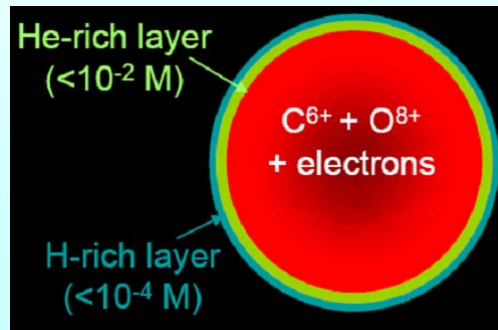
Joint self-consistent description of **thermodynamics** and **transport** for heat, mass and impulse transfer (diffusion, thermo-conductivity and equation of state)

$$e\mathbf{E} + m_e\mathbf{G} + \nabla\mu_e + d_T\nabla T = 0$$

## Simplified case

- **Total thermodynamic equilibrium** ( $T = \text{const}$ )
- No influence of magnetic field
- No relativistic effects
- No energy loss or deposition

for example:  
White Dwarfs



# General approach

## Variational formulation of equilibrium statistical mechanics

J.W. Gibbs // C. De Dominicis, 1962 // Hohenberg & Kohn, 1964 // R. Evans, 1979 etc..

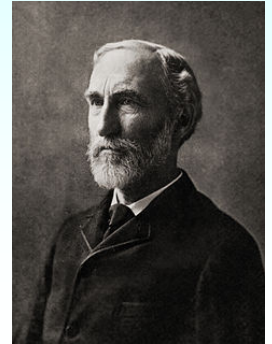
**NB!**

- **three small parameters**

$$x_m \equiv (m_e / m_i)$$

$$x_c \equiv \left( \frac{\partial n_e}{\partial p_e} \right)_T^{id} / \left( \frac{\partial n_i}{\partial p_i} \right)_T^{id}$$

$$\alpha \equiv \frac{Gm^2}{e^2} \sim 10^{-36}$$



**NB!**

- **two large parameters**

- Range of Coulomb forces
- Range of gravitational forces

# Integral form of thermodynamic equilibrium conditions

## Variational formulation (*multi-component version*)

$$F = \min_{\mathbf{F}} \left[ T, V, \{N\} \mid \{n_j(\mathbf{r})\} : \{n_{jk}(\mathbf{r}, \mathbf{r}')\} \dots \right]_{\substack{\{T=\text{const}, N_k=\text{const}\} \\ V_1(\mathbf{r}), V_{1,2}(\mathbf{r}, \mathbf{r}'), V_{1,2,3}(\mathbf{r}, \mathbf{r}', \mathbf{r}'') \dots = \text{const}}}$$

The main problem – ***strong non-locality*** of the **free energy functional** due to **long-range nature** of **Coulomb** and **gravitational interaction**

Standard approach: - separation of two main non-local parts in mean-field approximation:

$$\begin{aligned} F \{T, V(\mathbf{r}) / [\{n_i(\cdot)\} / \{n_{ij}(\cdot, \cdot)\}] \} &\equiv \\ &\equiv - \sum_{jk} \frac{Gm_j m_k}{2} \int \frac{n_j(\mathbf{r}) \cdot n_k(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} d\mathbf{r} d\mathbf{r}' + \sum_{jk} \frac{Z_j Z_k e^2}{2} \int \frac{n_j(\mathbf{r}) \cdot n_k(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} d\mathbf{r} d\mathbf{r}' + F^* [\{n_i(\cdot)\} // \{n_{ij}(\cdot, \cdot)\}] \end{aligned}$$

**NB!** The rest  $F^* \{ \dots \}$  is the free energy of new system on compensating background(s)

It's assumed that the rest free energy functional  $F^*[n_i // n_{ij}]$  is weakly non-local

Hence weakly non-local chemical potentials:  $\mu_j^{(\text{chem})}$  - could be introduced

$$\mu_j^{(\text{chem})} \equiv \left( \delta F^* [\dots] / \delta n_j(\cdot) \right)_{T, n_{k \neq j}}$$

# Local forms of thermodynamic equilibrium conditions

Heat exchange:

$$T(\mathbf{r}) = \text{const}$$

Impulse exchange:

$$\nabla P_{\Sigma} = -\rho(\mathbf{r})\nabla \varphi_G(\mathbf{r})$$

Particle exchange:

## In terms of potentials

Constancy of total (generalized) electro-chemical potential

$$m_j \varphi_G(\mathbf{r}) + q_j \varphi_E(\mathbf{r}) + \mu_j^{(\text{chem})} \{n_i(\mathbf{r}), n_e(\mathbf{r}), \{n_{ik}(\mathbf{x}, \mathbf{y})\} T\} = \text{const}$$

(j, k = electrons, ions)

## In terms of forces

Balance of forces including generalized “non-ideality” force

$$m_j \nabla \varphi_G(\mathbf{r}) + q_j \nabla \varphi_E(\mathbf{r}) + \nabla \mu_j^{(\text{chem})} \{n_i(\mathbf{r}), n_e(\mathbf{r}), \{n_{ik}(\mathbf{x}, \mathbf{y})\} T\} = 0$$

(j, k = electrons, ions)

$\varphi_G(\mathbf{r})$  и  $\varphi_E(\mathbf{r})$  – *gravitational and electrostatic potentials*

## NB !

The set of equations for electro-chemical potentials instead of the set of separate equations of “hydrostatic” equilibrium for partial pressures



# Quickly rotating star

(*centrifugal force addition*)

Constancy of total (generalized) electro-chemical potential

$$m_j \{ \varphi_G(\mathbf{r}) + \varphi_C(\mathbf{r}) \} + q_j \varphi_E(\mathbf{r}) + \mu_j^{(\text{chem})} \{ n_i(\mathbf{r}), n_e(\mathbf{r}), \{ n_{jk}(\mathbf{x}, \mathbf{y}) \} T \} = \text{const}$$

( $j, k = \text{electrons, ions}$ )

Balance of forces including generalized "non-ideality" force

$$m_j \{ \nabla \varphi_G(\mathbf{r}) + \nabla \varphi_C(\mathbf{r}) \} + q_j \nabla \varphi_E(\mathbf{r}) + \nabla \mu_j^{(\text{chem})} \{ n_i(\mathbf{r}), n_e(\mathbf{r}), \{ n_{jk}(\mathbf{x}, \mathbf{y}) \} T \} = 0$$

( $j, k = \text{electrons, ions}$ )

$\varphi_G(\mathbf{r})$ ,  $\varphi_C(\mathbf{r})$  and  $\varphi_E(\mathbf{r})$  – gravitational, *centrifugal* and electrostatic potentials

Polarization field should be equal to zero in the case of the rotation limit when the centrifugal force is equal to the gravitational one.

$$F = \min F(T, V, \{N_k\} / \{n_i(\cdot)\} / \{n_{ij}(\cdot, \cdot)\} / \{n_{ijk}(\cdot, \cdot, \cdot)\} \dots) \equiv$$

$$\equiv - \sum_{j,k} \frac{Gm_j m_k}{2} \int \frac{n_j(\mathbf{r}) \cdot n_k(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} d\mathbf{r} d\mathbf{r}' + \sum_{j,k} \frac{Z_j Z_k e^2}{2} \int \frac{n_j(\mathbf{r}) \cdot n_k(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} d\mathbf{r} d\mathbf{r}' + F^* [\{n_i(\cdot)\} / \{n_{ij}(\cdot, \cdot)\}]$$

small parameter!

$$\alpha \equiv \frac{Gm^2}{e^2} \sim 10^{-36}$$

Extremely small but non-zero violation of global electroneutrality!

**Total charge disbalance -  $\Delta Q$**

$$\Delta Q \sim \alpha N_{\Sigma}^{barion}$$

$$N_{\Sigma}^{barion} \approx 10^{57}$$

$$\Delta Q \sim \alpha \cdot 10^{57} \approx (10^{21} - 10^{22}) e \approx 100 Q$$

White Dwarfs:  $q_{ex} \approx 10 \text{ e/m}^3$

Neutron Stars:  $q_{ex} \approx 10 \text{ e/mm}^3$

CONTRIBUTIONS

Gravi-term  $\Leftrightarrow$  Coulomb term

FREE ENERGY: 1  $\Leftrightarrow$   $10^{-36}$

1<sup>st</sup> derivatives: 1  $\Leftrightarrow$  1

2<sup>nd</sup> derivatives: 1  $\Leftrightarrow$   $10^{+36}$

Thermodynamically equilibrium star is electroneutral almost everywhere

**NB!** Deviation from electroneutrality must not be uniform everywhere

**Exception:** - it could be **concentrated** on **discontinuity surfaces**

(phase boundaries, jump-like change in ionic composition etc.)

# Macroscopic Screening in Non-Ideal Plasma

Finally: **In electroneutrality regions one obtains:**

$$e\nabla \varphi_E(\mathbf{r}) = -\nabla \varphi_G(\mathbf{r}) \frac{\langle \mathbf{Z} | \mathbf{D}_\mu^n | \mathbf{M} \rangle}{\langle \mathbf{Z} | \mathbf{D}_\mu^n | \mathbf{Z} \rangle}$$

Here:

$\mathbf{D}_\mu^n(\mathbf{r})$   
matrix

$$\{\delta \mathbf{n}(\mathbf{r}) / \delta \boldsymbol{\mu}(\mathbf{r}')\}_T \equiv \left[ \left[ \delta n_j(\mathbf{r}) / \delta \mu_k(\mathbf{r}') \right] \right]_{T, \mu_i (i \neq k)} \equiv \left[ \left[ \delta^2 F^* / \delta \mu_j(\mathbf{r}) \delta \mu_k(\mathbf{r}') \right] \right]_{T, \mu_i (i \neq k)}$$

$\langle \mathbf{Z} | \equiv \{Z_j\}$   
 $| \mathbf{M} \rangle \equiv \{M_j\}$

$\mathbf{D}_\mu^n$

is inverse matrix to:

$$\mathbf{D}_n^\mu \equiv \left[ \left[ \delta^2 F^* / \delta n_j(\mathbf{r}) \delta n_k(\mathbf{r}') \right] \right]_{T, n_i (i \neq k)}$$

$$\mathbf{D}_\mu^n * \mathbf{D}_n^\mu = \mathbf{E}$$

**NB!**

Non-ideality effects are included naturally  $\Leftrightarrow$

$$\mathbf{D}_\mu^n = \left( \mathbf{D}_\mu^n \right)^{id} + \Delta \mathbf{D}_\mu^n$$

$$e\nabla \varphi_E(\mathbf{r}) = -\nabla \varphi_G(\mathbf{r}) \frac{\langle Z | \mathbf{D}_\mu^n | M \rangle}{\langle Z | \mathbf{D}_\mu^n | Z \rangle}$$

### Does not restricted by:

- *Spherical symmetry condition*
- *Nomenclature of ions*
- *Degree of ionization*
- *Degree of Coulomb non-ideality*
- *Degree of electronic degeneracy*

.....

**NB!** Matrix  $\mathbf{D}_\mu^n$  is still non-local

$$F = \min F(T, V, \{N_k\} / \{n_i(\cdot)\} / \{n_{ij}(\cdot, \cdot)\}) \equiv$$

$$\equiv -\sum_{jk} \frac{Gm_j m_k}{2} \int \frac{n_j(\mathbf{r}) \cdot n_k(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} d\mathbf{r} d\mathbf{r}' + \sum_{jk} \frac{Z_j Z_k e^2}{2} \int \frac{n_j(\mathbf{r}) \cdot n_k(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} d\mathbf{r} d\mathbf{r}' + F^*[\{n_i(\cdot)\} / \{n_{ij}(\cdot, \cdot)\}]$$

## “Quasi-uniformity” approximation

$$F = \min F(T, V, \{N_k\} / \{n_i(\cdot)\}) \equiv$$

$$\equiv -\sum_{jk} \frac{Gm_j m_k}{2} \int \frac{n_j(\mathbf{r}) \cdot n_k(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} d\mathbf{r} d\mathbf{r}' + \sum_{jk} \frac{Z_j Z_k e^2}{2} \int \frac{n_j(\mathbf{r}) \cdot n_k(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} d\mathbf{r} d\mathbf{r}' - \int f^*(\{n_i(\mathbf{r}) \dots n_k(\mathbf{r})\}) d\mathbf{r}$$

$\mu$  is a function, not functional

$$\mu_j^{(chem)}(\mathbf{r}) \equiv \left( \partial f^*[T, \{n_k(\mathbf{r})\}] / \partial n_j \right)_{T, n_{k \neq j}}$$

$$f^*(\{n\}) \equiv \lim_{\{N_k\}, V \rightarrow \infty} \left\{ \frac{F(N_i \dots N_k, V, T)}{V} \right\}_{N_k/V \rightarrow n_k}$$

### In terms of potentials

$$m_j \phi_G(\mathbf{r}) + q_j \phi_E(\mathbf{r}) + \mu_j^{(chem)}[\{n_k(\mathbf{r})\}, T] = \text{const} \quad (j, k = \text{electrons, ions})$$

### In terms of forces

$$m_j \nabla \phi_G(\mathbf{r}) + q_j \nabla \phi_E(\mathbf{r}) + \nabla \mu_j^{(chem)}[\{n_k(\mathbf{r})\}, T] = 0 \quad (j, k = \text{electrons, ions})$$

**NB!** The *local* free energy density  $f^*(\{n\})$  must be defined for *non-electroneutral* densities  $\{n_k\}$

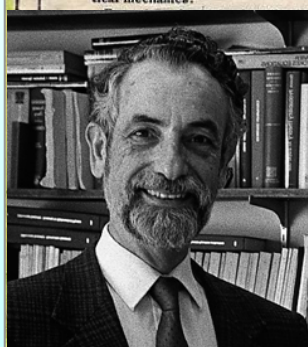
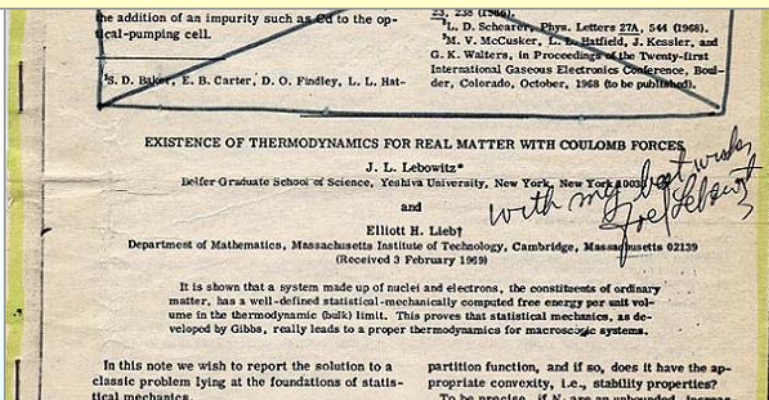
# The problem of thermodynamic limit in Coulomb systems

Lebowitz J.L. & Lieb E.H. *PRL*, **22** 631 (1969)

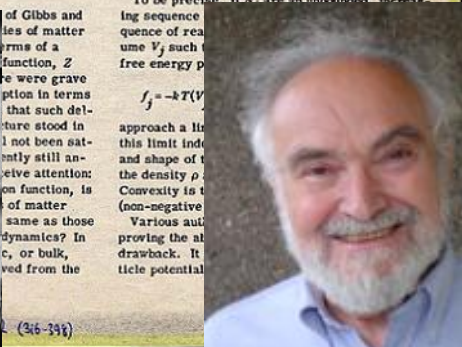
Lebowitz J.L. & Lieb E.H. *Adv. Math.*, **9** 317 (1972)



## Existence of Thermodynamics for Real Matter with Coulomb Forces



Elliot Lieb



Joel Lebowitz

$$f^* (\{n\}, T) \equiv \lim_{\{N_k\}, V \rightarrow \infty} \left\{ \frac{F(N_1 \dots N_k, V, T)}{V} \right\}_{N_k/V \rightarrow n_k}$$

Thermodynamic limit strongly depends on **disbalance** of net **electric charge**

$$Q \rightarrow 0$$

$$Q \sim N^\epsilon (< 2/3)$$

$$Q \sim N^\epsilon (> 2/3)$$

1

Could be avoided in Electroneutral Grand Canonical Ensemble

Thermodynamic limit still depends on **potential** of **surface dipole**

2

Duality:  
(local) **chemical**  $\Leftrightarrow$  (non-local) **electrochemical potentials**

Thermodynamic limit is still **non-local** !

?

# Macroscopic Screening in Non-Ideal Plasma

$$e\nabla \varphi_E(\mathbf{r}) = -\nabla \varphi_G(\mathbf{r}) \frac{\langle \mathbf{Z} | \mathbf{D}_\mu^n | \mathbf{M} \rangle}{\langle \mathbf{Z} | \mathbf{D}_\mu^n | \mathbf{Z} \rangle}$$

In “quasi-uniformity” approximation

Here:

$$\mathbf{D}_\mu^n(\mathbf{r}) \Leftrightarrow$$

matrix

$$\{\partial \mathbf{n}(\mathbf{r}) / \partial \boldsymbol{\mu}(\mathbf{r}')\}_T \equiv \left[ \left[ \partial n_j(\mathbf{r}) / \partial \mu_k(\mathbf{r}') \right] \right]_{T, \mu_i (i \neq k)} \equiv \left[ \left[ \partial^2 F^* / \partial \mu_j(\mathbf{r}) \partial \mu_k(\mathbf{r}') \right] \right]_{T, \mu_i (i \neq k)}$$

$$\langle \mathbf{Z} | \equiv \{Z_j\}$$

$$| \mathbf{M} \rangle \equiv \{M_j\}$$

$$\mathbf{D}_\mu^n$$

is inverse matrix to:

$$\mathbf{D}_n^\mu \equiv \left[ \left[ \partial^2 F^* / \partial n_j(\mathbf{r}) \partial n_k(\mathbf{r}') \right] \right]_{T, n_i (i \neq k)}$$

$$\mathbf{D}_\mu^n * \mathbf{D}_n^\mu = \mathbf{E}$$

Non-ideality effects are included naturally  $\Leftrightarrow$

$$\mathbf{D}_\mu^n = \left( \mathbf{D}_\mu^n \right)^{id} + \Delta \mathbf{D}_\mu^n$$

$$e\nabla \varphi_E(\mathbf{r}) = -\nabla \varphi_G(\mathbf{r}) \frac{\langle Z | \mathbf{D}_\mu^n | M \rangle}{\langle Z | \mathbf{D}_\mu^n | Z \rangle}$$

## Simplified cases:

### - **Ideal-mixture approximation**

*(multi-component "chemical picture")*

### - **Classical weakly non-ideal plasma**

*(Debye approximation in Grand Canonical Ensemble)*

### - **Strongly non-ideal classical ionic mixture on strongly degenerated ideal electrons**

*(switching-off the electron-ionic correlations)*

### - **Two-component electron-ionic system with arbitrary degree of degeneracy and non-ideality**

*(strongly correlated system)*



# Ideal-mixture approximation

$$\mathbf{D}_\mu^n = (\mathbf{D}_\mu^n)^{id}$$

(chemical picture: - a, b, ab, ab<sub>2</sub>, a<sub>2</sub>b, . . . a<sub>n</sub>b<sub>m</sub>)

$$e\nabla\varphi_E(\mathbf{r}) = -\nabla\varphi_G(\mathbf{r}) \frac{\langle \mathbf{Z} | \mathbf{D}_\mu^n | \mathbf{M} \rangle}{\langle \mathbf{Z} | \mathbf{D}_\mu^n | \mathbf{Z} \rangle}$$

↔

$$e\nabla\varphi_E(\mathbf{r}) = -\nabla\varphi_M(\mathbf{r}) \frac{\left( \sum_j \tilde{n}_j M_j Z_j \right)}{\left( \sum_j \tilde{n}_j Z_j^2 \right)}$$

$\langle \mathbf{Z} | \equiv \{Z_j\}$   
 $| \mathbf{M} \rangle \equiv \{M_j\}$

$$\tilde{n}_j \equiv kT \left( \partial n_j / \partial \mu_j \right)_{T, n_{k \neq j}}^{id. gas} \quad (j = 1, 2, 3, \dots)$$

$$\tilde{n}_e \rightarrow 0$$

$$(n_e \lambda_e^3 \gg 1)$$

**NB !** Electronic contribution falls out from  $\frac{\partial \langle \sum_j M_j Z_j \rangle}{\partial \mu_j}$  in the limit of strong electron degeneracy due to diminishing of ideal-gas electronic compressibility:

**Here:**

$$\mathbf{D}_\mu^n(\mathbf{r}) \Leftrightarrow \{ \delta \mathbf{n}(\mathbf{r}) / \delta \boldsymbol{\mu}(\mathbf{r}') \}_T \equiv \left[ \left[ \delta^2 F^* / \delta \mu_j(\mathbf{r}) \delta \mu_k(\mathbf{r}') \right] \right]_{T, \mu_i (i \neq k)}$$

$$\mathbf{D}_\mu^n * \mathbf{D}_n^\mu = \mathbf{E}$$

$\mathbf{D}_\mu^n$  is inverse to:  $\mathbf{D}_n^\mu \equiv \left[ \left[ \delta^2 F^* / \delta n_j(\mathbf{r}) \delta n_k(\mathbf{r}') \right] \right]_{T, n_i (i \neq k)}$

# Classical weakly non-ideal *i-e* plasma

(Debye approximation)

$$F_G^{(Z)} \approx -F_E^{(Z)} \left[ 1 + \frac{(1 - Z^2 \Gamma_D/4)}{Z(1 - \Gamma_D/4)} \right],$$

$$\Gamma_D \equiv (e^2/kTr_D) \ll 1, \quad \{r_D^{-2} \equiv (4\pi e^2(1 + Z^2)/kT)\}. \quad \zeta_e \equiv n_e \lambda_e^3 \ll 1$$

Coulomb “non-ideality force” moves positive ions *inside* the star in addition to gravitation  
“Non-ideality force” *increases* compensating electrostatic field  $\varphi_E(\mathbf{r})$  *in comparison with the ideal-gas approximation*

= = = = =  $\ll$  = = = = =

## Classical strongly non-ideal ionic plasma on strongly degenerated electrons

(no electron-ionic correlations)

(Quasi-crystal approximation)

$$F_G^{(Z)} \approx -F_E^{(Z)} \left[ 1 - \frac{a_M \Gamma_Z}{Z} x_c(\zeta_e) \right],$$

$$\{\Gamma_Z \equiv Z^2 e^2 (4\pi n_i/3)^{1/3} / kT \gg 1, \zeta_e \equiv n_e \lambda_e^3 \gg 1, a_M \approx 0.4\}.$$

# Non-ideality effects in two-component plasma

$\{+Z, e\}$

Equilibrium condition with “non-ideality force”

$$m_k \nabla \varphi_G(\mathbf{r}) + Z_k e \nabla \varphi_E(\mathbf{r}) + \nabla \mu_k^{(\text{chem})} \{n_i(\mathbf{r}), n_e(\mathbf{r}), T\} = 0 \quad (k = \text{electrons, ions})$$

Final equation for average electrostatic field

*(with taking into account non-ideality and degeneracy effects)*

$$m_i \nabla \varphi_G(\mathbf{r}) + Z_i e \nabla \varphi_E(\mathbf{r}) \left[ 1 + \frac{(\mu_{ii}^0 + \Delta_i^i + Z \Delta_e^i)}{Z(Z \mu_{ee}^0 + Z \Delta_e^e + \Delta_i^e)} \right] = 0$$

Here:

$\mu_j^0(n_j, T)$  – ideal-gas part of (*local*) chemical potential of specie  $j$

$\Delta \mu_j^{(\text{chem})}(n_j, n_i, \dots, n_k, T)$  – non-ideal-gas part of (*local*) chemical potential of specie  $j$

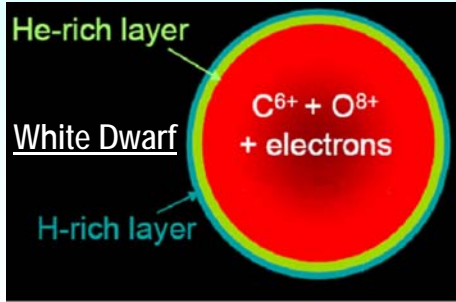
$$\mu_{jj}^0 \equiv \left( \frac{\partial \mu_j^0}{\partial n_j} \right) \quad \Delta_k^j \equiv \left( \frac{\partial \Delta \mu_j}{\partial n_k} \right)$$

“... As for **plasma polarization** in a star, it is **hardly possible to imagine any observable consequences** of this phenomenon ...”  
(polemics with NN)

**Observable consequences** *for* **plasma polarization**

# Well-known example - I

Accretion → diffusion → burning *of* hydrogen  
*in outer layer of compact stars*



Chang & Bildsten (2003) *Diffusive nuclear burning in neutron star envelopes*

Mixture  $_{12}C^{6+}$ ,  $_{16}O^{8+}$ ,  $_4He^{2+}$

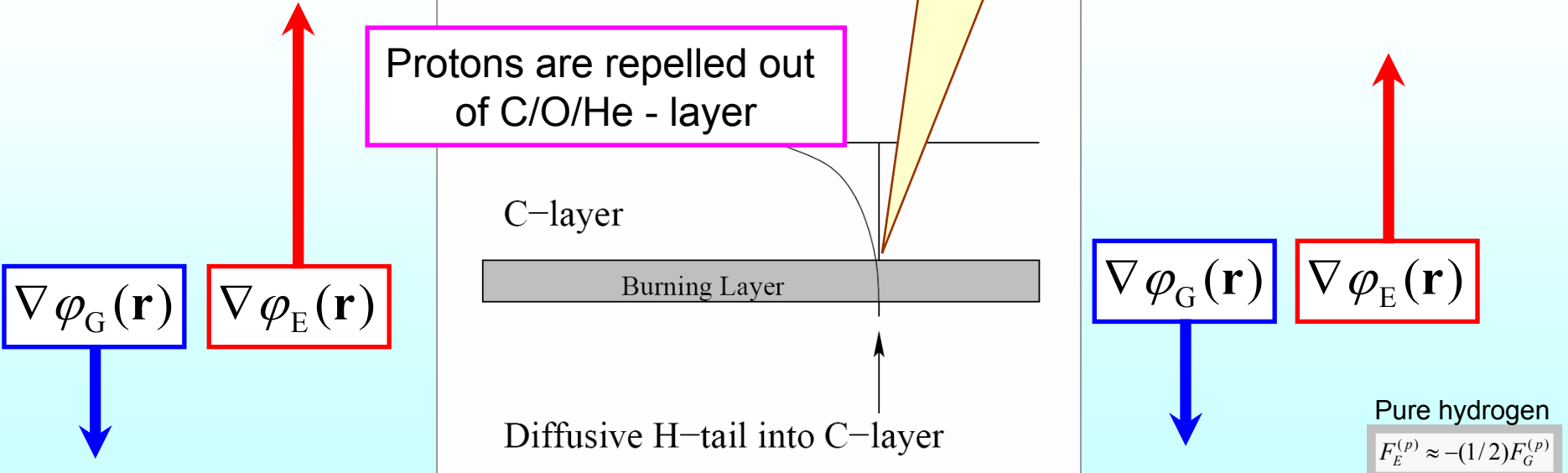
Mishaud & Fontain (1979)

$$F_E^{(p)} = -\frac{A}{(Z+1)} F_G^{(p)} \approx -(1.33 - 1.8) F_G^{(p)}$$

$$F_E^{(p)} = -\frac{A}{Z} F_G^{(p)} \approx -2 F_G^{(p)}$$

Ideal ions + degenerated electrons

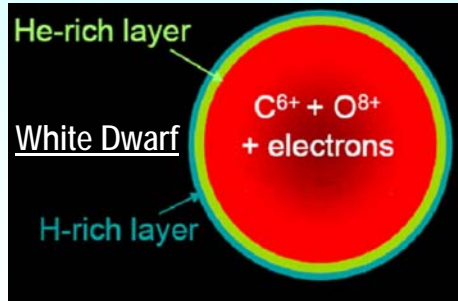
Ideal ions + non-degenerated electrons



# Well-known example - II

## Diffusion *and* sedimentation of $^{22}\text{Ne}$ in interior of WD

Bildsten & Hall (2001) *Gravitational settling of  $^{22}\text{Ne}$  in liquid white dwarf interior*



Mixture  $_{12}\text{C}^{6+}$ ,  $_{16}\text{O}^{8+}$ ,  $_{4}\text{He}^{2+}$

$$F_E^{(p)} = -\frac{A}{Z} F_G^{(p)} \approx -2F_G^{(p)}$$

The net force on  $^{22}\text{Ne}$

$$F = -22m_p g \hat{r} + 10eE \hat{r} = -2m_p g \hat{r}$$

.... The total increase in cooling age by the time the WD completely crystallizes ranges from 0.25-1.6 Gyr, depending on the value of  $D$  and the WD mass.

**NB !**

Coulomb non-ideality at *micro-level* discriminates  $_{16}\text{O}^{8+}$  in  $_{12}\text{C}^{6+}$ , and  $_{12}\text{C}^{6+}$  in  $_{4}\text{He}^{2+}$  ... and *accelerates* Rayleigh–Taylor hydrodynamic instability

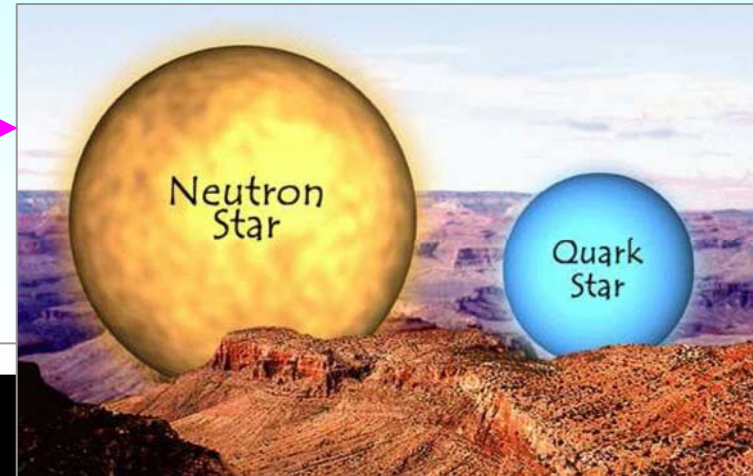
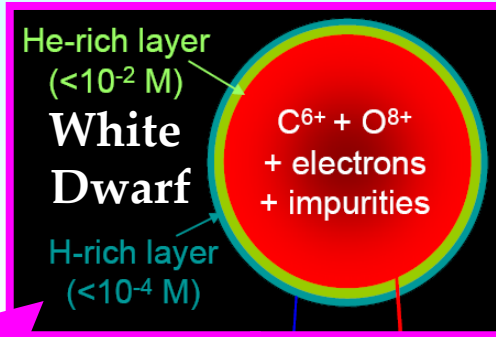
Coulomb non-ideality effect at *macro-level* (plasma polarization) *suppresses* Rayleigh–Taylor instability

**Plasma polarization *and* hydrodynamics**  
**in compact stars**

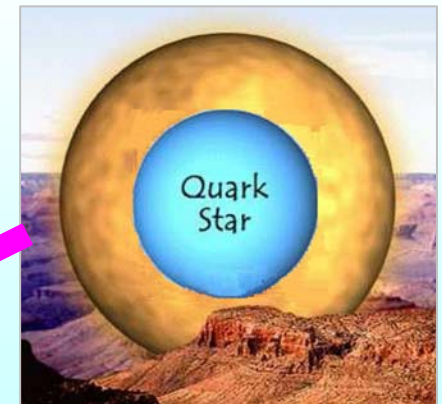
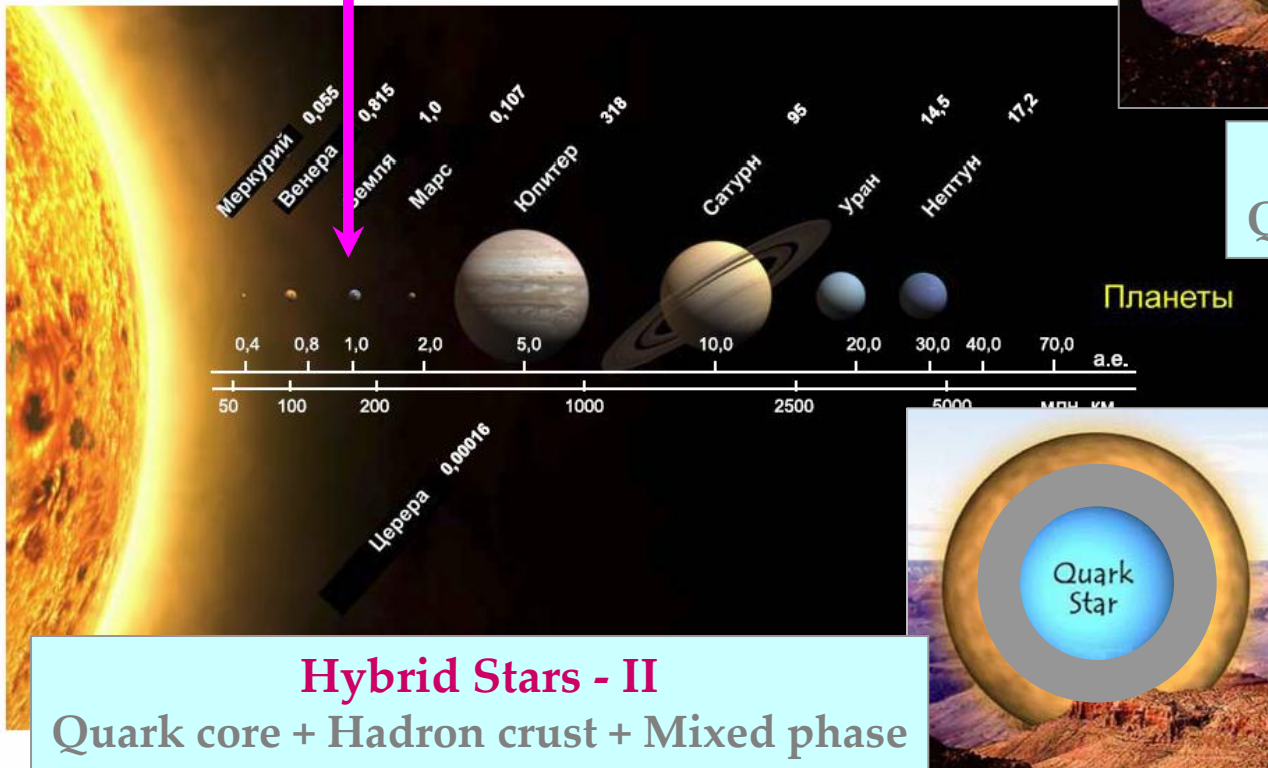
# Compact stars

White dwarfs, Neutron stars, "Strange" (quark) stars, Hybrid stars

Neutron and "Strange" Stars



Hybrid Stars  
 Quark core + Hadron crust



←  $R \sim 10 \text{ km}$  →

Hybrid Stars - II  
 Quark core + Hadron crust + Mixed phase

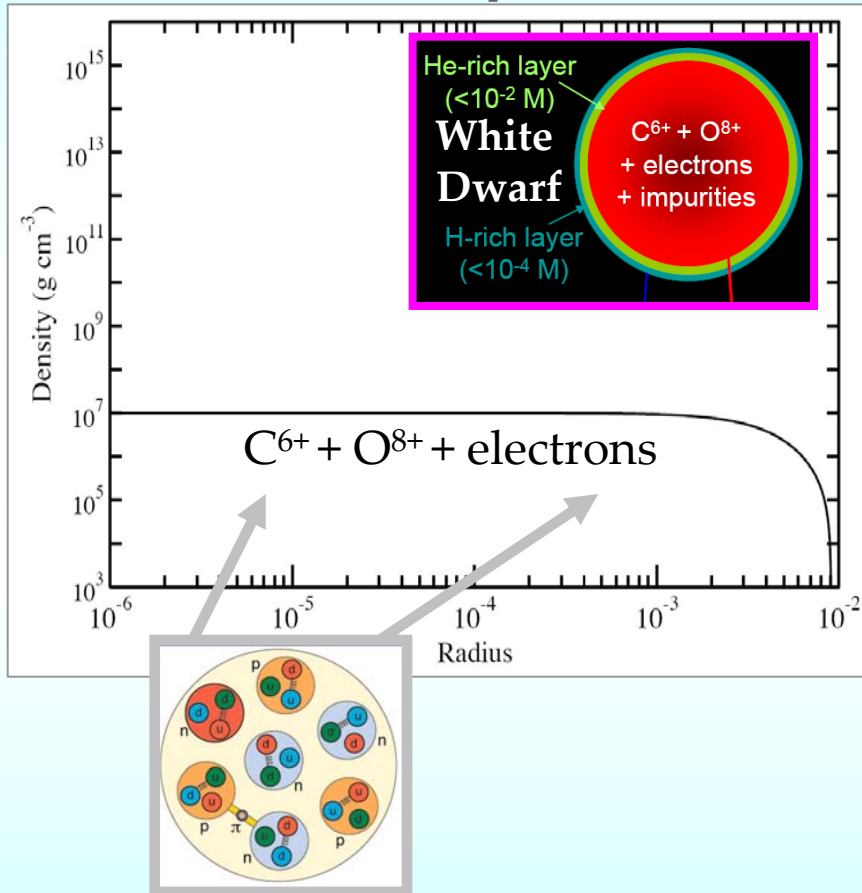
Рис. 66. Массы планет (в единицах массы Земли) и их среднее расстояние от Солнца [17].



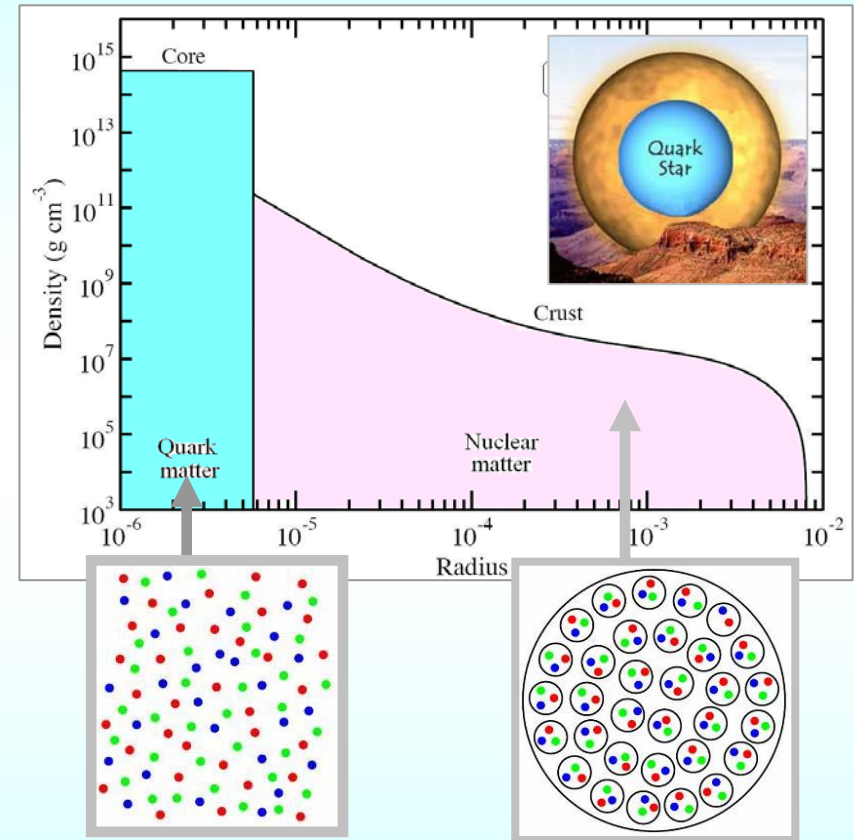
# Hybrid (*strange*) white dwarfs

G.Mathews., F.Weber *et al.* *J. Phys. G*, (2006) - *White dwarfs with strange-matter cores*

## Ordinary WD



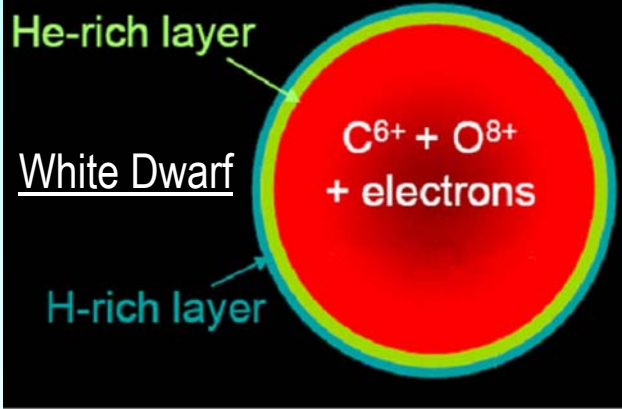
## Strange WD



**Exotics:** - Hybrid white dwarf with intermediate mixed phase  
*Quark core + Nuclear crust + Mixed phase (quark-nuclear emulsion)*

# White Dwarf

**Typical WD**  $\Leftrightarrow$  mixture  $_{12}\text{C}^{6+}$ ,  $_{16}\text{O}^{8+}$ ,  $_{4}\text{He}^{2+}$  +  
+ electronic background (*strongly degenerated*)



**WD – is strongly non-ideal** ( $\Gamma \sim 10^2 - 10^3 \gg 1$ )

$$e\nabla\varphi_E(\mathbf{r}) = -\nabla\varphi_G(\mathbf{r}) \frac{\langle Z|\mathbf{D}_\mu^n|M\rangle}{\langle Z|\mathbf{D}_\mu^n|Z\rangle} \rightarrow F_E^{(Z)} \approx -F_G^{(Z)} \left[ 1 - \frac{a_M \Gamma_Z}{Z} x_c(\zeta_e) \right]^{-1} \approx -F_G^{(Z)}$$

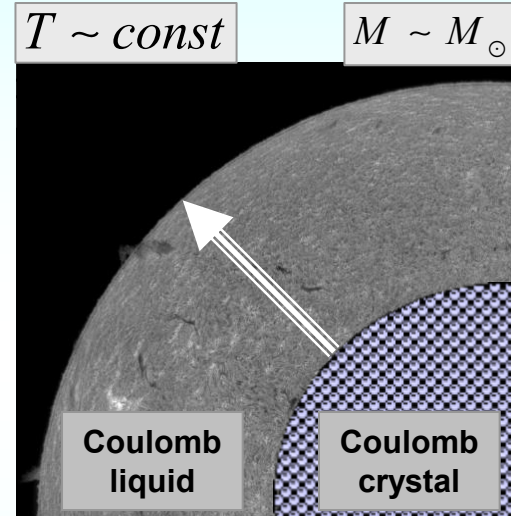
$$F_E^{(Z)} + F_G^{(Z)} \approx 0$$

**Total force** acting on every ion  
(nuclei:  $_{12}\text{C}^{6+}$ ,  $_{16}\text{O}^{8+}$ ,  $_{4}\text{He}^{2+}$ )  
is **equal to zero!**

**NB!**

**White Dwarf** is in **weightless state** in fact!

**What does it mean – hydrodynamics of a star in weightless state?**



$$T \sim 10^6 \div 10^7 \text{ K} \quad \rho \sim 10^6 \text{ g/cm}^3$$

$$n_c \sim 3 \cdot 10^{29} \div 3 \cdot 10^{32} \text{ cm}^{-3}$$

$$\zeta_e \equiv n_e \lambda_e^3 \sim 10^5$$

$$x_c(\zeta_e) \equiv \left( \frac{\mu_{ii}^0}{Z \mu_{ee}^0} \right) \sim 10^{-3} \div 3 \cdot 10^{-4}$$

# Hydrodynamics of WD in weightless state ?

$$F_E^{(Z)} + F_G^{(Z)} \approx 0$$

Carbon, oxygen and helium does **not sink** or **float** in each other!

Any hypothetical **layered structure** from  ${}_{12}\text{C}^{6+}$ ,  ${}_{16}\text{O}^{8+}$ ,  ${}_{4}\text{He}^{2+}$  is **hydrodynamically stable** as well as their homogeneous mixture

Rayleigh-Taylor **hydrodynamic instability** «**does not work**» in WD !

**R-T instability comes out of sources, which induce convection** in WD !

**NB !**

Plasma polarization due to gravitation and non-ideality can **suppress hydrodynamic instabilities** in interiors of compact stars !

**Plasma polarization *and* hydrodynamics**  
**in compact stars**

2

“... As far as plasma polarization in a star is concerned, it hardly possible to imagine any its observable consequences ...”  
(in polemics)

## Given:

**Total force** acting on every ion (nuclei:  ${}_{12}\text{C}^{6+}$ ,  ${}_{16}\text{O}^{8+}$ ,  ${}_{4}\text{He}^{2+}$ ) is **equal to zero !**

$$F_E^{(Z)} + F_G^{(Z)} \approx 0$$

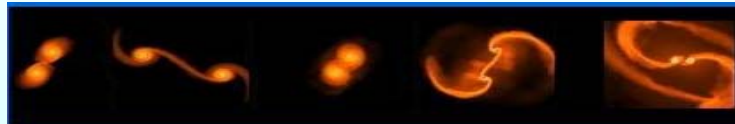
## Naive questions

**Why compact star is spherical ?**

**Why rotating star is spherical ?** (*pancake ? roll ? more complicated ?*)

**Why rotating binaries are spherical ?**

**What is the form of mergers** (*if polarization field is taken into account*) ?



**Are these questions meaningful or meaningless ?**

“... As far as plasma polarization in a star is concerned, it hardly possible to imagine any its observable consequences ...”  
(in polemics)

## Naive questions - II

**Structured Mixed Phase  $\Leftrightarrow$  “Pasta” plasma**

## Structured Mixed Phase $\Leftrightarrow$ "Pasta" plasma

'Pasta' plasma – quark-hadron phase transition in interior of neutron stars  
(‘Mixed phase’ of Glendenning *et al.*)

- Charged quark droplets (rods, slabs *etc*) in equilibrium hadron matter
- Charged hadron bubbles (tubes, slabs *etc*) in equilibrium quark matter

"Pasta" plasma

"Pasta" plasma

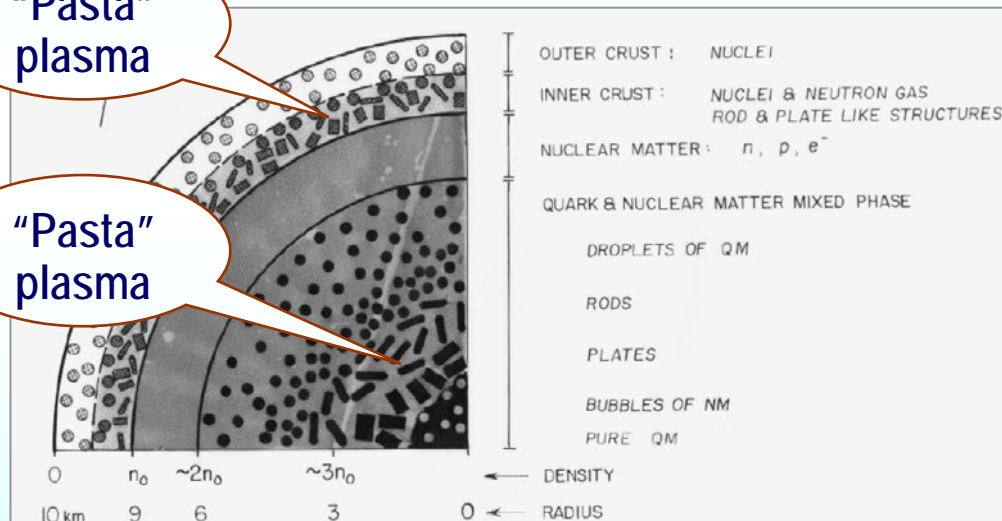
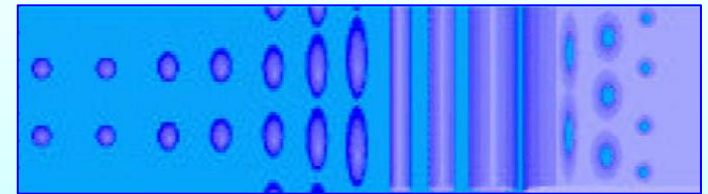


Fig. 1. Nuclear and quark matter structures in a  $\sim 1.4M_{\odot}$  neutron star. Typical sizes of structures are  $\sim 10^{-14}m$  but have been scaled up to be seen.

Ravenhall D., Pethick C. & Wilson J.  
*PRL* 50 (1983)

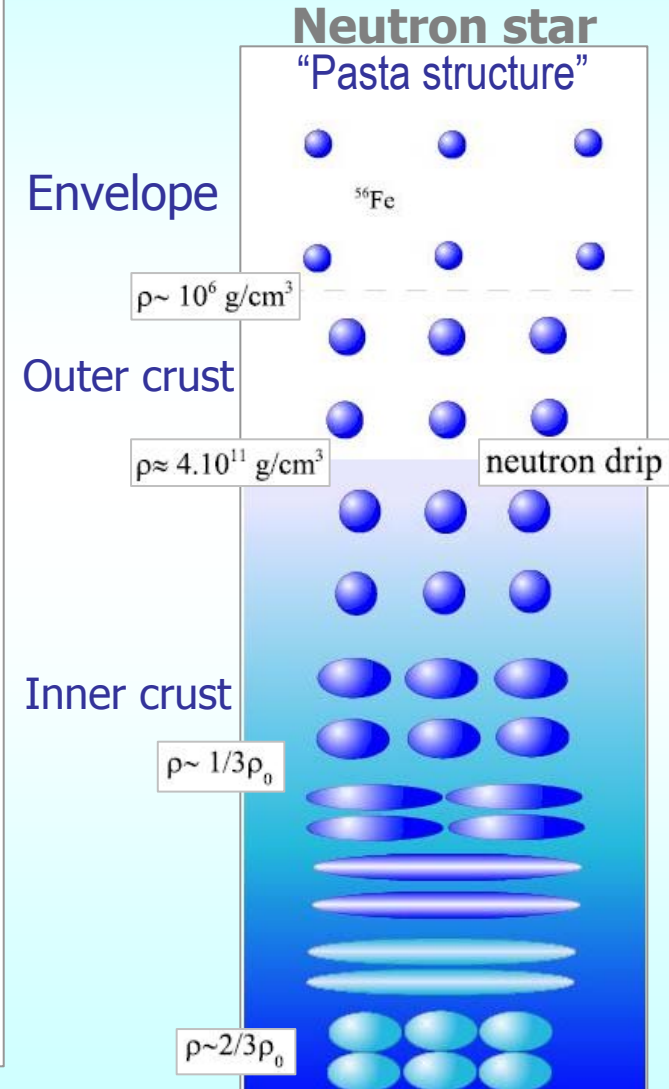
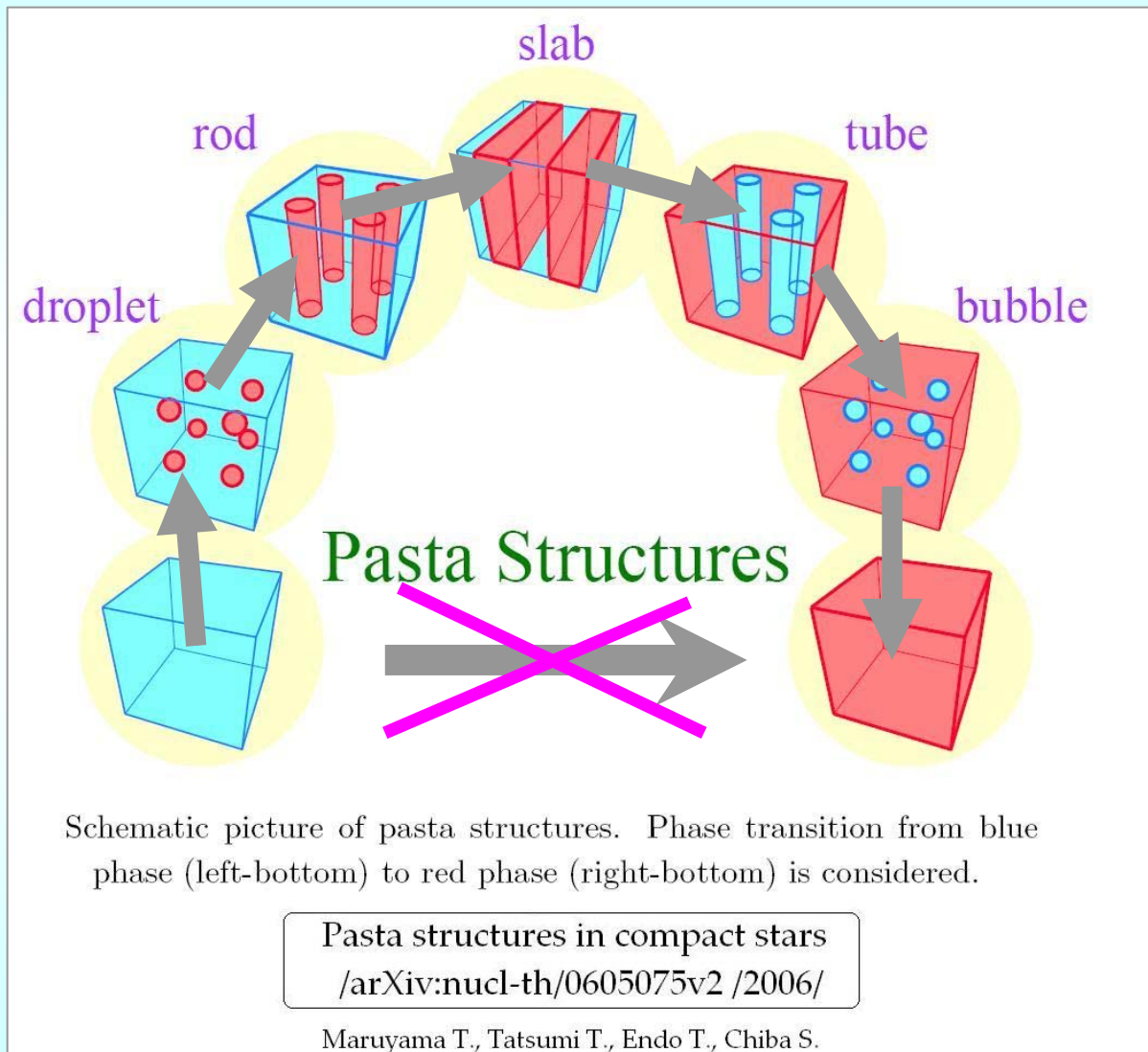
Heiselberg and Hjorth-Jensen  
*Phase Transitions in Neutron Stars*  
arXiv/9802028v1 (1998)

T.Maruyama, T.Tatsumi, T.Endo, S.Chiba  
*Pasta structures in compact stars*  
arXiv/0605075v2 31 (2006)



**"Pasta" plasma:- "Spaghetti" phase, "Lasagne" phase . . . . .**

# Structured Mixed Phase Concept $\Leftrightarrow$ "Pasta"



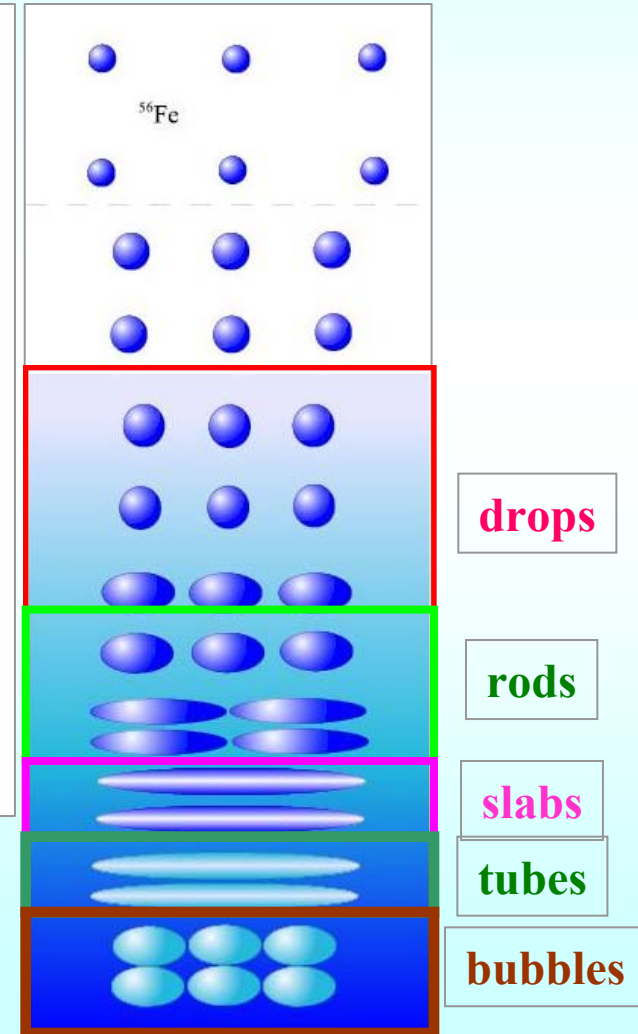
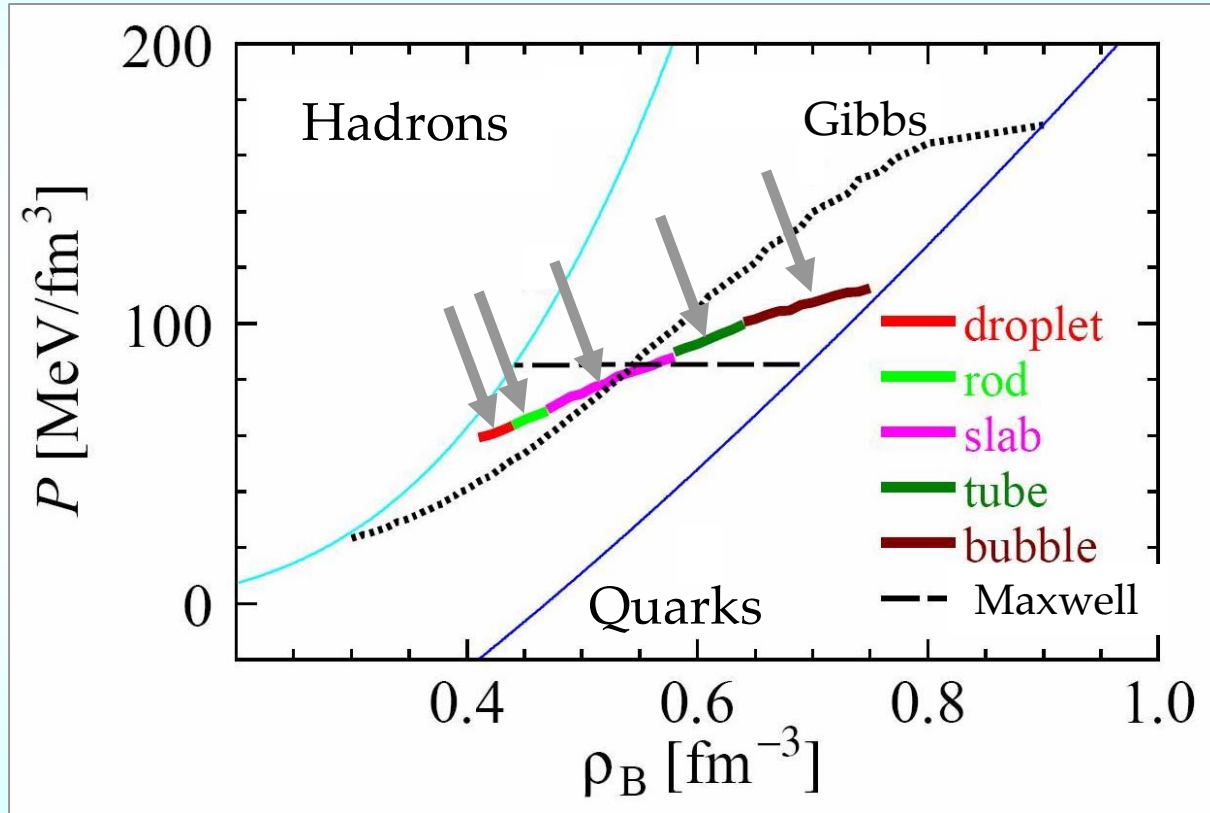
**"Pasta" plasma:- "Spaghetti" phase, "Lasagne" phase . . . . .**



# Structured Mixed Phase Concept $\Leftrightarrow$ "Pasta"

The sequence of five (or more ?) mini-phase transitions !

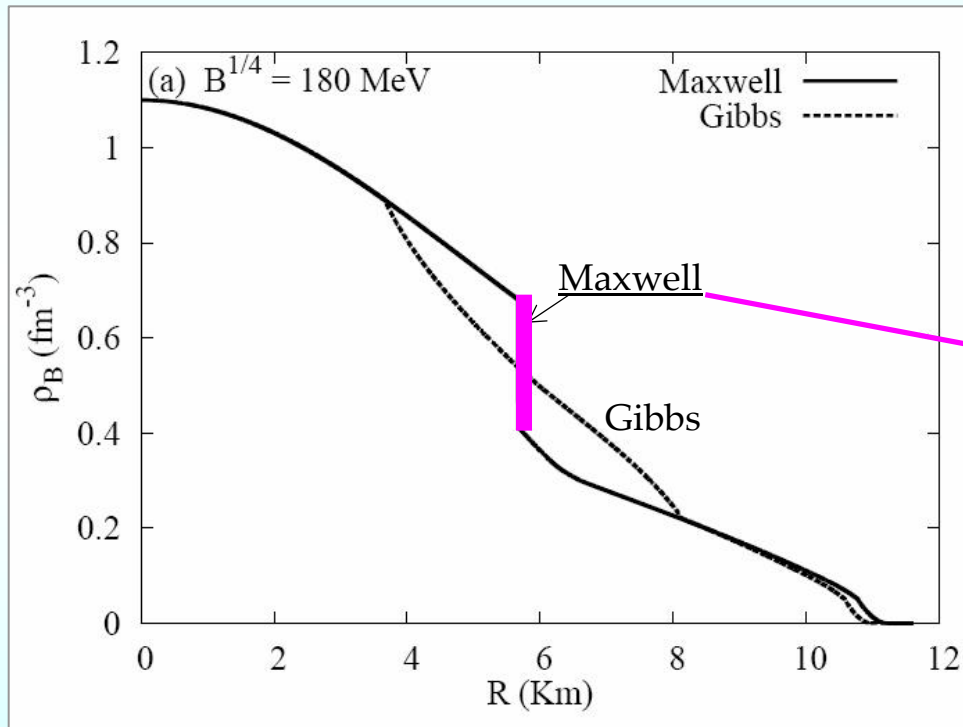
Uniform (nucleons)  $\rightarrow$  Drops  $\rightarrow$  Rods  $\rightarrow$  Slabs  $\rightarrow$  Bubbles  $\rightarrow$  Uniform (quarks)



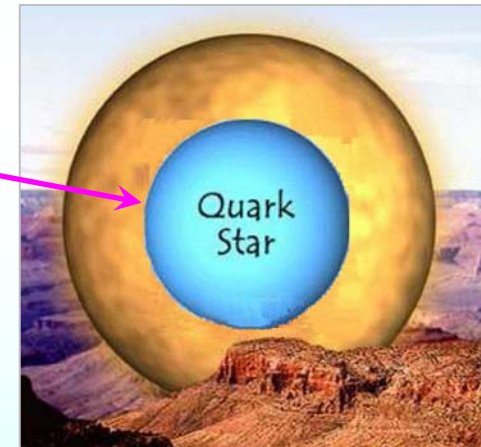
Maryuama T., Tatsumi T., Endo T., Chiba S.  
arXiv/0605075v2

# Hybrid star without mixed phase

*(Jump-like discontinuity in extensive parameters (density, entropy etc.))*



**Hybrid Star**  
Quark core + Hadron crust



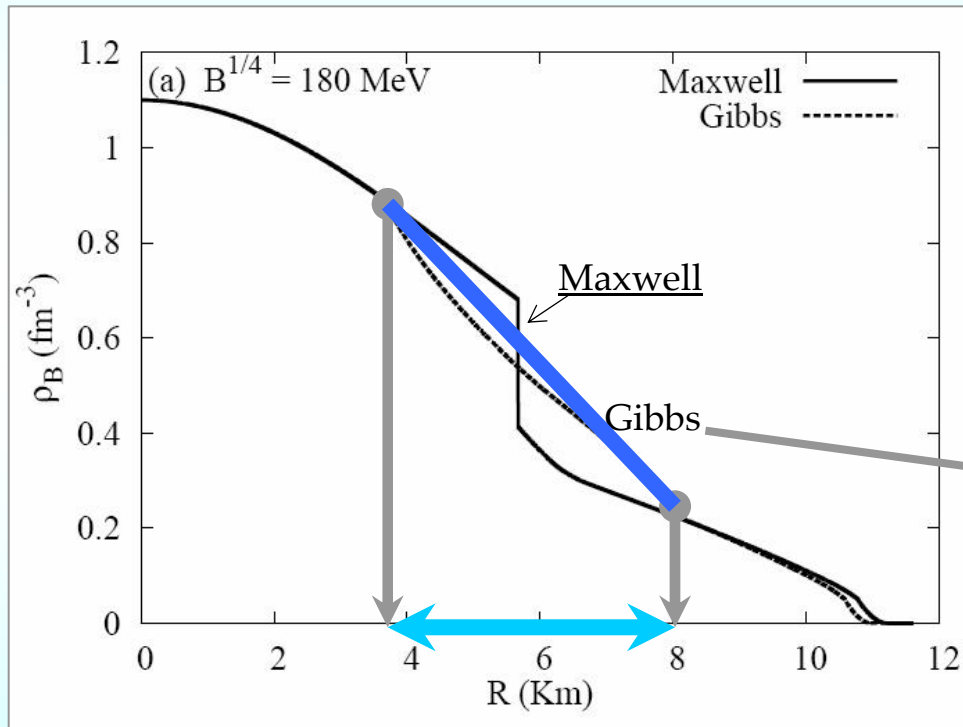
$|\leftarrow R \sim 10 \text{ km} \rightarrow|$

Bhattacharyya A., Mishustin I., Greiner W. <[arXiv0905.0352b](https://arxiv.org/abs/0905.0352b)> (2009)

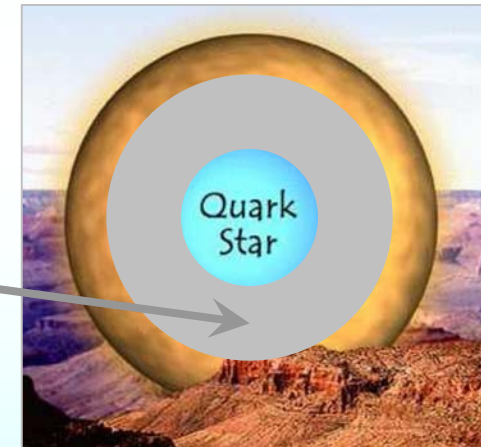
# Mixed phase layer in hybrid star

*(Highly dispersive mesoscopic phase – charged quark-hadron emulsion)*

Expected to be about 40% depth !



**Hybrid Stars**  
Quark core + Hadron crust



$|\leftarrow R \sim 10 \text{ km} \rightarrow|$

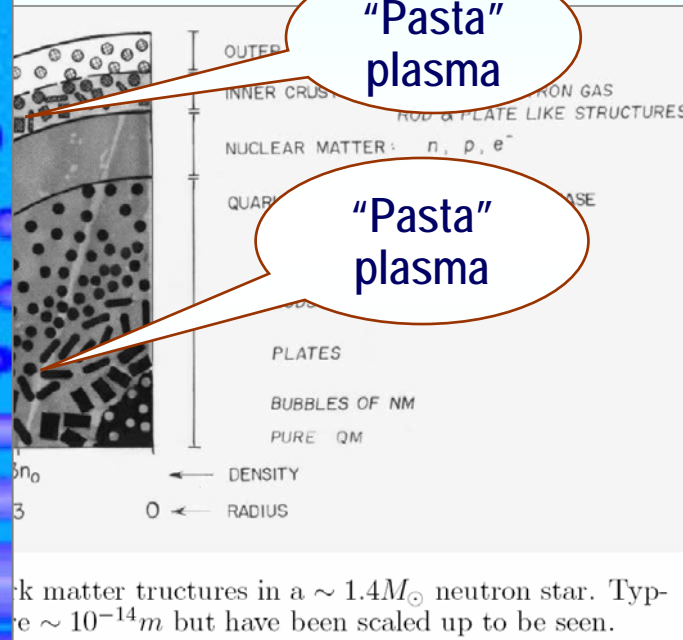
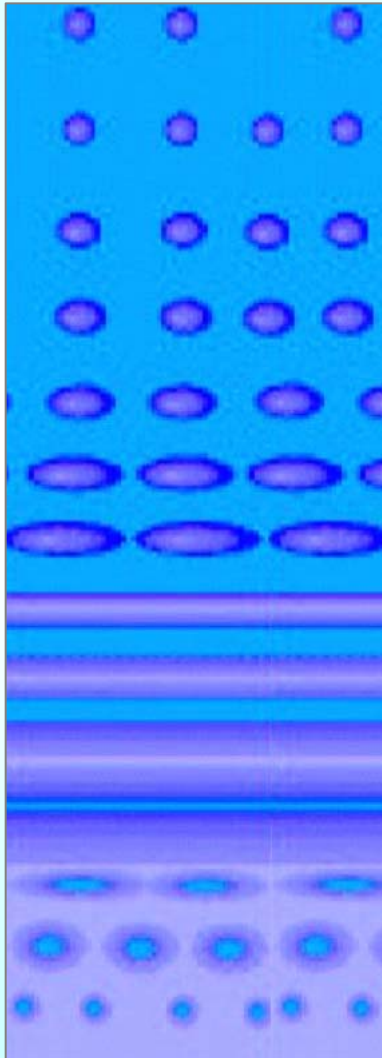
Bhattacharya A., Mishustin I., Greiner W. <[arXiv0905.0352b](https://arxiv.org/abs/0905.0352b)> (2009)

# Structured Mixed Phase $\Leftrightarrow$ "Pasta" plasma

Uniform-I  $\rightarrow$  Drops  $\rightarrow$  Rods  $\rightarrow$  Slabs  $\rightarrow$  Bubbles  $\rightarrow$  Uniform-II

'Pasta' plasma – hadron-quark phase transition in interior of neutron stars  
 ('Mixed phase' of Glendenning *et al.*)

quark droplets (rods, slabs) in equilibrium hadron matter  
 hadron bubbles (tubes, slabs) in equilibrium quark matter



quark matter structures in a  $\sim 1.4M_{\odot}$  neutron star. Typical size  $\sim 10^{-14}m$  but have been scaled up to be seen.

Heiselberg *and* Hjorth-Jensen  
*Phase Transitions in Neutron Stars*  
 arXiv/9802028v1 (1998)

T.Maruyama, T.Tatsumi, T.Endo, S.Chiba  
*Pasta structures in compact stars*  
 arXiv/0605075v2 31 (2006)

**"Pasta" plasma:-**

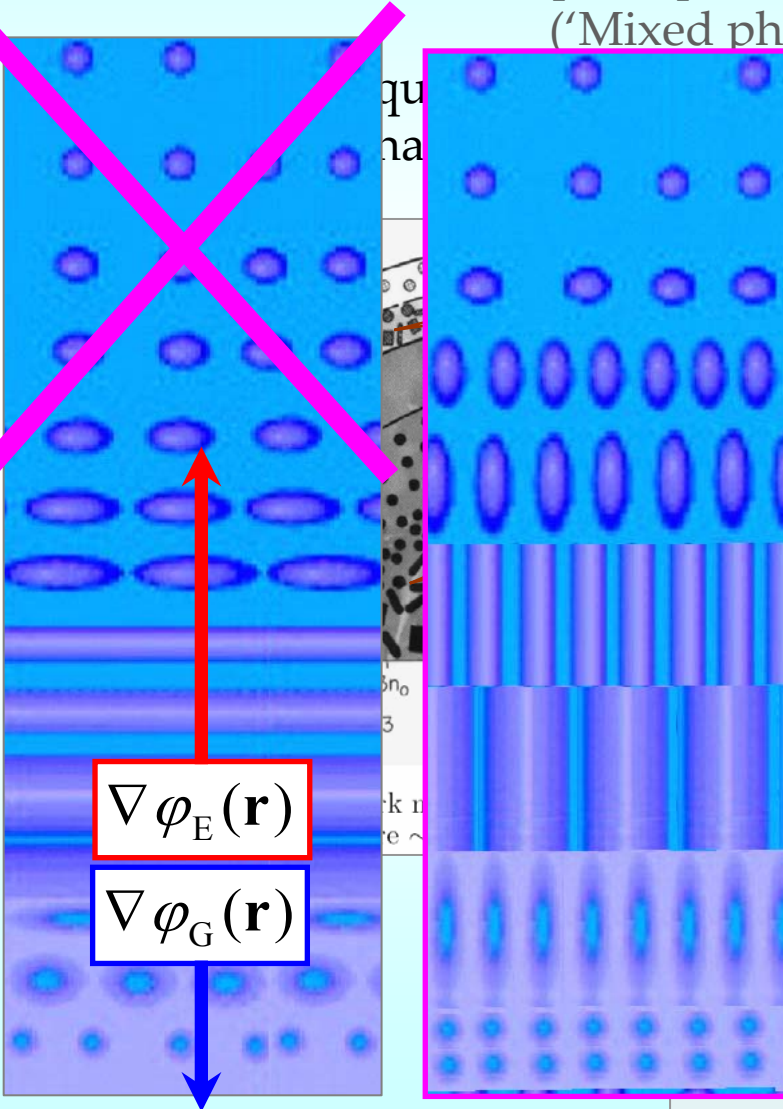
*"Spaghetti" phase, "Lasagne" phase ...  
 ... "Milk" phase, "Swiss cheese" phase...*

# Structured Mixed Phase $\Leftrightarrow$ "Pasta" plasma

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'Pasta' plasma – hadron-quark phase transition in interior of neutron stars  
 ('Mixed phase' of Glendenning *et al.*)

What is the **orientation**  
 of **spaghetti** and **lasagne** ?



$$\nabla \varphi_E(\mathbf{r})$$

$$\nabla \varphi_G(\mathbf{r})$$

Heis "North-Jensen  
 Phase" **?** Neutron Stars  
 71 (1998)

T.Maruyama, T. Endo, S. Chiba  
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"Pasta" plasma:-

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# Structured Mixed Phase $\Leftrightarrow$ "Pasta" plasma

Uniform-I  $\rightarrow$  Drops  $\rightarrow$  Rods  $\rightarrow$  Slabs  $\rightarrow$  Bubbles  $\rightarrow$  Uniform-II

'Pasta' plasma – hadron-quark phase transition in interior of neutron stars

(*'Mixed phase'* of Glendenning *et al.*)

(slabs) in equilibrium hadron matter

What is the **topology** (connectivity) of **spaghetti** and **lasagne** ?

Honeycomb



Hei... North-Jensen  
Phase **?** Neutron Stars  
1 (1998)

T.Maruyama, T.Tatsumi, T.Endo, S.Chiba  
Pasta structures in compact stars

Not spaghetti! Not lasagne! **Honeycomb?**

$$\nabla \varphi_E(\mathbf{r})$$

$$\nabla \varphi_G(\mathbf{r})$$

What are the **transport properties** of such **mist-net-foam-honeycomb** structure ?

# **Electrostatics *of* Phase Boundaries *in* Coulomb Systems**

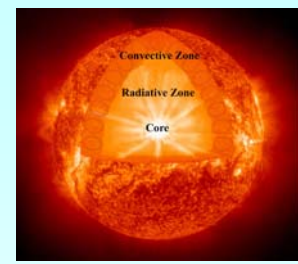
Iosilevskiy I. / Int. Conference "*Strongly Coupled Coulomb Systems*", St.-Malo, 1999

Iosilevskiy I. / Int. Conference "*Physics of Neutron Stars*", St.-Pb. Russia, 2008

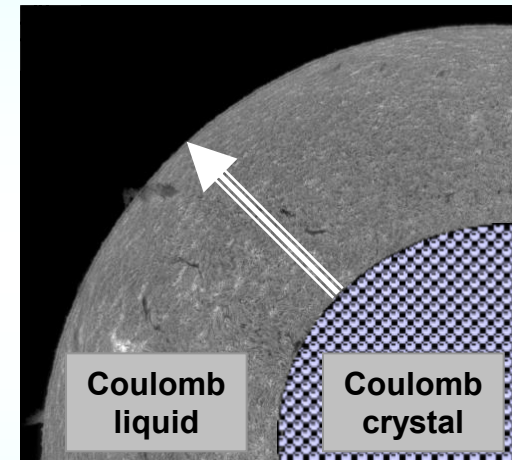
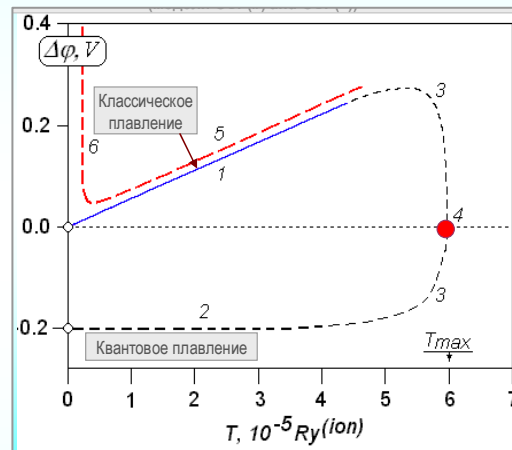
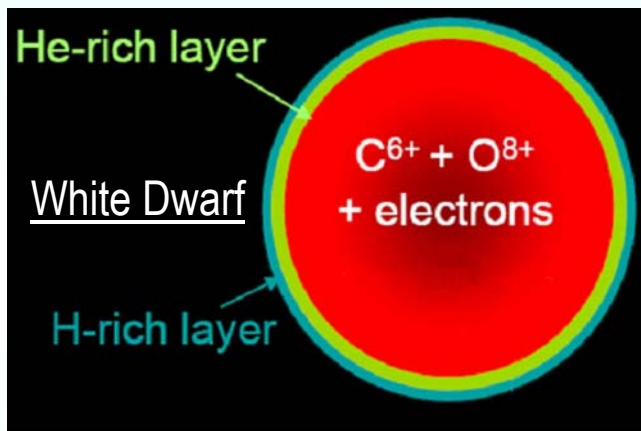
Iosilevskiy I. / Int. Conference "*Physics of Non-Ideal Plasmas*", Moscow, Russia, 2009



Strongly Coupled Coulomb Systems  
Sent-Malo, France, September, 1999



# Electrostatic Potential of Phase Boundaries in Coulomb Systems



**Igor Iosilevskiy** and **Alexander Chigvintsev**  
*Moscow Institute of Physics and Technology (State University)*

**Victor Gryaznov**  
*Institute of Problems of Chemical Physics RAS, Chernogolovka*



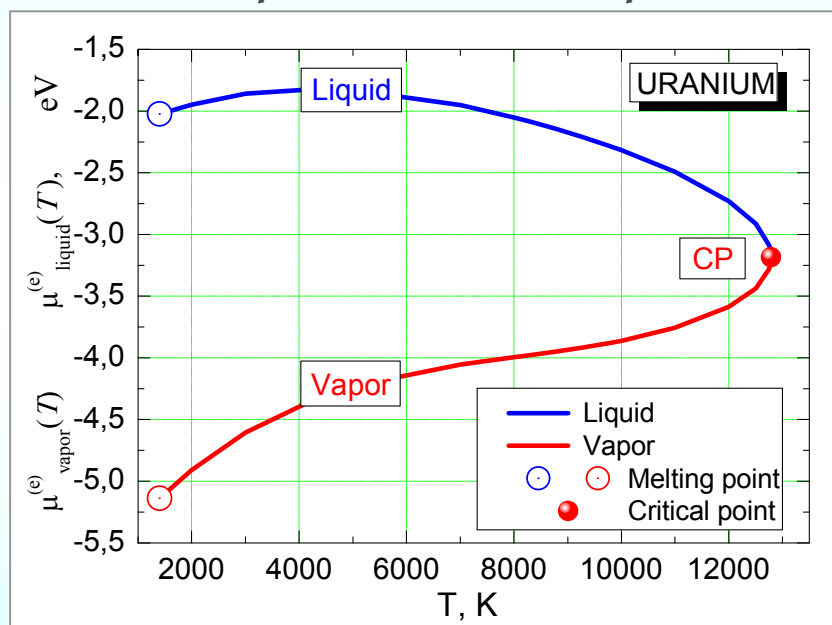


Any phase boundary in equilibrium Coulomb system is accompanied by existence of ***stationary electrostatic potential difference*** due to the **long-range nature of Coulomb** forces

(Iosilevskiy & Chigvintsev, *J. de Physique IV*, 2000)

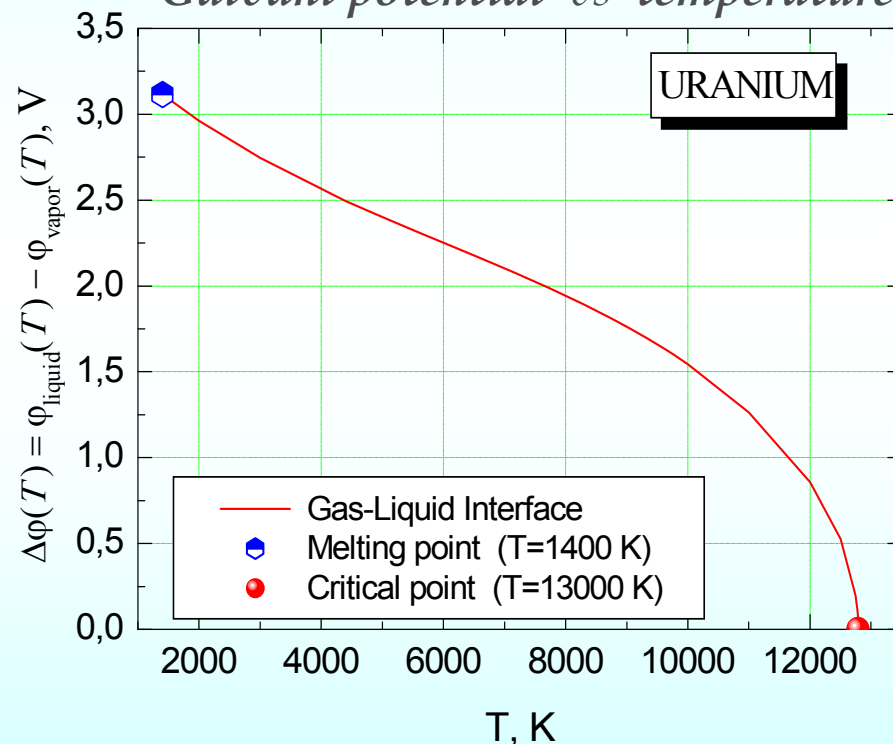
## Potential of gas-liquid interface in Uranium

*Chemical potential vs temperature*



**Electrochemical Phase Diagram**

*Galvani potential vs temperature*



Calculation of gas-liquid equilibrium via plasma model (code "SAHA-U")

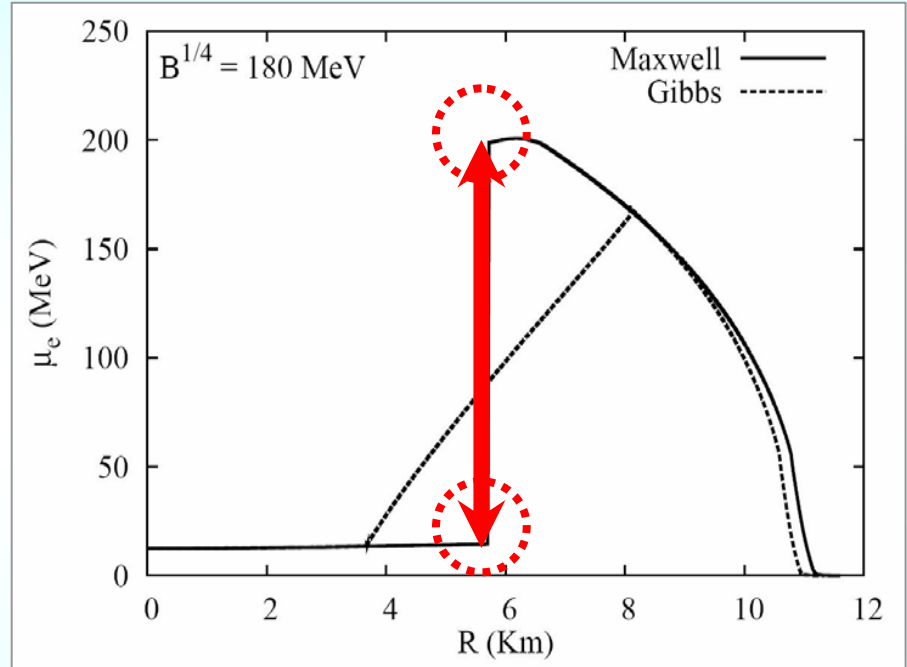
(Iosilevskiy & Gryaznov, *J. Nuclear Mater.* 2005)

$$e\Delta\phi = (\mu_e)_{\text{liquid}} - (\mu_e)_{\text{vapor}}$$

# Electrostatics of phase boundaries in Coulomb systems

## Quark-Hadron phase transition in Hybrid Star

Bhattacharyya A., Mishustin I., Greiner W.,  
arXiv:0905.0352v1 (2009)



$$e\Delta\phi_{HQ} = (\mu_e)_{\text{Hadron phase}} - (\mu_e)_{\text{Quark phase}}$$

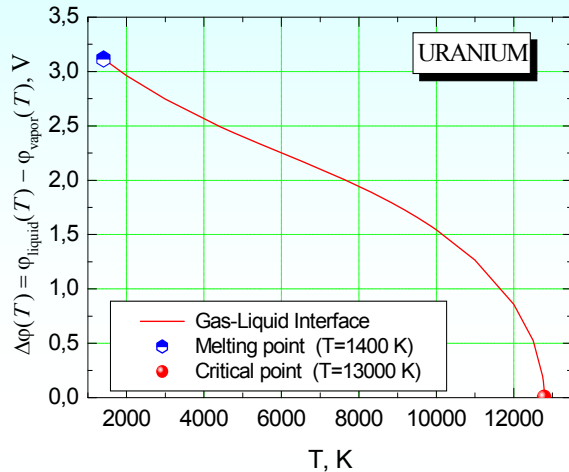
$$e\Delta\phi_{HQ} \approx 200 \text{ MeV} !$$

$$\delta_{HQ} \approx 10^3 \text{ fm} \rightarrow E \sim 10^{18} \text{ V/cm}$$

For comparison: Alcock et al., 1986:  $\rightarrow E \sim 10^{17} \text{ V/cm}$

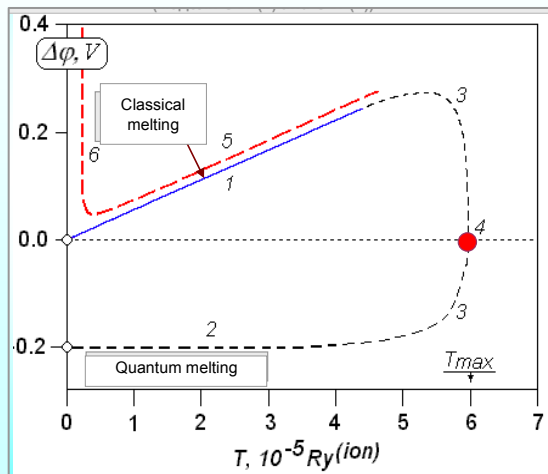
## Terrestrial applications

### Electrostatic (Galvani) potential



Iosilevskiy & Gryaznov, *J.Nucl.Mat.* 2005

### Electrostatic "portrait" of Wigner crystal in OCP

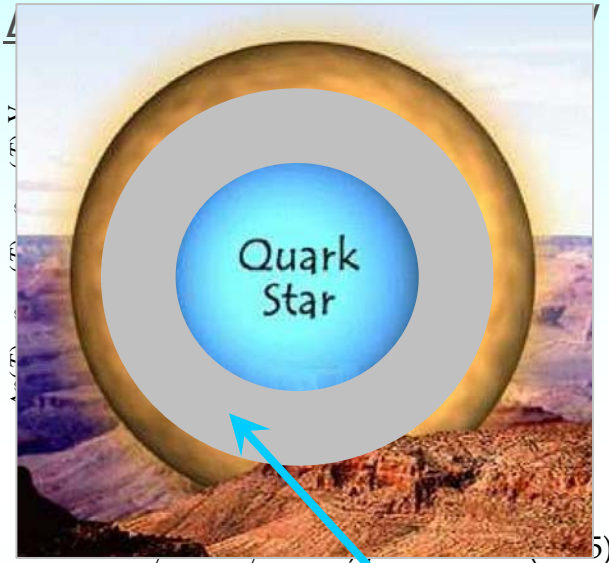


Iosilevskiy & Chigvintsev, *J. Physique*, 2000

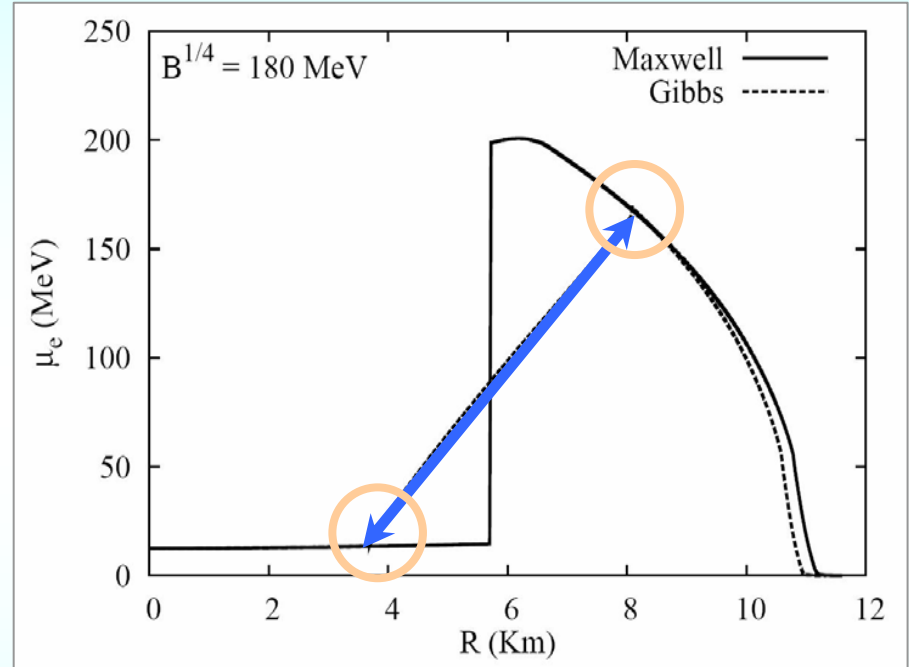
# Electrostatics of phase boundaries in Coulomb systems

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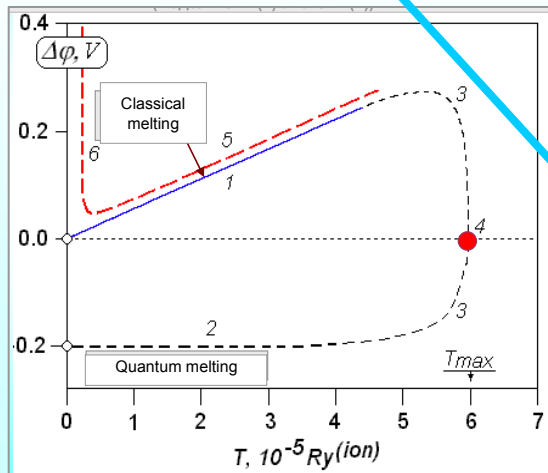
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Bhattacharyya A., Mishustin I., Greiner W.,  
arXiv:0905.0352v1 (2009)



Electrostatic "portrait" of Wigner crystal in OCP



Iosilevskiy & Chigvintsev, *J. de Physique* (2000)

$$e\Delta\phi_{HQ} = (\mu_e)_{\text{Hadron phase}} - (\mu_e)_{\text{Quark phase}}$$

$$e\Delta\phi_{HQ} \approx 200 \text{ MeV} !$$

$$\delta_{HQ} \approx 10^3 \text{ fm} \rightarrow E \sim 10^{18} \text{ V/cm}$$

$$\text{Mixed phase} - \delta_{HQ} \approx 10^4 \text{ m} \rightarrow E \sim 100 \text{ V/cm}$$

# Electrostatics of phase boundaries in Coulomb systems

## **Macroscopic charge *on* phase boundaries *in* Compact Stars**

# General Rule

$$e\nabla\varphi_E(\mathbf{r}) = -\nabla\varphi_G(\mathbf{r}) \frac{\langle Z | \mathbf{D}_\mu^n | M \rangle}{\langle Z | \mathbf{D}_\mu^n | Z \rangle}$$

Any **jump-like discontinuity** in thermodynamic parameters (**phase boundary**, **jump** in ionic **composition** *etc.*) must be accompanied with existing of **macroscopic charge** localized at this interface.

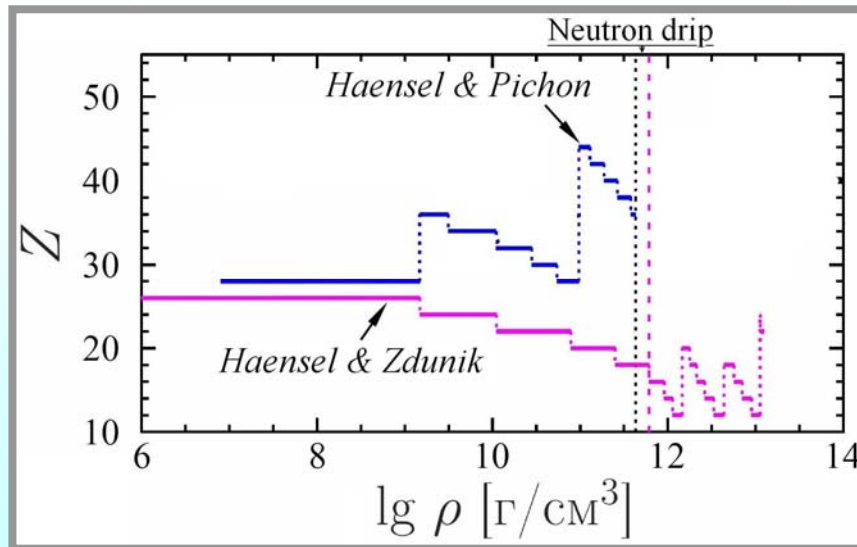
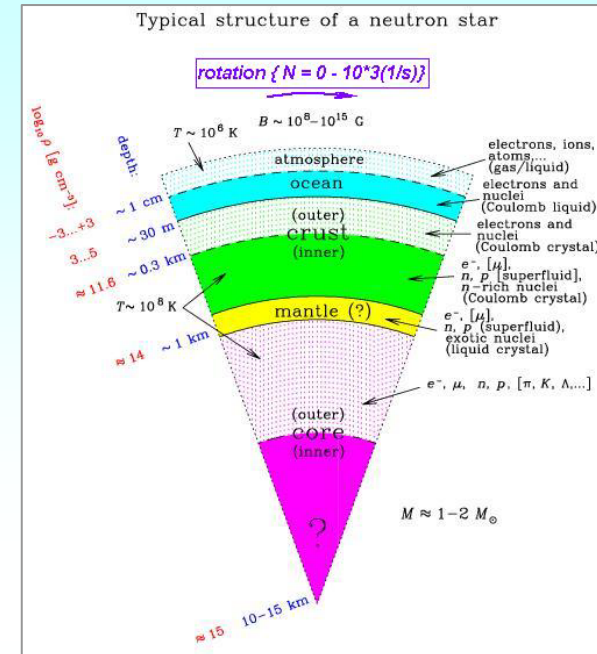
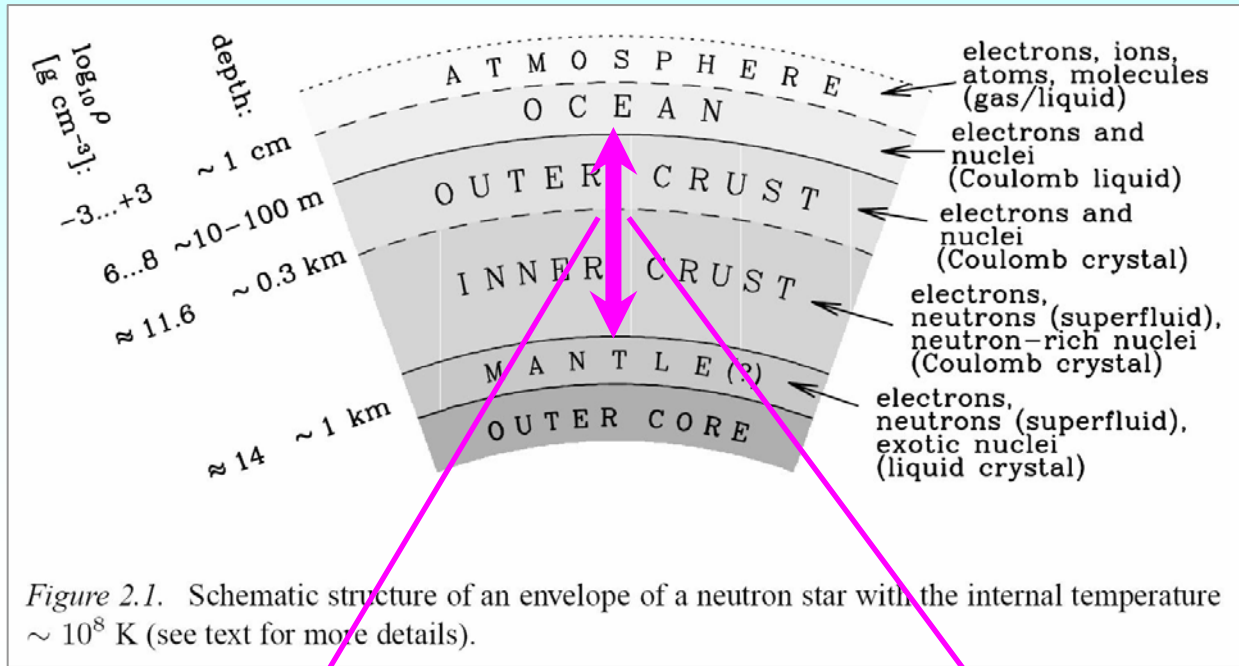
[astro-ph:0901.2547](#) / [astro-ph:0902.2386](#)

Iosilevskiy I. / Int. Conf. *"Physics of Neutron Stars"*, St.-Pb. Russia, 2008

Iosilevskiy I. / Int. Conf. *"Physics of Non-ideal Plasmas"*, Moscow, Russia, 2009

Iosilevskiy I. / Int. Conf. *"Plasma Physics"*, Zvenigorod, Russia, 2010

# Plasma polarization in thermodynamics of neutron stars

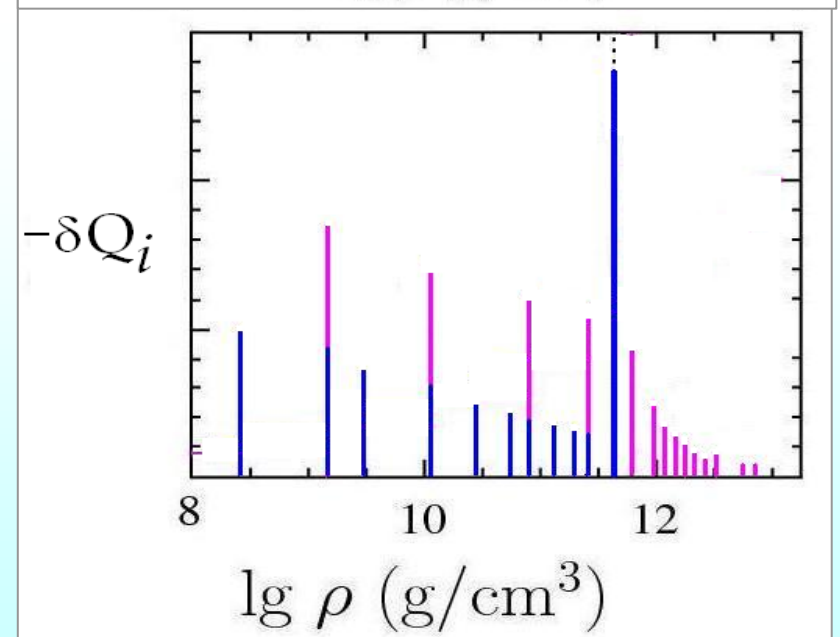
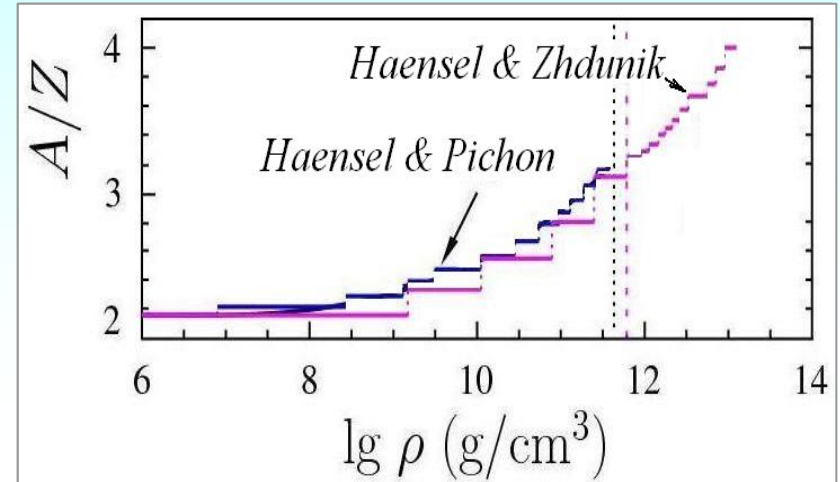
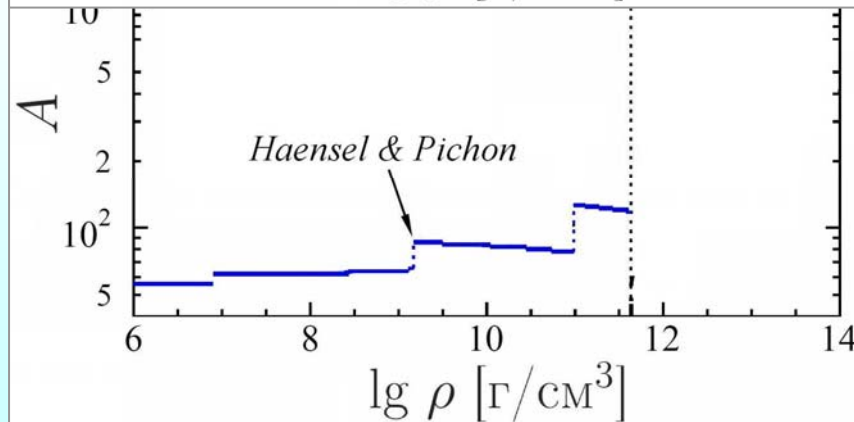
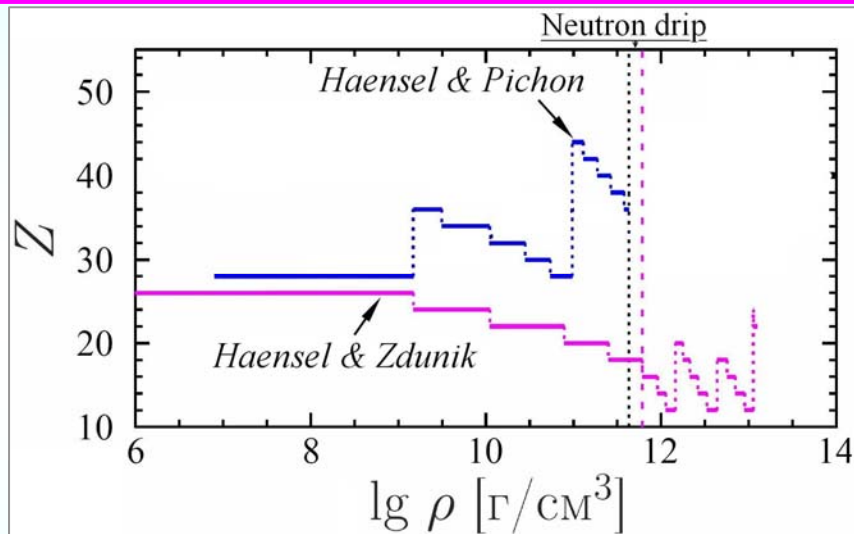


# Macroscopic charge on phase boundaries in MAO

Typically – ratio  $A/Z$  *increases* when we cross the interface toward the inner layer.

It means *decreasing* of electrostatic field, i.e. macroscopic *negative charge* localized on two-layer interface.

$$e\nabla\varphi_E(\mathbf{r}) = -\nabla\varphi_G(\mathbf{r}) \frac{\langle Z|\mathbf{D}_\mu^n|\mathbf{M}\rangle}{\langle Z|\mathbf{D}_\mu^n|\mathbf{Z}\rangle} \approx -m_p\nabla\varphi_G(\mathbf{r}) \frac{A}{Z}$$





# Cassini-Huygens

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## Conclusions and perspectives

- **Plasma polarization** in massive astrophysical bodies is **general** phenomenon
- **Plasma polarization** in massive astrophysical bodies is **universal** phenomenon
- **Plasma polarization** in massive astrophysical bodies is **interesting** phenomenon
- **Plasma polarization** in massive astrophysical bodies manifests itself in great number of **observable consequences** in **thermodynamics** of MAO
- **Plasma polarization** in massive astrophysical bodies manifests itself in great number of **observable consequences** in **hydrodynamics** of MAO
- **Coulomb non-ideality** effects at **micro**-level could **amplify hydrodynamic instability** in MAO, while **Coulomb non-ideality** at **macro**-level could **suppress hydrodynamic instability**





# Still unclear:

- Local and global thermodynamic stability of (*strongly non-ideal*) matter in MAO?

- Electrostatic potential (*micro and macro*) in “pasta plasma” inside compact star?

- Gravitational polarization inside QGP-plasma in Strange Stars?

- Inertial polarization in binaries and mergers ?

- Electrostatics of Supernova explosions ?

- Electrostatics of Black Holes ?

## Questions

- Gravitational polarization with relativistic effects ?

- What does it mean: gravitational polarization in media, where mass is not constant ?

- Polarization in compact star with strong magnetic field ?

# Cassini-Huygens

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There will be enough challenges to keep us all happily occupied for years to come...

*Hugh Van Horn (1990)*

*( Phase Transitions in Dense Astrophysical Plasmas )*

# Thank you!



**Support:** ISTC 3755 // CRDF MO-011-0, and by **RAS Scientific Programs**

“Physics and Chemistry of Extreme States of Matter” *and* “Physics of Compressed Matter and Interiors of Planets”,  
MIPT Education Center “Physics of High Energy Density Matter” and by **Extreme Matter Institute (EMMI)**